1

A New Mechanism for the Free-rider Problem

Sujay Sanghavi , *Member, IEEE,* and Bruce Hajek *Fellow, IEEE,*

***Abstract*—The free-rider problem arises in the provisioning of public resources, when users of the resource have to contribute towards the cost of production. Selfish users may have a tendency to misrepresent preferences – so as to reduce individual contributions – leading to inefficient levels of production of the resource. Groves and Loeb formulated a classic model capturing this problem, and proposed (what later came to be known as) the VCG mechanism as a solution. However, in the presence of heterogeneous users and communication constraints, or in decentralized settings, implementing this mechanism places an unrealistic communication burden. In this paper we propose a class of alternative mechanisms for the same problem as considered by Groves and Loeb, but with the added constraint of severely limited communication between users and the provisioning authority. When these mechanisms are used, efficient production is ensured as a Nash equilibrium outcome, for a broad class of users. Furthermore, a natural bid update strategy is shown to globally converge to efficient Nash equilibria. Also, upper bounds are provided for the revenue that can be generated by any individually rational mechanism that ensures efficient production at any Nash equilibrium. It is shown that there exist mechanisms in our class that achieve each of the bounds. An extension to multiple public goods with interrelated valuations is also presented.**

***Index Terms*—Free-riders, Public Goods, Game Theory, Convex Optimization,**

I. Introduction

This paper proposes a new class of mechanisms for addressing the free-rider problem that arises in the production of public goods. By *public good* we refer to a resource whose usage is non-exclusionary: it can be used simultaneously and equally by all users. This is in contrast to a *private good*, which has to be divided up among the users, each of whom has exclusive access to its portion after the auction. Common examples of public goods in everyday life are television / radio broadcasts, weather reports and public works such as libraries.

In proposing the mechanisms described in this paper we are motivated by public goods in modern communication and computation systems. Consider for example a large distributed database, containing information available to all users, without exclusion. Each user contributes towards the building / maintenance of this database, either in direct monetary terms or through contributed storage resources. Since the information in the database is assumed to be freely available to all users, each user has an incentive to minimize the amount of resources it contributes. However, if every user acts according to these selfish considerations, the net result could be a possibly severe under-provisioning of the resource. This is the classic "free-rider problem:" inefficient provisioning of a public good due to selfish behavior. Besides socially inefficient production, free-riding may also lead to budgetary shortfalls: user contributions may cover only a fraction of the cost of efficient, or even near-efficient, production.

Mechanisms for the production of public goods proceed as follows. Users are asked to submit bids to the producer. Based on the received bids the producer then decides, according to a pre-specified and globally known rule, the quantity of the public good to be produced and the contributions to be made by each of the users. Groves and Loeb [1] proposed a generic model capturing the free-rider problem in the production of a real-valued amount of a public good. The mechanism they proposed for solving the problem was one of the earliest instances of what later came to be known as the general class of VCG mechanisms. This paper proposes alternative mechanism designs for the same resource allocation problem as formulated in [1]: we are interested in mechanisms that ensure the production of the efficient real-valued quantity of a public good, in the presence of users whose objectives are the maximization of their individual net profits. The auctions presented in this paper are not suitable for combinatorial settings.

It is well known (see e.g. [2]) that VCG mechanisms are the only ones that ensure efficient production as dominant strategy outcomes in a wide variety of resource allocation problems, including the ones investigted in this paper. It is also increasingly apparent that in many settings the implementation of VCG mechanisms places a heavy communication and computational demand on the auctioneer and agents, even to the extent that they are deemed infeasible to implement. Another criticism of the VCG mechanism is that it asks for detailed private information, namely the entire

Sujay Sanghavi is with LIDS, MIT and can be reached at sanghavi@mit.edu. Bruce Hajek is with ECE and CSL, UIUC and can be reached at b-hajek@uiuc.edu. This work was done while both authors were at UIUC, and was supported by grants NSF ANI 99-80544 and ECS-0621416.

set of user preferences, to be made public for the purposes of resource allocation. Even when bids may be submitted anonymously, users may be unwilling or unable to completely revel their preferences.

In this paper we consider the same problem as was considered in [1], but add a severe communication constraint. Specifically, we require that each user's bid be a *single* real number. This is in contrast to the VCG implementation, which asks that the bid be an entire real-valued *function*. Since dominant strategy equilibria are unreasonable to expect in this setting, we settle for Nash strategies as the equilibrium concept. We propose ex-post individually rational mechanisms that result in the production of an optimal quantity of the public good at any Nash equilibrium. Furthermore, Nash equilibria are shown to always exist, and there is a one-to-one correspondance between the set of optimal quantities and the set of Nash equilibria. Revelation of single-valued bids implies that it is not possible to infer a user's private valuation information from its bid.

Such a mechanism design immediately raises the question of price discovery: how do users know / arrive at a Nash equilibrium? This is not a concern for VCG mechanisms as users are assumed to know their own value functions. For the mechanisms in this paper, myopic best response adjustments to bids in continuous time result in global convergence to Nash equilibria. Furthermore, these updates are easy to compute and need very little information – which can be provided by the mechanism designer – about the rest of the market.

VCG mechanisms are individually rational because when bidding optimally each user can ensure that its payment does not exceed the value it obtains from the good's production. In the mechanism proposed in this paper, the payment made by a user will always be less than its single-valued bid. This is true at all times, not just at the final equilibrium. Also, at Nash equilibrium, no user's payment will exceed the value it obtains from the good's production. The mechanism presented in this paper is thus also individually rational.

Nash implementations for public good problems have been proposed in the literature, primarily with the objective of addressing the issue of budgetary shortfall, which VCG mechanisms are susceptible to. Groves and Ledyard [3] devised the first Nash implementation, to be followed by Walker [4]. These mechanisms have no budgetary shortfall, but are not individually rational: even if, for example, a user values for all levels of the public good at ϵ, these mechanisms may still end up charging the user more than ϵ, *at equilibrium*. Also, the mechanisms as stated work only for the special case when the total cost of producing the public good is linear in the amount produced. Furthermore, the mechanisms in [3] and [4] require that the price of production be known, and use it as a parameter in the payment function. The mechanisms proposed in this paper are not budget balanced, but are individually rational and work for any convex production cost function. Also, the payment function in our mechanism does not depend on the cost function.

