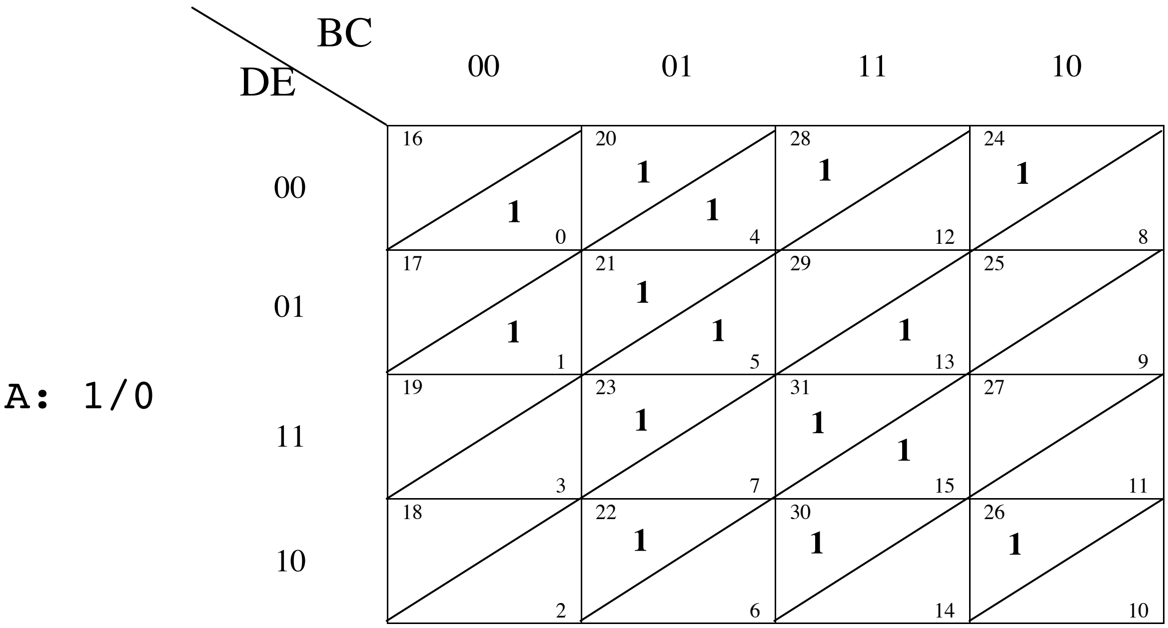
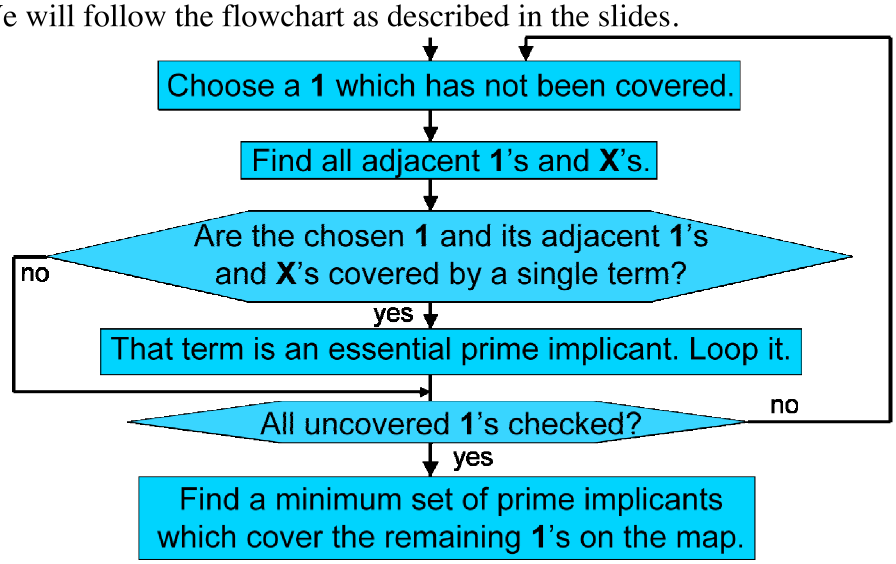
Karnaugh Maps: Detailed Workout

We will solve two different five variable problems using Karnaugh Maps by following the algorithm in its entirety as I described in class.

1. Given `F(ABCDE)` = Σm(0,1,4,5,13,15,20,21,22,23,24,26,28,30,31).
   First lets put this down in a Karnaugh Map



We will follow the flowchart as described in the slides.

