Gram Schmidt Based Greedy Hybrid Precoding for Frequency Selective Millimeter Wave MIMO Systems

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Hybrid architecture for mmWave MIMO

Large antenna arrays at TX/RX for sufficient received power

Large available bandwidth for Gbps data rates

Hardware constraints limit number of RF chains and ADC resolution

Prior work on the hybrid architecture is extensive

- Hybrid precoding with fully-connected arrays [Zha'05][EIA'14][Yu'16][Soh'15]
- Hybrid precoding with sub-array structures [Hab'15]
- Hybrid precoding/combining with switches [Men'16],[Alk'16]


Most prior work considered narrow-band channels, but mmWave is wideband.
System model – OFDM based hybrid precoding

Received signal at subcarrier $k$  

$$y[k] = H[k]F_{RF}F[k]s[k] + n[k]$$

RF precoding in done in the time domain – **common** for all subcarriers

Baseband precoding can be designed **per** subcarrier
Frequency domain

\[ H[d] = \sqrt{\frac{N_{\text{BS}}N_{\text{MS}}}{\rho_{\text{PL}}}} \sum_{\ell=1}^{L} \sum_{r_{\ell}=1}^{R_{\ell}} \alpha_{r_{\ell}} p_{r_{\ell}} (dT_S - \tau_{\ell} - \tau_{r_{\ell}}) a_{\text{MS}}(\theta_{\ell} - \vartheta_{r_{\ell}}) a_{\text{BS}}^{*}(\phi_{\ell} - \varphi_{r_{\ell}}) \]

Time domain

\[ H[k] = \sum_{d=0}^{D-1} H[d] e^{-j\frac{2\pi k}{K} d}. \]

Sparse channel in the angular domain – only a few clusters exist

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Problem formulation

Objective is to design the hybrid precoders to maximize mutual information

\[
\left\{ \mathbf{F}_\text{RF}^*, \{\mathbf{F}^*[k]\}_{k=1}^K \right\} = \arg \max_{\mathbf{F}_\text{RF}, \{\mathbf{F}[k]\}_{k=1}^K} \frac{1}{K} \sum_{k=1}^K \log_2 \left| \mathbf{I}_{\text{NMS}} + \frac{\rho}{\text{NS}} \mathbf{H}[k] \mathbf{F}_\text{RF} \mathbf{F}[k] \mathbf{F}[k]^* \mathbf{F}_\text{RF}^* \mathbf{H}[k]^* \right|
\]

s.t.
\[
\mathbf{F}_\text{RF}[r] \in \mathcal{F}_\text{RF}, \quad r = 1, \ldots, \text{N}_{\text{RF}}
\]
\[
\mathbf{F}_\text{RF} \mathbf{F}[k] \in \mathcal{U}_{\text{NS} \times \text{N}_{\text{RF}}}, \quad k = 1, 2, \ldots, K.
\]

RF precoders are taken from quantized codebooks

RF codebook represents hardware constraints (or due to limited feedback)

Assumes optimal combining at RX

Unitary power constraint (more results with other power constraints in [Alk'15])

Optimal hybrid precoders with a given RF codebook

Result #1 For a given RF precoder $\mathbf{F}_{RF}$, the optimal baseband precoder is

$$\mathbf{F}[k]^* = (\mathbf{F}_{RF}^* \mathbf{F}_{RF})^{-\frac{1}{2}} [\mathbf{V}[k]]_{1:N_s}^:, \quad k = 1, 2, ..., K.$$  

with the SVD decomposition

$$\mathbf{H}[k] = \mathbf{U}[k] \mathbf{\Sigma}[k] \mathbf{V}^*[k] \quad \mathbf{\Sigma}[k] \mathbf{V}^*[k] \mathbf{F}_{RF} (\mathbf{F}_{RF}^* \mathbf{F}_{RF})^{-\frac{1}{2}} = \mathbf{U}[k] \mathbf{\Sigma}[k] \mathbf{V}^*[k]$$