Allocation of continuous-valued (i.e. infinitely divisible) *private* goods based on single-valued bids, as well as the dynamics of convergence to equilibria, has received attention previously in networking contexts, with the good primarily being represented by bandwidth. Kelly [5] assumes users are price takers, and a primal-dual price selection algorithm is shown to converge to the optimal allocation. In [6–9], the price taking assumption is relaxed, with each user now being able to anticipate the effect of his/her own bid on the price. The work [10] proposes the Nash Bargaining solution as a natural mechanism, if the objective is pareto-optimality, that can be implemented in a distributed fashion in networks. In the above papers users submit bids indicating their willingness to pay, while in [11], the users send rates into the system and accept the resulting charges. In all of the above papers, each user has a utility function for its share of the good, but is constrained to bit a single real number (which can be varied over time). Myopic continuous-time updates are shown to converge to Nash equilibria in [8, 9]. Our paper can be viewed as an extension of these implementation ideas to the realm of continuous valued public good provisioning problems.

In a combinatorial private good setting, Blumrosen and Nisan [12] limit the strategy space even further, requiring that each bid be limited to a few bits. The problem of provisioning a public good differs from private good problems in the sense that the allocation decision, namely the real-valued quantity to be produced, is one-dimensional, while in the private good case the allocation is a vector including each user's quantity.

For public goods, under many mechanisms, inefficient production may occur as an equilibrium outcome. In Section II we give an example of a simple intuitive mechanism – the "pay as bid" mechanism – and show that inefficiencies in production may be quite severe even for reasonable scenarios of user value functions. In Section III we describe one example of the class of new mechanism we propose in this paper, and prove the existence, uniqueness, and optimality results for Nash equilibria in the resulting game. In Section IV we show that myopic gradient ascent updates in continuous time result in global convergence to a Nash equilibrium. In Section V we extend the example mechanism to the case when there are multiple public goods and users with joint value functions. We present the full class of mechanisms in Section VI, and conclude with some discussion in Section VII.

II. System Model

A certain quantity $Q \in \mathbb{R}_{++}$ of a public good has to be produced by a producer, where $\mathbb{R}_{++}$ is the set of strictly positive reals. The producer can produce a quantity Q at cost $C(Q)$. Once produced, it is available to n users, where $n \geq 2$. Each user i obtains a value $U_i(Q)$ from the good's production, and contributes a payment p_i towards its production. It is assumed that $C(Q)$ is strictly increasing and convex and each $U_i(Q)$ is strictly increasing and concave, and all functions are continuously differentiable. This is the same as the model in Groves and Loeb [1]. [1]

A quantity Q^* is said to be *efficient* if producing that quantity maximizes the net social benefit:

$$\sum_i U_i(Q^*) \, - \, C(Q^*) \; \geq \; \sum_i U_i(Q) \, - \, C(Q)$$

for all $Q \in \mathbb{R}_{++}$. If a quantity Q does not satisfy the above requirement, it is inefficient. It is assumed that there exists some finite $Q^* > 0$ that is efficient. Concavity implies that the efficiency of Q^* is characterized by the first-order conditions.

Lemma 2.1: A quantity Q^* is efficient if and only if $\sum_i U_i'(Q^*) = C'(Q^*)$.

Any mechanism for the production of the good proceeds as follows. First, each user i is asked to submit a *bid* b_i. Then, the producer maps the vector of bids $\underline{\mathbf{b}}$ into a produced quantity $f(\underline{\mathbf{b}})$ and a payment $p_i(\underline{\mathbf{b}})$ for each user i. We will call f the *production function* and the p_i's the *payment functions*. The production and payment functions are known by the users in advance, i.e. before they submit their bids. Specifying the space of allowed bids and the production and payment functions specifies the mechanism.

One example of such a mechanism is the classical VCG mechanism. A VCG mechanism requires users to submit bids that are functions on $\mathbb{R}_+$. Given these bid functions b_i, the production function is

$$f^{VCG}(\underline{\mathbf{b}}) \;=\; \arg\max_{Q \geq 0} \sum_i b_i(Q) - C(Q)$$

[1] Except that in [1] it is assumed that $Q \geq 0$ and $C(Q) = pQ$ for some $p > 0$. Also the U_i functions need not be differentiable.

while the payment function for user i is

$$\begin{aligned} p_i^{VCG}(\underline{\mathbf{b}}) \;=\; & \left(\max_{Q \geq 0} \sum_{j \neq i} b_j(Q) - C(Q) \right) \\ & - \left(\sum_{j \neq i} b_j(f^{VCG}(\underline{\mathbf{b}})) - C(f^{VCG}(\underline{\mathbf{b}})) \right) \end{aligned}$$

Given a mechanism and bid vector $\underline{\mathbf{b}}$, the *net reward* of user i is given by

$$R_i(\underline{\mathbf{b}}) \;=\; U_i(f(\underline{\mathbf{b}})) - p_i(\underline{\mathbf{b}}) \qquad (1)$$

Given the mechanism, the users play a non-zero-sum non-cooperative game, with each user trying to maximize its own net reward.

As an example of a mechanism susceptible to the free-rider problem, consider the *pay as bid* mechanism where user payments are the bids – $p_i(\underline{\mathbf{b}}) = b_i$ – and the production function is the one that balances the budget:

$$f^*(\underline{\mathbf{b}}) \;=\; X(B) \qquad (2)$$

where $X = C^{-1}$ is the inverse of the cost function and $B = \sum_i b_i$ is the total of all bids (and payments).

For this mechanism it can be seen that the first-order necessary condition for a bid vector $\widetilde{\underline{\mathbf{b}}}$ to be a Nash equilibrium is that

$$U_i'(X(\widetilde{B})) \;\leq\; C'(X(\widetilde{B}))$$

for *each* user i, with equality if $\widetilde{b}_i > 0$. It is easy to see from Lemma 2.1 that there will not be an efficient Nash equilibrium when more than two users are present, for *any* set of value functions U_i and cost function C – under the assumptions of convexity of C, concavity of U_i's and that any efficient quantity Q^* is non-zero.

III. A New Mechanism

In this section we present an example from the new class of mechanisms that ensure socially optimal production. To do so we need to specify the space of allowable bid vectors, the production function f, and the payment functions p_i.