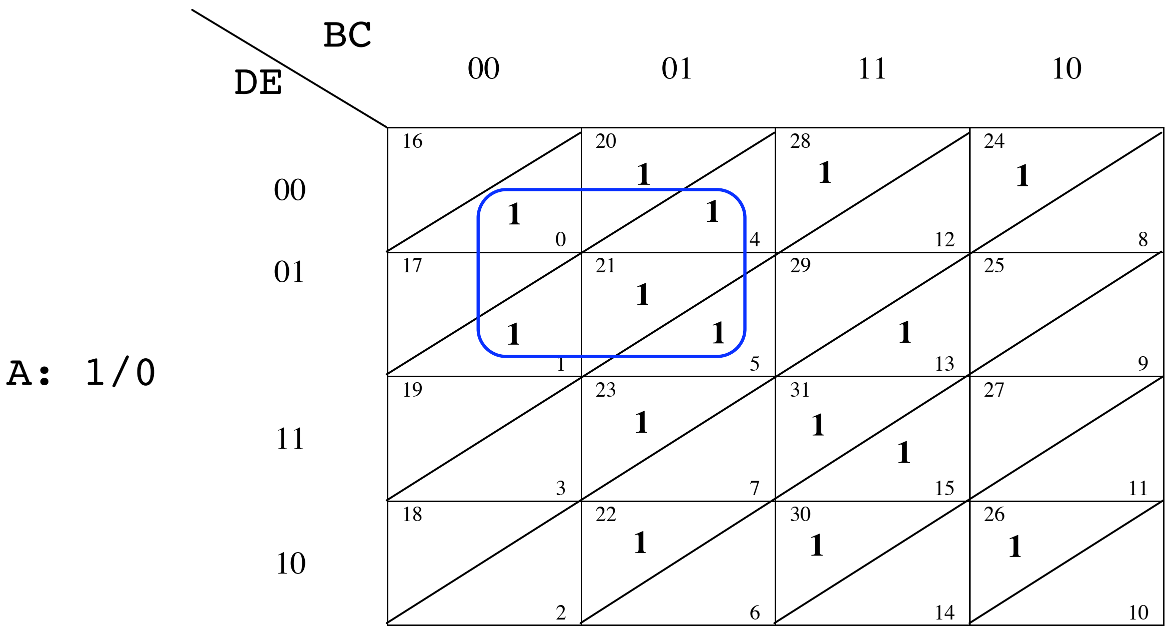


* Choose a 1 which has not been covered: Pick $\mathbf{1}_0$, This is the **1** in location 0.
* Find adjacent 1's and X's to $\mathbf{1}_0$: 1,4 (we will write the locations of these 1's and X's instead of using the subscript notation for convenience)

- Is there a single term that covers all these?
- Yes: Loop it; This corresponds to the Essential Prime Implicant term `A'B'D'`

Repeat this process. Lets do this in a tabular form as below

Chosen uncovered 1	Adjacent 1’s and X’s	Covered by a single term?	Essential Prime Implicant
1_0	1,4	YES	A’B’D’



Continue with the next uncovered 1:

Chosen uncovered 1	Adjacent 1’s and X’s	Covered by a single term?	Essential Prime Implicant
1_0	1,4	YES	A’B’D’
1_{20}	4,21,22,28	NO	--

Note1: Note that we are still considering 4 in the adjacent 1’s for 1_{20}. So, you must include ALL adjacent 1’s even those that have already been covered. This was a point I did not make clear in class.

Note2: Note that while 1_{20} and its neighbors 22, and 28 can combine (along with 30) this grouping does not include 4 and 21. Clearly there is no term that combines all of 20, 4, 21, 22 and 28 with possibly others to form a single term.

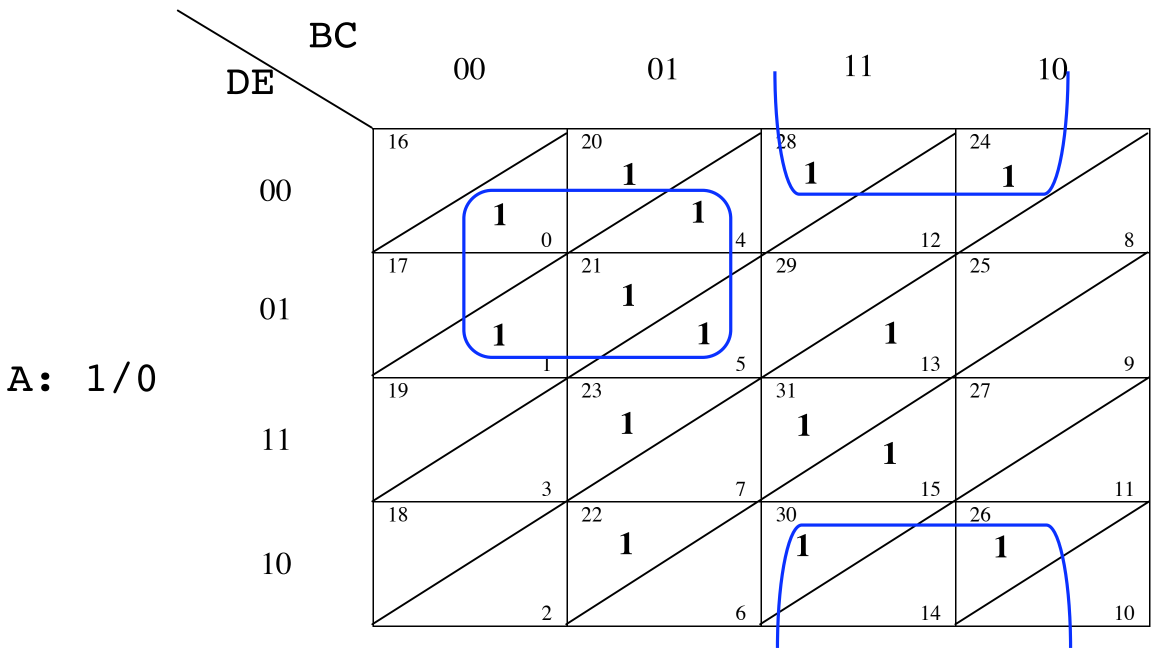
Lets continue the process as shown in the table below:

Chosen uncovered 1	Adjacent 1’s and X’s	Covered by a single term?	Essential Prime Implicant
1_0	1,4	YES	A’B’D’
1_{20}	4,21,22,28	NO	--
1_{21}	5,20,23	NO	--

Again the adjacent 1’s to 1_{21} include 5, which has already been covered. Also, note that there is a single term A’B’C that covers 1_{21},20,23 (along with 22) but we cannot use it because it does not cover 5. It does not matter that 5 has already been accounted for in an earlier Essential Prime Implicant.