Result #2 Optimal mutual information depends only on the RF precoders

$$\mathcal{I}^*_\text{HP} = \max_{\mathbf{F}_{RF} \in \mathcal{F}_{RF}} \frac{1}{K} \sum_{k=1}^{K} \log_2 \left( \mathbf{I}_{N_s} + \frac{\rho}{N_S} \left[ \mathbf{\Sigma}[k] \right]_{1:N_s,1:N_s}^2 \right)$$

* Gives a benchmark for any other heuristic hybrid precoding algorithm
* Requires an exhaustive search over the RF precoders codebook
Direct greedy hybrid precoding

Greedily select the RF beamforming vectors in $N_{RF}$ iterations

\[ I_{HP}^{(i)} = \max_{f_{RF}^{(i)} \in F_{RF}} \frac{1}{K} \sum_{k=1}^{K} \sum_{\ell=1}^{i} \log_2 \left( 1 + \frac{\rho}{N_{RF}} \lambda_{\ell} \left( H[k] F_{RF}^{(i,n)} \left( F_{RF}^{(i,n)} H[k] \right)^{-1} \right) \right) \]

\[ F_{RF}^{(i,n)} = \left[ F_{RF}^{(i-1)}, f_{n} \right] \]

Matrix inversion for each beamforming vector

Exhaustive search over the per-vector RF beamforming codebook

Selected RF vectors in previous i-1 iterations

Calculation of the eigenvalues

Can we do better (lower complexity) than direct greedy hybrid precoding?
**Gram Schmidt based greedy hybrid precoding (1/2)**

**Observation** \( F_{RF} (F_{RF}^* F_{RF})^{-\frac{1}{2}} \) has a semi-unitary structure

**Applying Gram-Schmidt orthogonalization in each iteration**

\[
I_{HP}^{(i)} = \max_{f_{RF}^* \in F_{RF}^*} \frac{1}{K} \sum_{k=1}^{K} \sum_{\ell=1}^{i} \log_2 \left( 1 + \frac{\rho}{N_{RF}} \lambda_\ell \left( H[k] F_{RF}^{(i,n)} \left( F_{RF}^{(i,n)*} F_{RF}^{(i,n)} \right)^{-1} F_{RF}^{(i,n)*} H[k]^* \right) \right)
\]

\[
F_{RF}^{(i,n)} = \left[ F_{RF}^{(i-1)}, P^{(i-1)} f_{RF} \right] \quad P^{(i-1)\perp} = \left( I_i - F_{RF}^{(i-1)} \left( F_{RF}^{(i-1)*} F_{RF}^{(i-1)} \right)^{-1} F_{RF}^{(i-1)*} \right).
\]

**What is the advantage of applying Gram-Schmidt?**
Gram Schmidt based greedy hybrid precoding (2/2)

Orthogonalization allows the simplification

\[
\mathcal{I}_{HP}^{(i)} = \max_{f_n \in \mathcal{F}_{RF}} \frac{1}{K} \sum_{k=1}^{K} \sum_{\ell=1}^{\ell} \log_2 \left( 1 + \frac{\rho}{N_{RF}} \lambda_\ell \left( T^{(i-1)} + H[k] P^{(i-1)\perp} f_n \ f_n^* P^{(i-1)\perp} H^*[k] \right) \right)
\]

Constant per iteration

rank-1 matrix

Only one matrix inverse and rank-one SVD update per iteration

Theorem
GS and direct hybrid precoding achieve exactly the same mutual information

Much lower complexity solution with no loss
Approximate GS-based greedy hybrid precoding