Each user's bid is a strictly positive real number: $b_i \in \mathbb{R}_{++}$. Given the vector of bids $\underline{\mathbf{b}}$, we propose the using the same production function

$$f^*(\underline{\mathbf{b}}) \;=\; X(B)$$

as in (2) above. Note that X is increasing, concave and differentiable, with $X'(C(Q)) = \frac{1}{C'(Q)}$. For each user i, denote the total bid of users other than i by $B_{-i} = \sum_{j \neq i} b_j$. We propose the following payment functions

$$p_i^*(\underline{\mathbf{b}}) \;=\; b_i - B_{-i} \log\left(1 + \frac{b_i}{B_{-i}}\right) \qquad (3)$$

The mechanism is thus fully specified. The other mechanisms we propose in Section VI use the same allocation function f^*, but different choices for the payment functions p_i.

The following properties are easy to see:

1) The reward function $R_i(b_i, B_{-i})$ is concave in b_i for all fixed values of b_j, $j \neq i$.
2) $0 < p_i(b_i, B_{-i}) < b_i$ for all $\underline{\mathbf{b}}$ and i: a user is never asked to pay more than its bid.

Theorem 3.1: For the public good model described in the previous section if the mechanism (f^*, p_i^*) is used, there is a one-to-one correspondence between the set of efficient quantities and the set of Nash equilibria for the game. Also, at any of these Nash equilibria the corresponding efficient quantity is provisioned.

Note: Rosen's theorem [13] cannot be directly used to show the existence of Nash equilibria in this game since the users' strategy spaces are all $\mathbb{R}_{++}$, which is not closed.

Proof of Theorem 3.1:

Since each user's reward function (1) is concave in the users' own bid, the simultaneous satisfaction of the following first-order conditions by all users at a bid vector $\widetilde{\underline{\mathbf{b}}}$ is necessary and sufficient for $\widetilde{\underline{\mathbf{b}}}$ to be a Nash equilibrium:

$$U_i'(X(\widetilde{B}))X'(\widetilde{B}) \,-\, 1 \,+\, \frac{\widetilde{B}_{-i}}{\widetilde{b}_i + \widetilde{B}_{-i}} \;=\; 0 \;\;\text{ for all } i$$

If $\widetilde{Q} \;=\; X(\widetilde{B})$ then $X'(\widetilde{B}) \;=\; \frac{1}{C'(\widetilde{Q})}$ and so the above conditions can be rewritten as

$$U_i'(\widetilde{Q}) \;=\; \frac{\widetilde{b}_i}{\widetilde{B}} C'(\widetilde{Q}) \;\;\text{ for all } i \qquad (4)$$

Suppose now that $\widetilde{\underline{\mathbf{b}}}$ is a Nash equilibrium, and $\widetilde{Q} = f^*(\widetilde{\underline{\mathbf{b}}})$ is the corresponding quantity that is produced. Summing the conditions in (4) over the set of users yields

$$\sum_i U_i'(\widetilde{Q}) \;=\; C'(\widetilde{Q})$$

By Lemma 2.1, this means that $\widetilde{Q}$ is efficient. Thus efficient quantities are provisioned at Nash equilibria.

For showing the existence of Nash equilibria, we simply turn the above argument around. Let Q^* be efficient – by assumption there exists at least one such quantity that is finite. Define for each user the bid

$$\widetilde{b}_i \;\triangleq\; \frac{U_i'(Q^*)}{C'(Q^*)} C(Q^*) \qquad (5)$$

Then, by Lemma 2.1, $\sum_i U_i'(Q^*) \;=\; C'(Q^*)$ and hence the total bid satisfies $\widetilde{B} = C(Q^*)$. Thus (5) can be rewritten as

$$U_i'(Q^*) \;=\; \frac{\widetilde{b}_i}{\widetilde{B}} C'(Q^*) \;\;\text{ for all } i$$

Since $Q^* \;=\; X(\widetilde{B})$, $\widetilde{\underline{\mathbf{b}}}$ satisfies the necessary and sufficient conditions of (4), and hence is a Nash equilibrium. Therefore $\widetilde{\underline{\mathbf{b}}}$ corresponds to the efficient quantity Q^*. $\square$

It is clear that the mechanism is not budget-balanced. However, it is possible to bound the subsidy $B - \sum_i p_i(\underline{\mathbf{b}})$ as a fraction of the total cost.

Proposition 3.1: When n users are present,

$$\frac{B - \sum_i p_i(\underline{\mathbf{b}})}{B} \;\leq\; (n-1)\log\frac{n}{n-1}$$

This is tight if the n bids are equal.

Proof of Proposition 3.1: For a bid vector $\underline{\mathbf{b}}$, the subsidy is given by

$$\begin{aligned}
B - \sum_i p_i(\underline{\mathbf{b}}) \;&=\; \sum_i B_{-i} \log\frac{B}{B_{-i}} \\
&=\; (n-1)B\sum_i \frac{B_{-i}}{(n-1)B}\log\frac{(n-1)B}{B_{-i}} \\
&\quad\; - (n-1)B\log(n-1) \\
&=\; (n-1)B\,H(B_-) - (n-1)B\log(n-1)
\end{aligned}$$

where $H(B_-)$ is the entropy of the n-length probability vector whose i^th element is $\frac{B_{-i}}{(n-1)B}$. Now, for any B_-, $H(B_-) \leq \log n$, with equality if and only if the elements of B_- are all equal. Hence

$$\frac{B - \sum_i p_i(\underline{\mathbf{b}})}{B} \;\leq\; (n-1)\log\frac{n}{n-1}$$

Thus proved. $\square$

The tightness of the above proposition implies that for a large number of identical users the subsidy will form a large portion of the total cost. As shown in Section VI, significant subsidies cannot be avoided in a large class of mechanisms that ensure optimal provisioning at Nash equilibria and use $f(\underline{\mathbf{b}}) \;=\; X(B)$ as the production function. In spirit, this result for our mechanism is similr to results for the VCG mechanism: large subsidies may be required.

IV. Dynamics

The above section shows that for the mechanism presented, Nash equilibria always exist and are efficient. However, users still have to find out these Nash equilibria. In this section we show that if users follow a natural bid update strategy, then the vector of bids converges to a Nash equilibrium from any valid initial condition.

Specifically, consider the user update rule when each user attempts gradient ascent, in continuous time, of its reward function (1):

$$\begin{aligned}\frac{d}{dt}b_i &= \frac{\partial}{\partial b_i}R_i(b_i, B_{-i}) \\ &= U_i'(X(B))X'(B) - \frac{b_i}{B} \end{aligned} \tag{6}$$

To follow this bid update procedure at a given time, the user only needs to know the amount currently provisioned, the cost function's derivative and the total of all the users' bids. The user does not need detailed information about what each user's bid is, or even how many users are present.