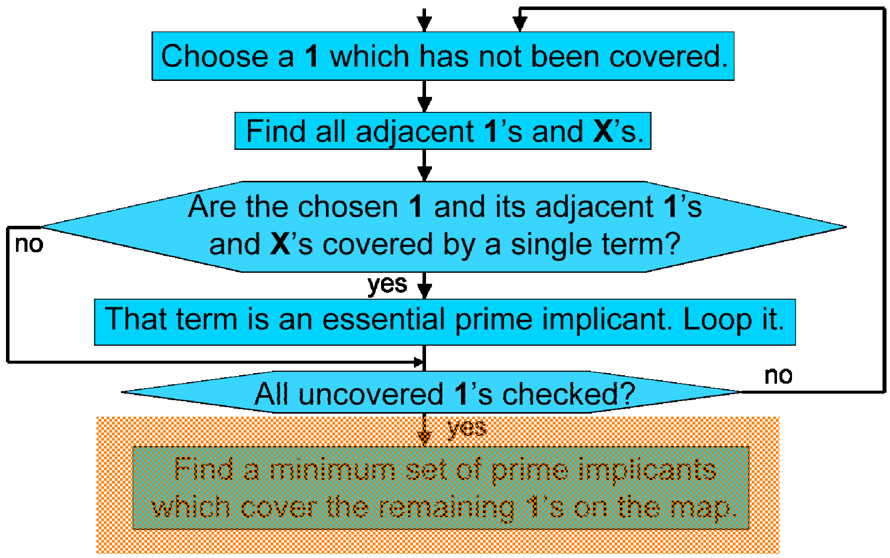
Continuing the process:

Chosen uncovered 1	Adjacent 1's and X's	Covered by a single term?	Essential Prime Implicant
1_0	1,4	YES	A'B'D'
1_{20}	4,21,22,28	NO	--
1_{21}	5,20,23	NO	--
1_{23}	21,22,31	NO	--
1_{28}	20,24,30	NO	--
1_{13}	5,15	NO	--
1_{15}	13,31	NO	--
1_{31}	15,23,30	NO	--
1_{30}	22,26,28,31	NO	--
1_{24}	26,28	YES	ABE'

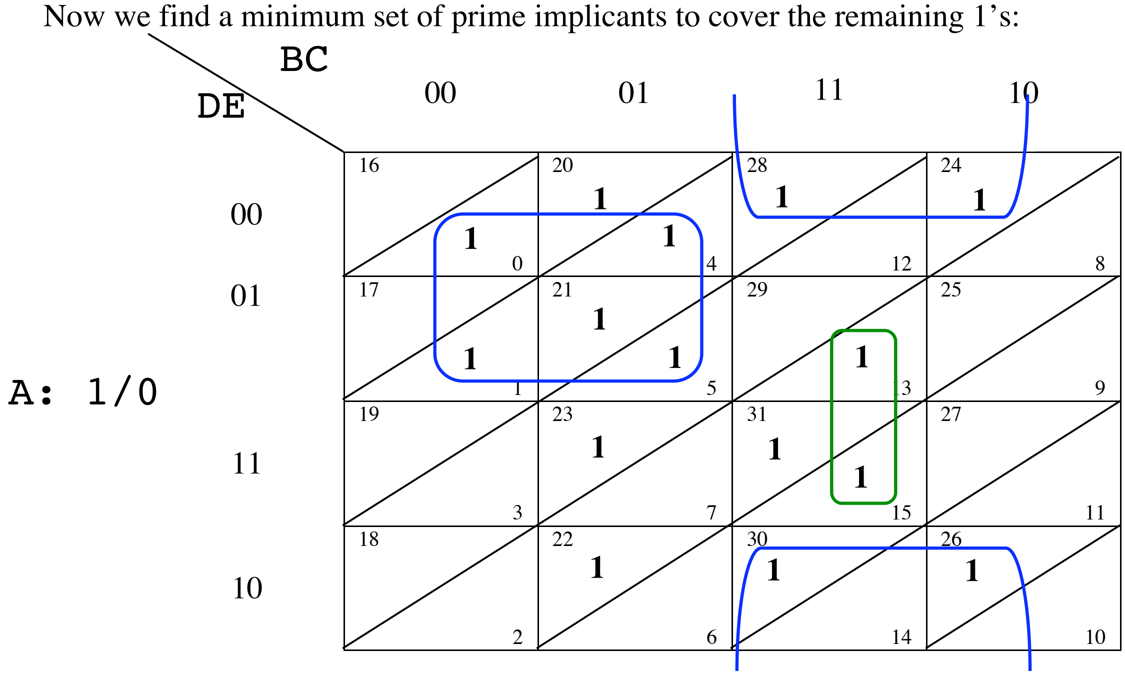


Note again that the Flowchart only asks if there is a single term that covers all adjacent 1's. This single term can include other 1's also (like 30 here in the second loop) as long as it covers all the 1's in the adjacency list.

