# Adaptive Induced Fluctuations for Multiuser Diversity

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## I. System Model and Problem Formulation

Consider a wireless downlink with 2 antennae at the base station (BS), 1 antenna at each of the K mobiles (MS) and independent proper stationary Rayleigh channels  $\{h_{n,k}(t)\}_{\substack{n=2,k=K\\n=1,k=1}}^{n=2,k=K}$ , where time t is slotted. Channels remain constant over one slot and change Gauss-Markov from slot to slot:  $h_{n,k}(t+1) = ah_{n,k}(t) + \sqrt{1-a^2}w_{n,k}(t)$ , where  $w_{n,k}(t)$  are mutually independent  $\mathcal{CN}(0, 2\sigma^2)$  random variables and  $a \leq 1$ . If the BS transmits signals x(t) and  $x(t)e^{j\theta(t)}$ , the received signal at the  $k^{th}$  MS will be

$$r_k(t) = rac{1}{\sqrt{2}} \left( h_{1,k}(t) + h_{2,k}(t) e^{j heta(t)} 
ight) x(t) + W_k(t)$$

where the  $W_k$ s are mutually independent  $\mathcal{CN}(0, 1)$  processes, and are independent of the h's and across time. In each time slot a value of  $\theta(t)$  is fixed by the base station and the users report their SNRs  $\frac{1}{2} |h_{1,k}(t) + h_{2,k}(t)e^{j\theta(t)}|^2$ . The user with the maximum SNR in a particular slot is chosen for service for that slot, and its SNR is the *SNR of service* for that slot.  $\theta(t)$  is changed from slot to slot.

The BS can induce channel fluctuations by varying  $\theta$  with time [1]. A trajectory for  $\theta$  that does not depend on the SNR feedback (and is thus independent of the channel) is called *open loop*. The problem we consider is to control  $\theta(t)$ , using only the SNR feedback, so that the SNR at which service is given is (on average) better than that with an open loop trajectory.

If the phases of each of the *h* were known at the BS, a value of  $\theta^*(t) = \arg h_{1,k}(t) - \arg h_{2,k}(t)$  is the best for user *k* at *t*. The service SNR using  $\theta^*(t)$  of the best user at *t* (beamforming in phase) provides an upper bound on the performance of any trajectory, but is unachievable with the feedback scenario under consideration.

### II. THE ADAPTIVE SCHEME

Intuitively, given that the channels possess sufficient memory, a value of  $\theta(t)$  that results in a good value of SNR of service at time t will result in a good SNR of service at t + 1too, but may not work well (say) 100 slots later. A trajectory which makes  $\theta$  cycle around the phase space and "slow down when it is doing well, and speed up when it is not" will do better than a trajectory that just takes a uniform sampling of the phase space, with no use of feedback.

The adaptive trajectory for  $\theta(t)$  that we propose to exploit this phenomenon follows the update equation

$$\theta(t+1) - \theta(t) = Be^{-\gamma SNR_t^*(\theta(t))}$$

where  $SNR_t^*(\theta(t))$  is the SNR of service at time t and B and  $\gamma$  are the parameters of the scheme.

### **\_\_\_\_II**. Results

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The performance of this adaptive scheme is compared to that of an open loop scheme and to the upper bound of beamforming in phase for the case of infinitely backlogged users without delay constraints. The average throughputs as a function of the number of MS users is shown in the first figure for two channel update speeds. The scheme performs better for channels with higher memory (i.e. higher a), but manages to beat the open loop one even for a channel coherence time of approximately 10 slots (a = 0.9).

We also studied the performance of the scheme for delayconstrained users with finite queues fed by Bernoulli arrivals. The adaptive scheme can support higher arrival rates than a non-adaptive one. The addition of a fairness mechanism (we used the exponential rule [2]) to this scenario results in further gains for both the adaptive and non-adaptive schemes. The results are shown in the second figure.

#### References

- [1] P. Viswanath, D. Tse and R. Laroia, "Opportunistic Beamforming using Dumb Antennas," Preprint, Sept 2001.
- [2] S. Shakkottai and A. Stolyar, "Scheduling for Multiple flows Sharing a Time-Varying Channel: The Exponential Rule," Bell Labs Tech. Memo, Jul 2000.

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