If the above gradient ascent update procedure is followed by each of the users, then the sum B of their bids is seen to follow a gradient ascent of the social utility function. This observation, formalized in the lemma below, is used to show global convergence of each of the bids to the corresponding Nash equilibrium bids. For ease of exposition we will assume that there is a unique optimal quantity Q^*.

Lemma 4.1: Let $\{U_i\}$ and C be such that there is a unique optimal quantity Q^*. Then, for any initial starting bid vector $\underline{\mathbf{b}}_0$ having $b_i > 0$ for at least two i, if the users follow the updates given by (6), then the sum of the bids B converges to $B^* = C(Q^*)$.

Proof of Lemma 4.1:

Adding the update equations (6) over the set of all users i, we see that

$$\begin{aligned}\frac{d}{dt}B &= X'(B)\left(\sum_i U_i'(X(B))\right) - 1 \\ &= \frac{d}{dB}\left(\sum_i U_i(X(B)) - B\right)\end{aligned}$$

Now, $\sum_i U_i(X(B)) - B$ is a concave increasing continuously differentiable function of B, and is maximized at one point, B^*, by assumption. Hence the above update equation implies that $B \to B^*$. □

Theorem 4.1: Let U_i and C be such that there is a unique optimal quantity Q^*. When the mechanism given by (2) and (3) is used, let $\underline{\mathbf{b}}^*$ be the corresponding Nash equilibrium bid vector. Then, for the bid updates given by (6), the vector of bids will converge to $\underline{\mathbf{b}}^*$ from any initial condition in $\mathbb{R}_{++}$

Proof of Theorem 4.1: The proof involves breaking time into two phases. In the first phase the sum B of the bids gets close to the optimal $B^* = C(Q^*)$, and in the next phase each of the individual bids b_i get close to b_i^*.

Given $\delta > 0$, we want to show the existence of a finite time instant T_δ such that $|b_i - b_i^*| < \delta$ for all i and all $t \geq T_\delta$. Towards this end, let $\epsilon_0 > 0$ be such that $\epsilon_0 B^* < \frac{\delta}{4}$ and $\epsilon_0 < \frac{b_i^*}{B^*}$ for all i. Then let $\epsilon_1 > 0$ be such that for all i the following holds

$$\left| U_i'(X(B))X'(B) - \frac{b_i^*}{B^*} \right| \leq \epsilon_0 \quad \text{whenever } |B - B^*| \leq \epsilon_1$$

Such an ϵ_1 exists by the continuity of $U_i'(X(B))X'(B)$ and the fact that $U_i'(X(B^*))X'(B^*) = \frac{b_i^*}{B^*}$, for all i. For this choice of ϵ_0 and ϵ_1, let T_1 be a finite time such that $|B(t) - B^*| < \epsilon_1$ for all $t > T_1$. Lemma 4.1 ensures the existence of such a T_1. T_1 is the end of phase one.

Since the update equation (6) holds for all $t > T_1$, this means that $b_i(t) \in [\underline{\mathbf{b}}_i(t), \bar{\mathbf{b}}_i(t)]$ for all $t > T_1$, where the upper and lower bounds satisfy $\underline{b}_i(T_1) = \bar{b}_i(T_1) = b_i(T_1)$ and for $t > T_1$ are updated according to the equations

$$\begin{aligned}\frac{d\underline{b}_i}{dt} &= \frac{b_i^*}{B^*} - \epsilon_0 - \frac{\underline{b}_i}{B^* - \epsilon_1} \\ \frac{d\bar{b}_i}{dt} &= \frac{b_i^*}{B^*} + \epsilon_0 - \frac{\bar{b}_i}{B^* + \epsilon_1}\end{aligned}$$

Solving the first of the above two equations yields

$$\begin{aligned}\underline{b}_i(t) &= \left(\frac{b_i^*}{B^*} - \epsilon_0\right)(B^* - \epsilon_1) \\ &\quad + \left(b_i(T_1) - \left(\frac{b_i^*}{B^*} - \epsilon_0\right)(B^* - \epsilon_1)\right) e^{\frac{t - T_1}{B^* - \epsilon_1}}\end{aligned}$$

Now by the assumptions made on T_1, δ, ϵ_0 and ϵ_1, this means that for t large enough, $\underline{b}_i(t) > b_i^* - \delta$. Similarly it can be shown that $\bar{b}_i(t) < b_i^* + \delta$, and hence that $b_i(t) \to b_i^*$ for all i. This finishes the proof. □

Although we have assumed that $Q^* > 0$, the above dynamics also work if the efficient allocation is $Q^* = 0$. Indeed, if $\sum_i U_i(0) - C(0) > \sum_i U_i(Q) - C(Q)$ for all $Q > 0$, then the above dynamics will result in $B \to 0$. Hence production will be efficient in the limit.

The continuous time dynamics of (6) have a natural discrete-time version:

$$b_i(k+1) \;=\; b_i(k) + \gamma \left(U_i'(X(B(k)))X'(B(k)) - \frac{b_i(k)}{B(k)} \right) \tag{7}$$

where $\gamma > 0$ is the step size. For these dynamics, we have the following theorem.

Theorem 4.2: Let U_i and C be twice continuously differentiable and such that there is a unique optimal quantity Q^*, and there are at least two users in the system. When the mechanism given by (2) and (3) is used, let $\underline{\mathbf{b}}^*$ be the corresponding Nash equilibirum bid vector. Then, for the discrete-time updates given by (7), the vector of bids converges to $\underline{\mathbf{b}}^*$ from any initial condition, as long as γ is small enough to ensure that $|\gamma \left(\sum_i U_i''(X(B))[X'(B)]^2 + U_i'(X(B))X''(B)\right)| < 1$ for $B^* \leq B \leq B^* + \gamma$.

Proof of Theorem 4.2:

Adding (7) over the set of users gives

$$B(k+1) \;=\; B(k) + \gamma \left(\sum_i U_i'(X(B(k)))X'(B(k)) - 1 \right)$$

Let $G(B) = \sum_i U_i'(X(B))X'(B) - 1$. Note that $G(B)$ is a strictly decreasing function of B, that $G(B^*) = 0$, and that $G(B) \geq -1$ for all B. Note also that the above condition on γ can be restated as $|\gamma G'(B)| < 1$ for $B^* \leq B \leq B^* + \gamma$.

Suppose now that $B(k) > B^*$ for some k. The fact that $|\gamma G'(B)| < 1$ for $B^* \leq B \leq B^* + \gamma$ implies that $B(k+1)$ will be such that $B^* \leq B(k+1) < B(k)$, which means that $B \to B^*$ for this case.