At this point we have found all the Essential Prime Implicants. Also, we have checked all the uncovered 1's in the Map. You can verify this by looking at the table and finding that all unlooped 1's have a corresponding row indicating that there are no more uncovered 1's to check. This bring us to the bottom of our flowchart:

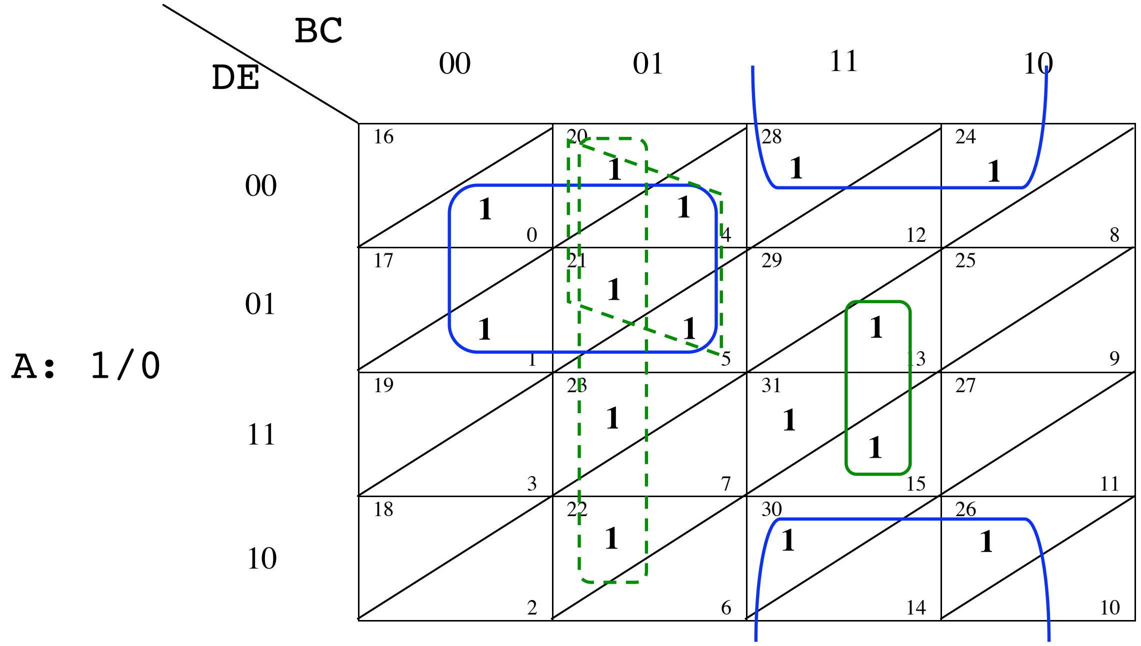


Now we find a minimum set of prime implicants to cover the remaining 1's:

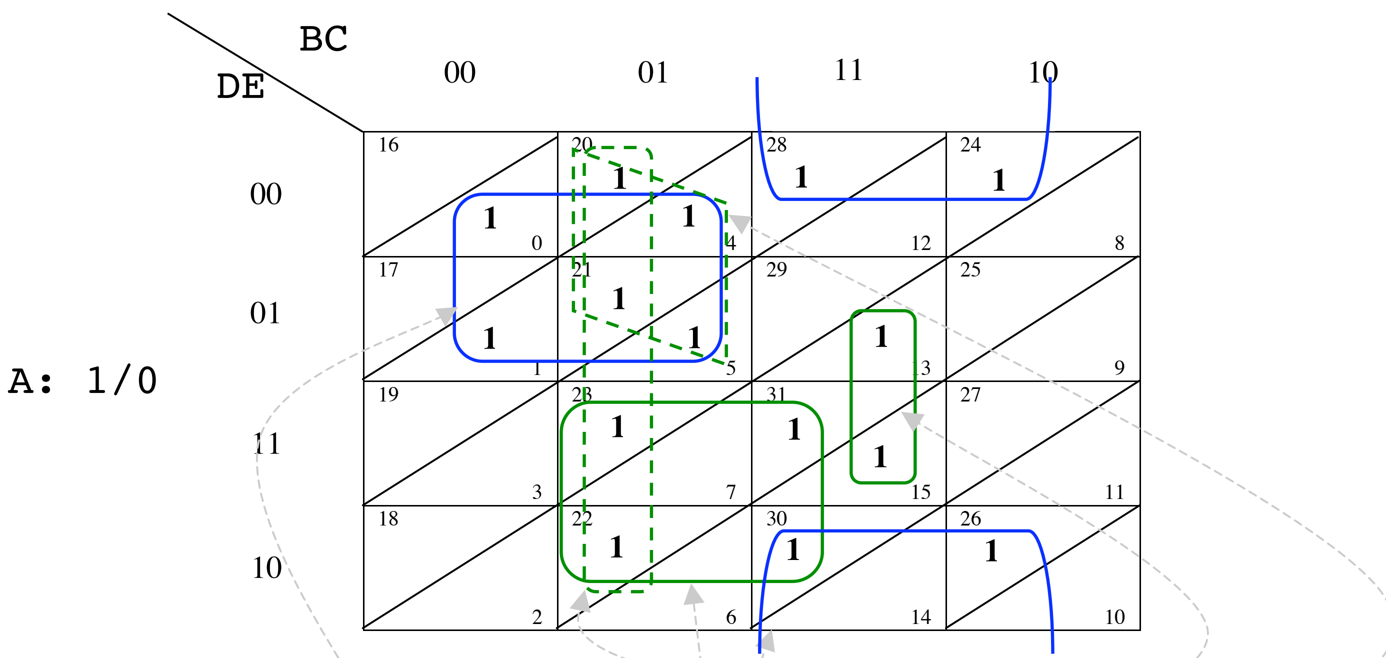


Note that we are looking for Prime Implicants for the remaining terms: 13, 15, 20, 21, 22, 23 and 31. We can see that 13 can be looped with 5 or 15 to form Prime Implicants: `A'CD'E` and `A'BCE`. Verify that these are Prime Implicants because neither can be part of a larger loop to eliminate a variable and also they are the only Prime Implicants that 13 is a part of. Since we are interested in choosing a minimum set of prime implicants we choose to pair 13 with 15 thus taking care of two uncovered 1's rather than just one.