With some relaxations, can further simplify to remove the rank one update

\[
\tilde{f}_{n}^{\text{RF}} = \arg \max_{\tilde{f}_{n}^{\text{RF}} \in \mathcal{F}_{\text{RF}}} \left| \tilde{\Sigma}_H \tilde{V}_H^* P^{(i-1)} \tilde{f}_{n}^{\text{RF}} \right|^2
\]

\[
\tilde{\Sigma}_H = \left[ \tilde{\Sigma}_1, \ldots, \tilde{\Sigma}_K \right] \quad \text{Concatenation of singular values of the K subcarriers}
\]

\[
\tilde{V}_H = \left[ \tilde{V}_1, \ldots, \tilde{V}_K \right] \quad \text{Concatenation of right singular vectors of the K subcarriers}
\]

Even lower complexity solution with low loss
Approximate GS-based greedy hybrid precoding

**Initialization**

Construct $\Pi = \tilde{\Sigma}_H \tilde{V}_H$, with $\tilde{\Sigma}_H = [\tilde{\Sigma}_1, ..., \tilde{\Sigma}_K]$ and $\tilde{V}_H = [\tilde{V}_1, ..., \tilde{V}_K]$. Set $F_{RF} =$ Empty Matrix.

**RF precoder design**

For $i, i = 1, ..., N_{RF}$

a) $\Psi = \Pi^* A_{CB}$

b) $n^* = \text{arg max}_{n=1,2,...,N_{CB}} \| [\Psi]_{:,n} \|_2$.

c) $F_{RF}^{(i)} = \begin{bmatrix} F_{RF}^{(i-1)} f_{n^*}^{RF} \end{bmatrix}$

d) $\Pi = \Pi \left( I_i - F_{RF}^{(i)} \left( F_{RF}^{(i)*} F_{RF}^{(i)} \right)^{-1} F_{RF}^{(i)*} \right)$

**Baseband precoder design**

$F[k] = F_{RF}^{(N_{RF})} \left( F_{RF}^{(N_{RF})*} F_{RF}^{(N_{RF})} \right)^{-\frac{1}{2}} \tilde{V}[k]_{:,1:N_B}, k = 1, ..., K$

Optimal baseband precoder given an RF codeword

Separable design of the RF/ baseband precoders
Simulation results

Setup
- BS has 32 antennas, MS has 16 antennas
- BS has 3 RF chains, and 3 transmitted streams
- Channel has 6 clusters, 5 rays per cluster
- AoA’s/AoD’s have angle spread of 10°
- Path delays uniformly distributed in [0, 128]
- OFDM has 512 subcarriers, cyclic prefix length=128, raised cosine pulshape
- RF beamforming vectors are selected from a beamsteering codebook with 64 vectors [beamforming vectors are matched to array response vectors at quantized angles]

Achievable upper bound for any hybrid precoding with the same RF codebook

Greedy algorithms are very close to optimal-exhaustive search-hybrid precoding

Gram-Schmidt and direct greedy hybrid precoding has the same performance

Simulation results

Setup
- BS has 64 antennas, MS has 16 antennas
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- Channel has 6 clusters, 5 rays per cluster
- AoA’s/AoD’s have angle spread of 10°
- Path delays uniformly distributed in [0, 128]
- OFDM has 512 subcarriers, cyclic prefix length=128, raised cosine pulseshape
- RF beamforming vectors are selected from a beamsteering codebook with 128 or 32 vectors [beamforming vectors are matched to array response vectors at quantized angles]

Gram-Schmidt hybrid precoding has good gain for different antenna sizes

Larger codebooks are needed for large antenna sizes

Conclusions

- We designed a Gram-Schmidt based greedy hybrid precoding algorithm
  - Separable design of the RF and baseband precoders
  - Low complexity selection of the RF beamforming vectors
  - Obtains a near-optimal performance in frequency selective mmWave channels
  - Achieves the same performance of direct (more-complex) greedy hybrid precoding

- Future work
  - Comparing OFDM and SC-FDE in mmWave systems
  - Extension to multi-user systems

- Extensions in the journal version [Alk’15]
  - Codebook designs for the RF and baseband precoders
  - Provide insights into performance and overhead with limited feedback

Questions?

Professor Robert W. Heath Jr.

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