So suppose now that $B(k) < B^*$ for some k. Then, $G(B(k)) > 0$ and hence $B(k+1) > B(k)$. If $B(k+1)$ is also greater than B^*, then by the argument above $B \to B^*$. Else we have that $B(k) < B(k+1) \leq B^*$, i.e. B has gotten closer to B^*. Thus, in the next step B either *(a)* gets closer to (but remains less than) B^*, or *(b)* exceeds B^* and subsequently decreases monotonically to B^*. Thus $B \to B^*$ for this case as well.

Using the convergence of $B \to B^*$, we can show the convergence of the individual bids $b_i \to b$, as was done in the continuous case. □

V. Multiple Goods

The mechanism presented above has a natural extension to the case when there are multiple public goods to be produced for users who have joint valuation functions. In this section we show that if the production costs are decoupled, then using the mechanism proposed in this paper separately for each good results in efficient joint provisioning of all goods.

Suppose now that there are M public goods, with the vector of quantities denoted by $Q = [Q_1, \ldots, Q_M]$. Each user has a value function $U_i(Q)$, which is assumed to be jointly continuous, differentiable, concave and strictly increasing [2] in each coordinate. The production of each good incurs a cost, as specified by the cost functions $C_m(Q_m)$ for $1 \leq m \leq M$. Each cost function C_m is assumed to be convex, strictly increasing and differentiable. $Q \in \mathbb{R}^M_{++}$ means that each coordinate is strictly positive: $Q_m \in \mathbb{R}_{++}$ for all m.

A vector of quantities Q^* is said to be *efficient* if it maximizes the net social benefit:

$$\sum_i U_i(Q^*) - \sum_m C_m(Q^*_m) \;\geq\; \sum_i U_i(Q) - \sum_m C_m(Q_m)$$

for all $Q \in \mathbb{R}^M_{++}$. It is assumed that there is at least one efficient $Q^* \in \mathbb{R}^M_{++}$ in which each quantity is finite.

With these assumptions, running a separate market for each good results in efficient production. Each user i now submits a vector of bids $b_i = [b_i^1 \ldots b_i^M]$: thus each bid is an M-dimensional vector $b_i \in \mathbb{R}^M_{++}$. As before define, for each good m, the bid sums $B^m \triangleq \sum_i b_i^m$ and $B^m_{-i} \triangleq \sum_{j \neq i} b_j^m$. Let X_m be the inverse function of C_m as in the single good case. For notational brevity, denote the vector of total bids by $B = [B^1, \ldots, B^M]$ and the vector production function by X. Thus $X(B)$ stands for the vector of quantities $[X_1(B^1), \ldots, X_M(B^M)]$. Also, $B_{-i} = [B^1_{-i}, \ldots, B^M_{-i}]$.

Consider the mechanism that, given all the bids, produces quantity $X_m(B^m)$ of each good m and charges user i an amount $\sum_m p_i^m$, where

$$p_i^m(b_i^m, B^m_{-i}) \;=\; b_i^m - B^m_{-i} \log\left(1 + \frac{b_i^m}{B^m_{-i}}\right)$$

is the payment user i makes towards the provisioning of good m. The level of production of each good is thus a local decision, with the users balancing payments across goods so as to maximize their net rewards. For the mechanism as described, the net reward for user i is given by

$$R_i(b_i, B_{-i}) \;=\; U_i(X(b_i + B_{-i})) - \sum_m p_i^m(b_i^m, B^m_{-i})$$

A vector of bid vectors $(\widetilde{b}_1, \ldots, \widetilde{b}_n)$ is a Nash equilibrium if

$$R_i(\widetilde{b}_i, \widetilde{B}_{-i}) \;\geq\; R_i(b_i, \widetilde{B}_{-i}) \quad \text{for all } b_i \in \mathbb{R}^M_{++}$$

[2] The strictly increasing requirement for U_i can be relaxed somewhat, but we will not discuss it here for brevity

As in the single good case, efficient allocations and Nash equilibria are fully characterized by first-order conditions. Thus Q^* is optimal if and only if

$$\sum_i \frac{\partial}{\partial Q_m} U_i(Q^*) \;=\; C'_m(Q^*_m) \;\; \text{for each } m$$

Thus, $(\widetilde{b}_1, \ldots, \widetilde{b}_n)$ is a Nash equilibrium if and only if

$$\frac{\partial}{\partial Q_m} U_i(\widetilde{Q}) \;=\; \frac{\widetilde{b}_i^m}{\widetilde{B}^m} C'_m(\widetilde{Q}_m) \;\; \text{for all } i \text{ and } m$$

where $\widetilde{Q} = X(\widetilde{B})$. This is the multiple-goods analogue of (4), and we can prove the existence and efficiency of Nash equilibria for the multiple goods case in the same way as was done for a single good. We state this as a theorem below and omit the proof.

Theorem 5.1: Consider the model with multiple public goods described in this section with the mechanism $(f^*, p^*_i : 1 \le i \le n)$ used for the provisioning of each good. Then there is a one-to-one correspondence between the set of efficient quantity vectors and the set of Nash equilibria for the game, such that at any of these Nash equilibria the corresponding efficient quantity vector is provisioned.

As in the single good case, if each user updates its bid vector according to gradient ascent in continuous time then there is global convergence to an efficient Nash equilibrium. The update equations are now given by the gradient

$$\frac{d}{dt} b_i \;=\; \bigtriangledown_{b_i} \left(U_i(X(B)) - \sum_m p^m_i(b^m_i, B^m_{-i}) \right) \qquad (8)$$

which is the same as

$$\frac{d}{dt} b^m_i \;=\; \left(\frac{\partial}{\partial Q_m} U_i(X(B)) \right) X'_m(B^m) - \frac{b^m_i}{B^m}$$

The proof of global convergence is similar to that for a single good. We state the theorem below and omit the proof.

Theorem 5.2: For the bid updates given by (8), the vector of bids converges to a Nash equilibrium from any initial condition where for each good m there are at least two users with strictly positive bids for that good.

VI. More General Efficient Mechanisms and Revenue Bounds

The mechanism presented in Section III is one example of a more general class of individually rational mechanisms that all guarantee the existence and efficiency of Nash equilibria, and use $f^*(\underline{\mathbf{b}})$ as given in (2) as their production function. In this section we present this more general class of mechanisms.