Next, consider 20, which can be looped with (22,28,30) to form Prime Implicant: `ACE'`, with (4,5,21) to form Prime Implicant: `B'CD'`, or with (21,22,23) to form Prime Implicant: AB`'C`. All these have the same number of variables and we could pick either: {`ACE', B'CD', AB'C`}



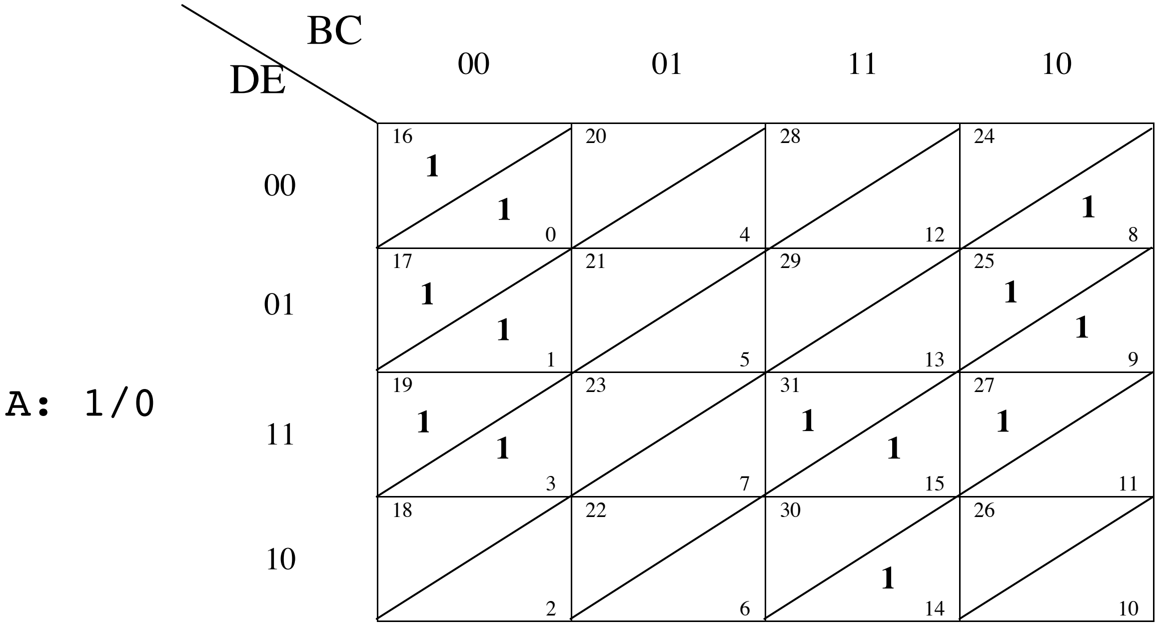
The only remaining 1 is in location 31. This can be looped with 15 to form the Prime Implicant `BCDE` or with (23, 22, 30) to form the Prime Implicant `ACD`. We choose `ACD` because it has fewer variables.



Picking ACE' from the first list above leaves 20 and 21 uncovered, the other two do not leave any uncovered 1's. So the final minimized expression for `F(ABCDE)` is:

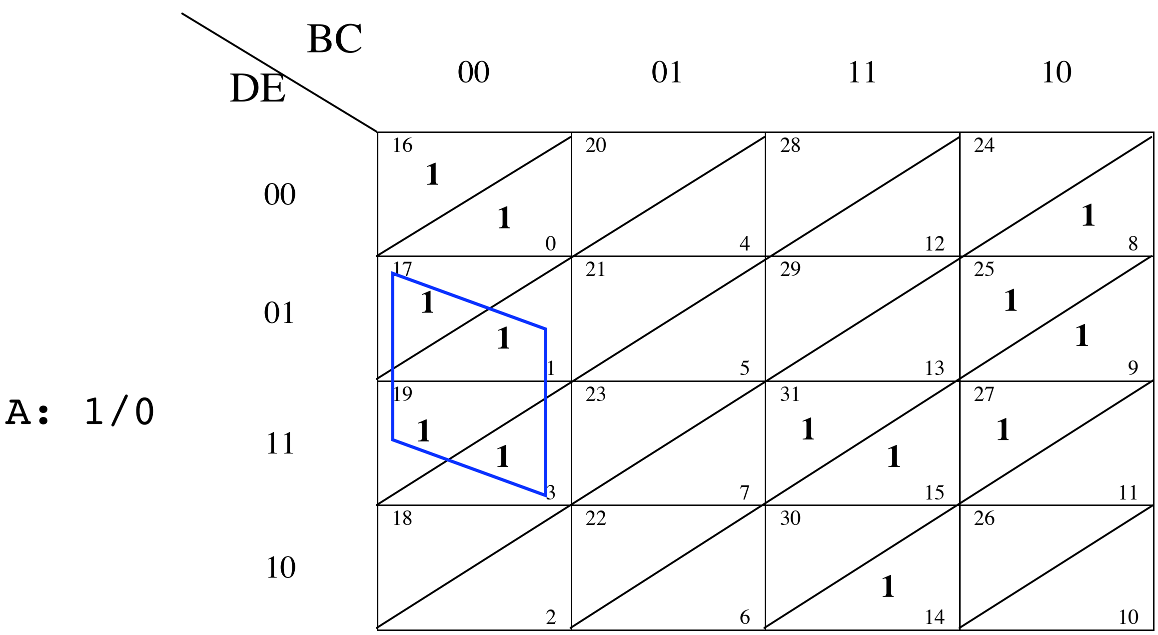
`F(ABCDE) = A'B'D' + ABE' + ACD + A'BCE + {B'CD'` or `AB'C}`