One primary reason for exploring a more general class of mechanisms is to optimize the revenue generated at Nash equilibirum. The following proposition gives an upper bound on the revenue of a broad class of mechanisms that use f^* as the production function.

Proposition 6.1: Let user value functions be U_i, C be the production cost function, and consider any mechanism that *(i)* uses $f^*(\underline{\mathbf{b}})$ (as given in (2)) as its production function, and *(ii)* has payment functions such that $p_i(0, \underline{\mathbf{b}}_{-i}) = 0$, and such that $p_i(\underline{\mathbf{b}})$ is convex and continuously differentiable in b_i for fixed $\underline{\mathbf{b}}_{-i}$. If $\widetilde{\underline{\mathbf{b}}}$ is a Nash equilibrium then the payments have to satisfy

$$\frac{\sum_i p_i(\widetilde{\underline{\mathbf{b}}})}{\widetilde{B}} \;\le\; \max_i \frac{U'_i(\widetilde{Q})}{C'(\widetilde{Q})}$$

where $\widetilde{Q} = X(\widetilde{B})$ is the quantity produced at the Nash equilibrium.

Remark: The condition $p_i(0, \underline{\mathbf{b}}_{-i}) = 0$ above ensures that the mechanism is individually rational, and the convexity of $p_i(\underline{\mathbf{b}})$ in b_i ensures that Nash equilibria exist.

Proof of Prop. 6.1:

For each user i,

$$\frac{p_i(\widetilde{\underline{\mathbf{b}}})}{\widetilde{b}_i} \;\le\; \frac{\partial p_i}{\partial b_i}(\widetilde{\underline{\mathbf{b}}}) \;=\; \frac{U'_i(\widetilde{Q})}{C'(\widetilde{Q})}$$

where the inequality holds because $p_i(b_i, \widetilde{\underline{\mathbf{b}}}_{-i})$ is a convex increasing function of b_i with $p_i(0, \widetilde{\underline{\mathbf{b}}}_{-i}) = 0$, and the equality is the first order necessary condition for $\widetilde{b}_i$ to be a Nash bid for user i. The proof of the proposition follows immediately from these relations. □

Consider now a mechanism that satisfies the conditions of Proposition 6.1 and has a Nash equilibrium $\widetilde{\underline{\mathbf{b}}}$ at which an efficient quantity Q^* is produced. Efficiency requires that $\sum_i U'_i(Q^*) = C'(Q^*)$. This immediately implies that if there are two or more users, it is not possible to recover the entire cost $B^* = C(Q^*)$ from the payments. Furthermore, if there are n users with identical value functions then at most $\frac{B^*}{n}$ of the total cost B^* is recovered. The above upper bound can thus be quite small if the number of users is large. This also indicates that efficient mechanisms that require budget-balance must either sacrifice individual rationality, or be applicable to more restricted classes of user value functions.

The design of the class of mechanisms presented below is based on the following realization: if in addition to the conditions of Proposition 6.1 the payment functions satisfy

$$\sum_i \frac{\partial p_i}{\partial b_i}(\underline{\mathbf{b}}) \;=\; 1$$

for all $\underline{\mathbf{b}} \in \mathbb{R}^n_{++}$, then any Nash equilibrium will result in efficient production. Of course, this does not guarantee the existence of Nash equilibria, which needs to be established seperately.

Let $\psi(s)$ be a strictly increasing continuous function from $[0,\infty)$ to $[0,\infty)$, with $\psi(0)=0$. Given a vector $\underline{\mathbf{b}}$ of bids, with $b_i \in \mathbb{R}_{++}$, consider the mechanism given by

$$f^*(\underline{\mathbf{b}}) = X(B) \tag{9}$$

$$p_i(\underline{\mathbf{b}}) = \int_0^{b_i} \frac{\psi(s)}{\psi(s)+\sum_{j\neq i}\psi(b_j)}\,ds \tag{10}$$

In terms of this notation, the mechanism given by (2) and (3) corresponds to $\psi(x)=x$. It can be shown that the reward function R_i is concave in the player's bid b_i, and that $p_i < b_i$, for all i. The following theorem is an analogue of Theore 3.1 and can be proved in a similar way.

Theorem 6.1: For the mechanism given by (9)-(10), there is a one-to-one correspondence between the set of efficient quantities and the set of Nash equilibria. Also, at any of these Nash equilibria the corresponding efficient quantity is provisioned.

The mechanism designer may be interested in maximizing revenue. Alternatively, it may be the designer's objective to provision optimally at minimal cost to the users. The following theorem shows that either objective can be achieved by a suitably designed mechanism from the class described in this section. The result is stated and proved for the case when there is a unique optimal quantity, but it generalizes to the case of multiple optima.

Theorem 6.2: Let user value functions $U_1 \ldots, U_n$ and production cost function C be such that there is a unique optimal quanitity $Q^* > 0$ costing $B^* = C(Q^*)$. Let $p_i^\alpha(\underline{\mathbf{b}})$ be the payment function for the mechanism having $\psi(s)=1-e^{-\alpha s}$, and $p_i^\beta(\underline{\mathbf{b}})$ be the payment function for the mechanism having $\psi(s)=e^{\beta s}-1$. Then, if $\underline{\mathbf{b}}^\alpha$ and $\underline{\mathbf{b}}^\beta$ are the corresponding Nash equilibria,

$$\lim_{\alpha\to\infty}\frac{\sum_i p_i^\alpha(\underline{\mathbf{b}}^\alpha)}{B^*} = \max_i \frac{U_i'(Q^*)}{C'(Q^*)} \tag{11}$$

$$\lim_{\beta\to\infty}\frac{\sum_i p_i^\beta(\underline{\mathbf{b}}^\beta)}{B^*} = 0 \tag{12}$$

Note however that the convergence of either of the above limits is not uniform in the choice of the user value functions.

Proof of theorem 6.2:

If p is a payment function based on ψ from the class given by (9)–(10), then it will have a unique Nash equilibrium $\widetilde{\underline{\mathbf{b}}}$ at which Q^* will be produced. The first-order conditions imply that, for each i

$$\frac{\partial p_i(\widetilde{\underline{\mathbf{b}}})}{\partial b_i} = \frac{\psi(\widetilde{b}_i)}{\psi(\widetilde{b}_i)+\sum_{j\neq i}\psi(\widetilde{b}_j)} = \frac{U_i'(Q^*)}{C'(Q^*)} \tag{13}$$

We will first prove (11). Let $\mathcal{I}$ be the set of users with the highest marginal value at Q^*: $U_i'(Q^*) = U_j'(Q^*) > U_k'(Q^*)$ for all $i,j\in\mathcal{I}$ and $k\notin\mathcal{I}$. For $\psi(s)=1-e^{-\alpha s}$, the above condition means that for all i,j

$$\frac{1-e^{-\alpha b_j^\alpha}}{1-e^{-\alpha b_i^\alpha}} = \frac{U_j'(Q^*)}{U_i'(Q^*)}$$

Since the sum B^α of the bids is equal to B^* for all α, if $i\in\mathcal{I}$ then $b_i^\alpha \nrightarrow 0$. Thus, if $i,j\in\mathcal{I}$ and $k\notin\mathcal{I}$ then $b_i^\alpha = b_j^\alpha$ for all α, and as $\alpha\to\infty$, the bids $\frac{b_k^\alpha}{b_i^\alpha}\to 0$ while $b_i^\alpha \to \frac{B^\alpha}{|\mathcal{I}|}$: in the limit, the users with the highest marginal valuations will contribute equally to the payment, while all others will have no contribution.

More formally, given $\epsilon>0$ there exists an α_1 such that

$$\frac{B^*}{|\mathcal{I}|}-\frac{\epsilon}{2} \le b_i^\alpha \le \frac{B^*}{|\mathcal{I}|}+\frac{\epsilon}{2} \quad \text{for all } i\in\mathcal{I} \text{ and } \alpha>\alpha_1$$

Also, there exists α_2 such that for all $\alpha>\alpha_2$,

$$\frac{1-e^{-\alpha s}}{1-e^{-\alpha\left(\frac{B^*}{|\mathcal{I}|}-\frac{\epsilon}{2}\right)}} \ge 1-\epsilon \quad \text{for all } s\ge\frac{\epsilon}{2}$$

Thus, if $\alpha>\max\{\alpha_1,\alpha_2\}$ then for $i\in\mathcal{I}$,

$$\begin{aligned}
p_i^\alpha(\underline{\mathbf{b}}^\alpha) &= \int_0^{b_i^\alpha}\frac{1-e^{-\alpha s}}{1-e^{-\alpha s}+\sum_{j\neq i}(1-e^{-\alpha b_j^\alpha})}\,ds\\
&= \int_0^{b_i^\alpha}\frac{\frac{1-e^{-\alpha s}}{1-e^{-\alpha b_i^\alpha}}}{\frac{1-e^{-\alpha s}}{1-e^{-\alpha b_i^\alpha}}+\sum_{j\neq i}\frac{U_j'(Q^*)}{U_i'(Q^*)}}\,ds\\
&\ge \int_{\frac{\epsilon}{2}}^{\frac{B^*}{|\mathcal{I}|}-\frac{\epsilon}{2}}\frac{\frac{1-e^{-\alpha s}}{1-e^{-\alpha\left(\frac{B^*}{|\mathcal{I}|}-\frac{\epsilon}{2}\right)}}}{1+\sum_{j\neq i}\frac{U_j'(Q^*)}{U_i'(Q^*)}}\,ds\\
&\ge \int_{\frac{\epsilon}{2}}^{\frac{B^*}{|\mathcal{I}|}-\frac{\epsilon}{2}}\frac{1-\epsilon}{1+\sum_{j\neq i}\frac{U_j'(Q^*)}{U_i'(Q^*)}}\,ds\\
&= \frac{U_i'(Q^*)}{\sum_j U_j'(Q^*)}\left(\frac{B^*}{|\mathcal{I}|}-\epsilon\right)(1-\epsilon)\\
&\ge \frac{U_i'(Q^*)}{C'(Q^*)}\frac{B^*}{|\mathcal{I}|}(1-\epsilon)-\epsilon
\end{aligned}$$

By symmetry, if $i,j\in\mathcal{I}$ then $p_i^\alpha(\underline{\mathbf{b}}^\alpha)=p_j^\alpha(\underline{\mathbf{b}}^\alpha)$. Hence, for

$\alpha > \max\{\alpha_1, \alpha_2\}$,

$$\begin{aligned}\frac{\sum_j p_j^\alpha(\underline{\mathbf{b}}^\alpha)}{B^*} &\geq \frac{\sum_{i\in\mathcal{I}} p_i^\alpha(\underline{\mathbf{b}}^\alpha)}{B^*} \\ &\geq \left(\max_i \frac{U_i'(Q^*)}{C'(Q^*)}\right)(1-\epsilon) - \epsilon\frac{|\mathcal{I}|}{B^*} \\ &\geq \left(\max_i \frac{U_i'(Q^*)}{C'(Q^*)}\right) - \epsilon - \epsilon\frac{|\mathcal{I}|}{B^*}\end{aligned}$$

Thus, given $\delta > 0$, choose ϵ such that $\epsilon\left(1 + \frac{|\mathcal{I}|}{B^*}\right) < \delta$. Then, there exists α_δ such that

$$\frac{\sum_i p_i^\alpha(\underline{\mathbf{b}}^\alpha)}{B^*} \geq \left(\max_i \frac{U_i'(Q^*)}{C'(Q^*)}\right) - \delta$$

for $\alpha \geq \alpha_\delta$ Since proposition 6.1 already establishes the upper bound, this proves (11).

The proof of (12) follows along similar lines. By substituting $\psi(x) = e^{\beta x} - 1$ in (13), we get

$$\frac{e^{\beta b_j^\beta} - 1}{e^{\beta b_i^\beta} - 1} = \frac{U_j'(Q^*)}{U_i'(Q^*)} \text{ for all } i, j$$

As $\beta \to \infty$, this means that $|b_j^\beta - b_i^\beta| \to 0$. However, the sum of the bids, B^β, is equalt to B^* for all β. Hence given ϵ there exists β_1 such that for all $\beta > \beta_1$,

$$\frac{B^*}{n} - \epsilon \leq b_i^\beta \leq \frac{B^*}{n} + \epsilon \text{ for all } i$$

Also, there exists β_2 such that for all $\beta > \beta_2$,

$$e^{\beta s} \leq \epsilon\left(e^{\beta(\frac{B^*}{n} - \epsilon)} - 1\right) \text{ for all } s \leq \frac{B^*}{n} - 2\epsilon$$