Problem 2:
Given `F(ABCDE)` = Σm(0,1,3,8,9,14,15,16,17,19,25,27,31)



Lets work the flowchart in a tabular format:

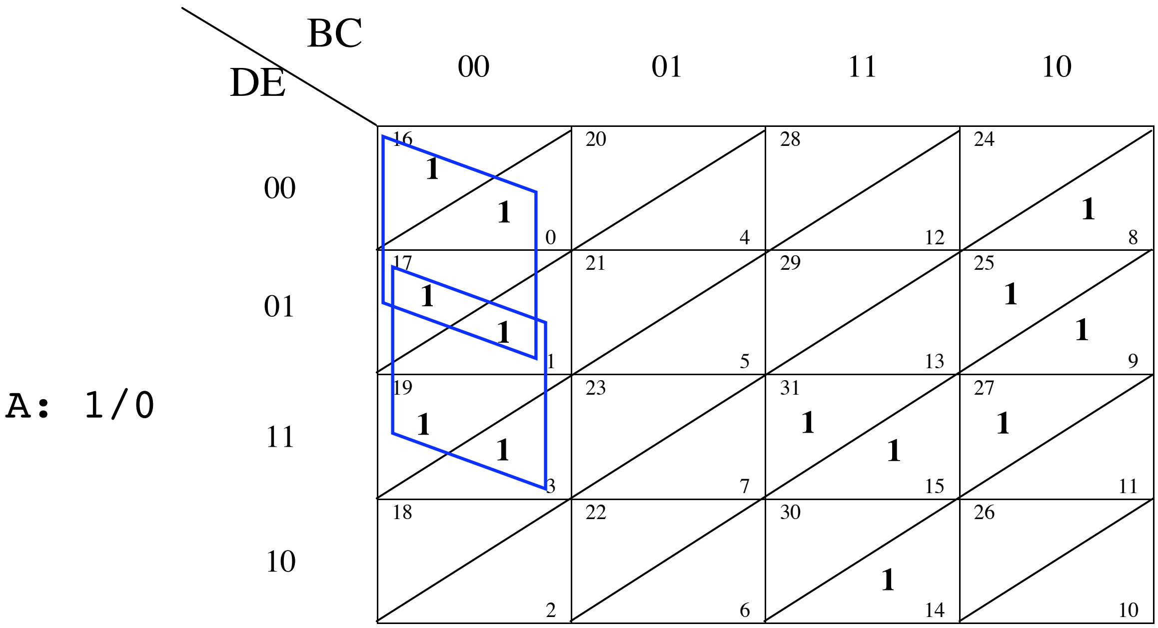
Chosen uncovered 1	**Adjacent 1's and X's**	**Covered by a single term?**	**Essential Prime Implicant**
1_0	1,8,16	NO	--
1_1	0,3,9,17	NO	--
1_3	1,19	YES	B'C'E



Lets continue with the process by checking the next unchecked 1:

Chosen uncovered 1	Adjacent 1’s and X’s	Covered by a single term?	Essential Prime Implicant
1_0	1,8,16	NO	--
1_1	0,3,9,17	NO	--
1_3	1,19	YES	B’C’E
1_{16}	0,17	YES	B’C’D

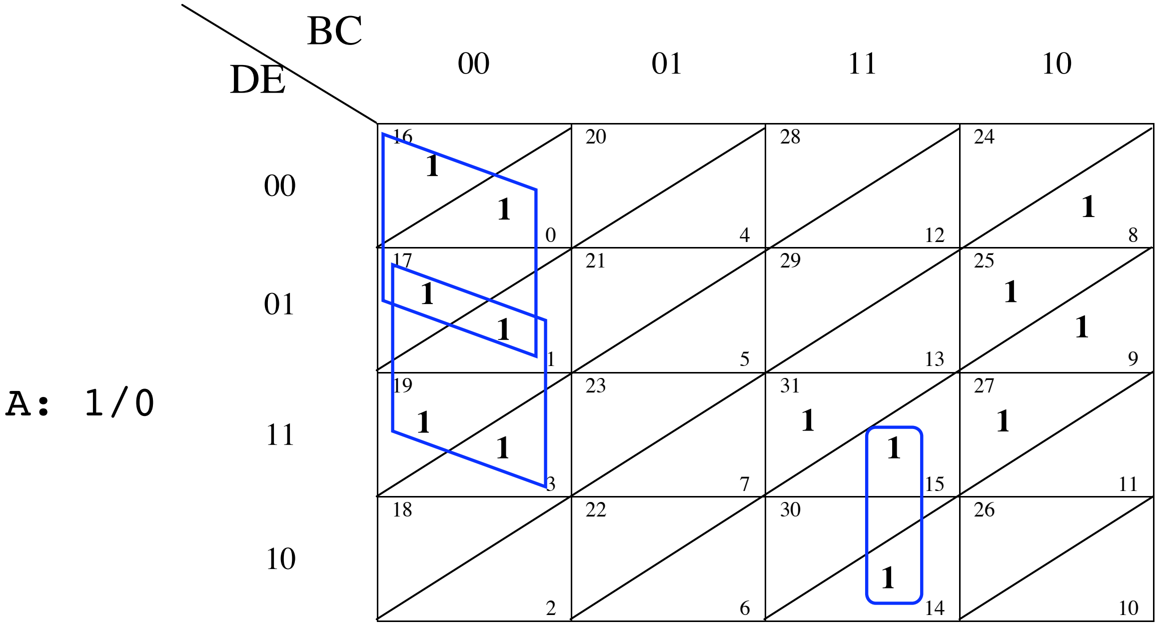
Loop it:



Lets continue with the process by checking the next unchecked 1s:

Chosen uncovered 1	Adjacent 1’s and X’s	Covered by a single term?	Essential Prime Implicant
1_0	1,8,16	NO	--
1_1	0,3,9,17	NO	--
1_3	1,19	YES	B’C’E
1_{16}	0,17	YES	B’C’D
1_{31}	15,27	NO	--
1_{15}	14,31	NO	--
1_{14}	15	YES	A’BCD

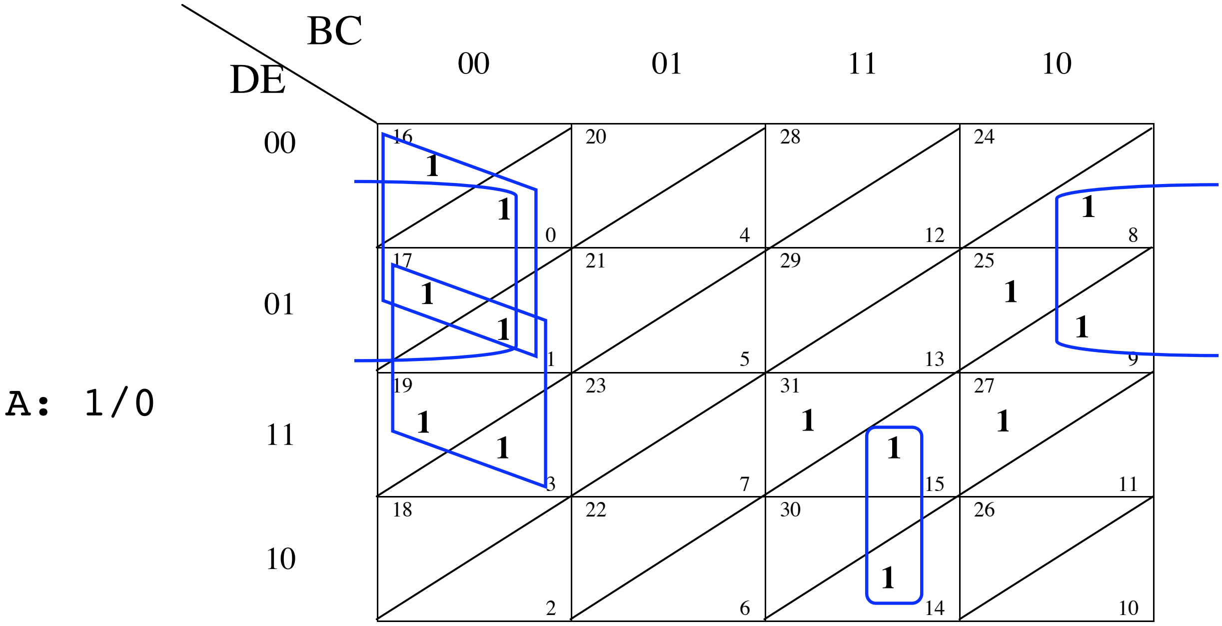
Loop it:



Lets continue with the process by checking the next unchecked 1s:

Chosen uncovered 1	Adjacent 1's and X's	Covered by a single term?	Essential Prime Implicant
1_0	1,8,16	NO	--
1_1	0,3,9,17	NO	--
1_3	1,19	YES	B'C'E
1_{16}	0,17	YES	B'C'D
1_{31}	15,27	NO	--
1_{15}	14,31	NO	--
1_{14}	15	YES	A'BCD
1_8	0,8	YES	A'C'D'

Loop it:

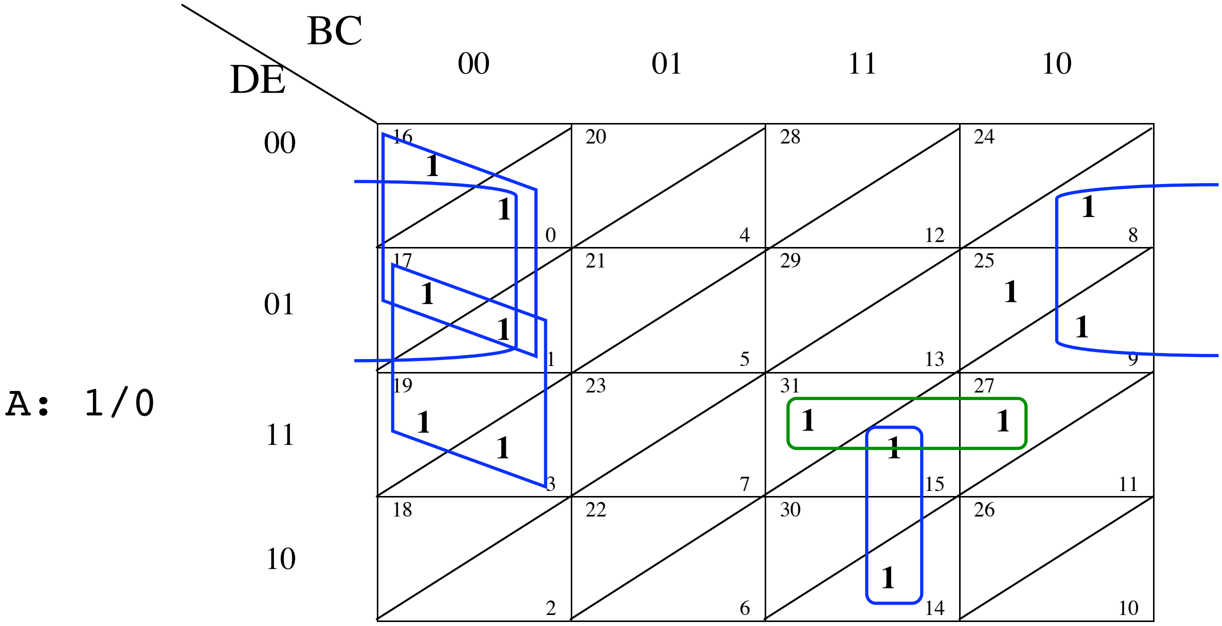


Lets continue with the process by checking the next unchecked 1s:

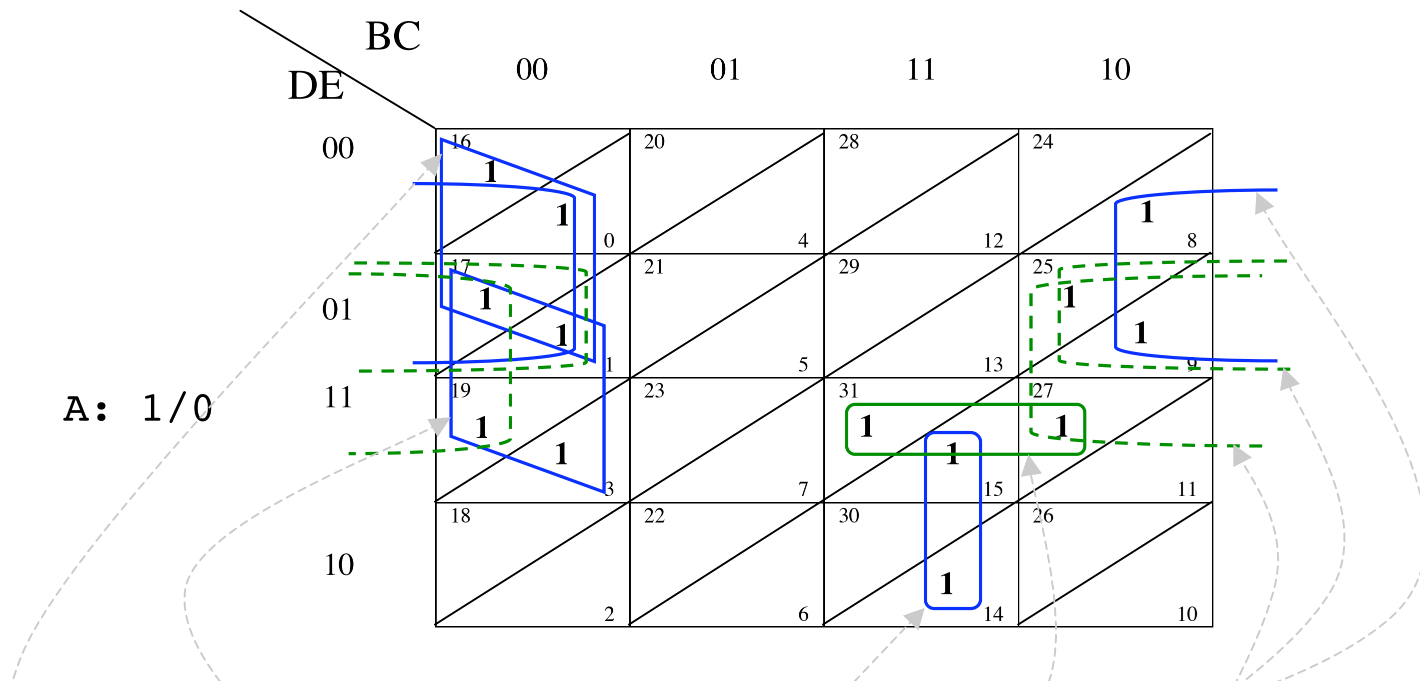
Chosen uncovered 1	Adjacent 1's and X's	Covered by a single term?	Essential Prime Implicant
1_{0}	1,8,16	NO	--
1_{1}	0,3,9,17	NO	--
1_{3}	1,19	YES	B'C'E
1_{16}	0,17	YES	B'C'D
1_{31}	15,27	NO	--
1_{15}	14,31	NO	--
1_{14}	15	YES	A'BCD
1_{8}	0,8	YES	A'C'D'
1_{25}	9,17,27	NO	--
1_{27}	19,25,31	NO	--

At this point we found all the Essential Prime Implicants. We are still left with three uncovered 1's in locations 25, 27 and 31.

Consider 31, which we can combine with 27 to form the Prime Implicant ABDE, or with 15 to form the Prime Implicant BCDE. We obviously choose the former ABDE because it eliminates two uncovered 1's.



Now consider 25, which we can combine with (1,9,17) to form the Prime Implicant `C'D'E`, or we can combine it with (17,19,27) to form the Prime Implicant `AC'E'`. Since both these prime implicants have the same number of terms we could choose either.



Now that all 1's are covered we are done:

`F(ABCDE) =`

`B'C'D' + B'C'E + A'C'D' + A'BCD + ABDE + {C'D'E or AC'E}`