Thus, for $\beta > \beta_1, \beta_2$, and any user i,

$$\begin{aligned}p_i^\beta(\underline{\mathbf{b}}^\beta) &= \int_0^{b_i^\beta} \frac{e^{\beta s} - 1}{e^{\beta s} - 1 + \sum_{j\neq i}(e^{\beta b_j^\beta} - 1)}\,ds \\ &\leq \int_0^{\frac{B^*}{n}+\epsilon} \frac{e^{\beta s} - 1}{e^{\beta s} - 1 + \sum_{j\neq i}(e^{\beta b_j^\beta} - 1)}\,ds \\ &\leq \int_0^{\frac{B^*}{n}-2\epsilon} \frac{e^{\beta s}}{\sum_{j\neq i}(e^{\beta b_j^\beta} - 1)}\,ds + 3\epsilon \\ &\leq \int_0^{\frac{B^*}{n}-2\epsilon} \frac{e^{\beta s}}{e^{\beta(\frac{B^*}{n}-\epsilon)} - 1}\,ds + 3\epsilon \\ &\leq \epsilon\left(\frac{B^*}{n} - 2\epsilon\right) + 3\epsilon\end{aligned}$$

Thus the total payment received is bounded by

$$\sum_i p_i^\beta(\underline{\mathbf{b}}^\beta) \leq (B^* + 3n)\epsilon$$

Given δ, choose ϵ so that $(1 + \frac{3n}{B^*})\epsilon < \delta$. Then, there exists β_δ such that for all $\beta > \beta_\delta$,

$$\frac{\sum_i p_i^\beta(\underline{\mathbf{b}}^\beta)}{B^*} \leq \delta$$

The theorem is thus proved. □

VII. Discussion

This paper proposes a class of mechanisms to alleviate the free-rider problem by ensuring efficiency at Nash equilibria of a static game. It then shows that user bids converge to Nash equilibria globally, provided they use myopic update strategies. Using iterative price and bid update procedures for computationally infeasible problems in auctions and resource allocation have been proposed recently for multi-unit auctions where users have bundle bids [14, 15], as well as in the allocation of divisible goods [8, 9]. All these mechanisms give efficiency and truthful revelation guarantees only when users are assumed to follow myopic best response bid updates. The analysis of user dynamics as repeated games in the true sense is hard. Furthermore, in general, it seems unlikely that the efficiency properties shown for static mechanisms will hold when the dynamics of convergence are repeated games. The issue of dynamics is thus a genuine point of criticism for this approach. In the settings of modern information systems however, two comments can be made to partially address this issue. Firstly, it may be that the mechanism is not an honest auction, but rather an implementable algorithm to find efficient allocations in the presence of communication constraints. Secondly, in large distributed settings, finding a viable alternative to best-response dynamics may be hard.

Acknowledgments

The authors would like to thank Prof. Steven Williams for helpful comments and suggestions, and for pointing out relevant existing literature.

References

[1] T. Groves and M. Loeb, "Incentives and public inputs," *J. of Public Economics*, 1975.

[2] J. Green and J. J. Laffont, "Characterization of satisfactory mechanisms for the revelation of preferences for public goods," *Econometrica*, 1977.

[3] T. Groves and J. Ledyard, "Optimal allocation of public goods: A solution to the "free-rider" problem," *Econometrica*, vol. 45, no. 4, 1977.

[4] M. Walker, "A simple incentive compatible scheme for attaining lindahl allocations," *Econometrica*, vol. 49, pp. 65–71, 1981.

[5] F. Kelly, "Charging and rate control for elastic traffic," *European Trans. on Telecom*, 1997.

[6] R. Johari and J. Tsitsiklis, "Efficiency loss in a network resource allocation game," *Mathematics of Operations Research*, 2004.

[7] S. Sanghavi and B. Hajek, "Optimal allocation of a divisible good to strategic buyers," *IEEE Conference on decision and control*, 2004.

[8] S. Yang and B.Hajek, "Revenue and stability of a mechanism for efficient allocation of a divisible good," preprint.

[9] R. T. Maheswaran and T. Basar, "Social welfare of selfish agents: motivating efficiency for divisible resources," *Proceedings of 43rd IEEE Conference on Decision and Control*, 2004.

[10] H. Yaiche, R. Mazumdar, and C. Rosenberg, "A game theoretic framework for bandwidth allocation and pricing in broadband networks," *IEEE/ACM Transactions on Networking*, vol. 8, pp. 667–678, Oct. 2000.

[11] S. Low and D. Lapsely, "Optimization flow control i. basic algorithm and convergence," *IEEE/ACM Transactions on Networking*, vol. 7, pp. 861–874, 1999.

[12] L. Blumrosen and N. Nisan, "Auctions with severely bounded communication," in *Proc. of the 43rd Annual Symposium on Foundations of Computer Science (FOCS'02)*, 2002.

[13] J. Rosen, "Existence and uniqueness of equilibrium points for concave n-person games," *Econometrica*, 1965.

[14] D. Parkes, "ibundle: An efficient ascending price bundle auction," *Proc. ACM Conference on Electronic Commerce (EC-99)*, 1999.

[15] L. Ausubel, P. Cramton, and P. Milgrom, "The clock-proxy auction: A practical combinatorial auction design," *Combinatorial Auctions*, MIT Press, 2005.

PLACE PHOTO HERE

Sujay Sanghavi Sujay has a PhD in ECE, an MS in Math and an MS in ECE, all from the University of Illinois at Urbana-Champaign. He is currently a postdoctoral researcher at LIDS, MIT. He is interested in probability, optimization and algorithms, and their applications to large-scale problems in communications and signal processing.

PLACE PHOTO HERE

Bruce Hajek Bruce Hajek (M'79-SM'84-F'89) received a B.S. in Mathematics and an M.S. in Electrical Engineering from the University of Illinois at Urbana-Champaign in 1976 and 1977, and a Ph. D. in Electrical Engineering from the University of California at Berkeley in 1979. He is a Professor in the Department of Electrical and Computer Engineering and in the Coordinated Science Laboratory at the University of Illinois at Urbana-Champaign, where he has been since 1979. His research interests include communication and computer networks, stochastic systems, combinatorial and nonlinear optimization, and information theory. He served as Associate Editor for Communication Networks and Computer Networks for the IEEE Transactions on Information Theory, as Editor-in-Chief of the same Transactions, and as President of the IEEE Information Theory Society. He is a member of the US National Academy of Engineering and he was a winner of the 1973 USA Mathematical Olympiad. He received the Eckman Award of the American Automatic Control Council, an NSF Presidential Young Investigator Award, an Outstanding Paper Award from the IEEE Control Systems Society, and the IEEE Kobayashi Computer Communications Award.