

Scheduling in Densified Networks: Algorithms and Performance

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Abstract—With increasing data demand, wireless networks are evolving to a hierarchical architecture where coverage is provided by both wide-area base-stations (BS) and dense deployments of short-range access nodes (AN) (e.g., small cells). The dense scale and mobility of users provide new challenges for scheduling: (i) High flux in mobile-to-AN associations, where mobile nodes quickly change associations with access nodes (time-scale of seconds) due to their small footprint, and (ii) multi-point connectivity, where mobile nodes are simultaneously connected to several access nodes at any time.

We study such a densified scenario with multi-channel wireless links (e.g., multi-channel OFDM) between nodes (BS/AN/mobile). We first show that traditional algorithms that forward each packet at most once, either to a single access node or a mobile user, do not have good delay performance. We argue that the fast association dynamics between access nodes and mobile users necessitate a multi-point relaying strategy, where multiple access nodes have duplicate copies of the data, and coordinate to deliver data to the mobile user. Surprisingly, despite data replication and no coordination between ANs, we show that our algorithm (a distributed scheduler – DIST) can approximately stabilize the system in large-scale instantiations of this setting, and further, performs well from a queue-length/delay perspective (shown via large deviation bounds).

Index Terms—Wireless Scheduling, Dense Networks, Multi-point Connectivity.

I. INTRODUCTION

The wireless industry is undergoing a sea change in cellular deployment. From a well-planned macro-cellular setting, the network is evolving to a hierarchical setting with cellular base-stations provide macro coverage (footprint of 1 km or more) and a dense deployment of access nodes (e.g., small cells [31] or femto cells [2], [1]) whose coverage range may be as little as 50 – 100 meters, provides short-range coverage. This combination – macro + dense short-range coverage – popularly referred to as *network densification*, leads to new challenges in network resource allocation.¹

First, the access nodes' small footprints imply that mobile nodes associate and disassociate with them at a much higher rate than previously seen. A car moving at just 30 mph results in hand-offs between ANs at the time-scale of seconds. This will likely worsen with emerging technologies for 5G systems such as millimeter wave (mmWave) Broadband [27], where the radio propagation environment results in highly non-isotropic and direction-dependent short-range

coverage². Thus, to ensure universal coverage, operators have no recourse but to provide a very dense deployment of ANs (especially in locations with high data demand). This leads to second challenge: mobile nodes have the opportunity to associate with several possible ANs at any given time (however, this set changes rapidly over time due to mobility and coverage directionality).

In this paper, we argue that operating these dense networks in a traditional manner, where mobile nodes associate with one AN at any time, and then hands-off to a new one as the environment/location changes, can be inefficient. Instead, we study an approach where data packets are *replicated* at a collection of ANs whose footprints most-likely cover the mobile node, and these ANs deliver packets to the mobile user by making decisions in a decentralized manner using local information. We communicate directly between the base-station and the mobile node *only as a last resort* when the ANs are unable to reach the mobile node (e.g., due to uncertainty in tracking the mobile node, poor location, poor channel rates due to fading). We propose a formal model to capture this setting and analytically show the performance benefits.

Coordination in wireless communication has been studied in various contexts like Distributed/Virtual MIMO [26], [25], Network Coding [20], [18] etc. Importantly, these techniques require coordination at the packet or time-slot level. As discussed in [12], backhaul delays could be much larger than the duration of a time-slot; further with densification, heterogeneity in backhaul delays will likely worsen. Thus, the key differentiating aspect from the above literature is that we consider the setting with delayed or sloppy coordination among the various access nodes. In our setting, access nodes do not have current knowledge of nearby nodes' instantaneous states, or indeed, even knowledge of which mobile nodes are connected to them.

A. Contributions

We study scheduling algorithms for networks with a base-station (BS) and multiple densified access nodes (AN) and multiple mobile users. We assume that the ANs are dense

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²For instance, in a mmWave Broadband system, the human body completely blocks radio propagation [19], [28]. Thus even slight movement (e.g., rotation of the human with the phone) can completely block the mmWave access node from communicating with a mobile node, thus leading to association changes that can occur within fractions of a second.

enough to support multi-point connectivity, i.e., each user can associate with multiple ANs at any given time. We propose an algorithm (DIST) for scheduling and evaluate its performance as detailed below.

- 1) **Algorithm DIST:** We propose a distributed algorithm called DIST where the BS and the ANs make their scheduling decisions independently, based only on local channel and queue-length information. Under the DIST algorithm, the BS forwards each packet to an AN that is currently connected to the intended user. If an AN cannot forward a received packet to the corresponding user because the user is no longer connected to it, unlike traditional algorithms, under the DIST algorithm, the AN forwards copies of packets to multiple ANs around it. In addition, if the ANs fail to deliver a packet to the user within a fixed number of time-slots, it is forwarded directly from the BS to the mobile user.
- 2) **Stability:** Under general arrival and bounded channel processes, we show that if the system scale is large enough, the DIST algorithm keeps the system stable (i.e., Markovian assumptions imply positive recurrence of the queues).
- 3) **Performance:** We have two performance results: (i) We first show that traditional algorithms like the BackPressure algorithm [32] in which the base-station forwards each packet at most once, either to a single access node or a mobile user, do not have good delay performance for mobile users, i.e., the delay rate functions are zero. (ii) For the proposed DIST algorithm, we show that for bounded i.i.d. arrivals and channels, the maximum queue-length rate function is strictly positive and therefore, the queue-length tails decay exponentially. Further, via simulations, we show that the DIST algorithm significantly outperforms the BackPressure algorithm in terms of the delay performance.

B. Related Work

Since the work by Tassiulas and Ephremides [32], there has been great interest in queue-length based scheduling in wireless networks (see [10] for a survey). In the many users/channels context (as in this paper), there has been recent activity to characterize stability, queue-length and delay performance, with and without relays (however without user mobility) [5], [6], [7], [30], [24], [16]. A key insight in these works has been the use of iterative allocations, where queues are updated to account for (partial) channel allocations even within a time-slot.

This paper focuses on the benefits of data replication and multi-point connectivity in a mobile cellular setting (i.e., multiple access nodes maintaining active communications with a mobile user). Such access has had a long history, starting from CDMA soft-handoff (to enable make-before-break voice connections) [33], [34]. More recently, in the setting of COordinated Multi-Point (COMP) [22], there has

been much work at the physical layer to develop cooperative communication strategies between a collection of base-stations and a mobile user. This is especially useful in densified settings, with increased opportunities (many base-stations/access points for coordination) and challenges (more complex interference management). These issues have been studied in various ways including simulations [8], field trials [15], [3], and information-theoretic techniques [11] (see [22] for a survey). In this paper, we focus on network level attributes – queue-lengths and delays – and show that even *local* scheduling algorithms that replicate data can significantly outperform more traditional scheduling algorithms.

Finally, as discussed in the introduction, coordination in wireless networks has a rich history and has been studied in various contexts like Network Coding, Multi-homing, virtual/distributed MIMO etc. See [35] for a detailed discussion of network challenges due to coordination. In this paper, we propose an algorithm which uses only local information, thus obviating the need for coordination between different ANs.

II. SYSTEM MODEL

We consider a two-tiered downlink communication system with a base-station, a large number of ANs and mobile users as shown in Figure 1. We study a multi-channel (e.g. OFDM) setting with a large number of orthogonal channels that can be used for communication simultaneously. This multi-channel setting, but without user mobility, was the focus in [24]. However, the fact that users are mobile and that the network is densified implies that the set of ANs that a mobile node is associated with is not time-invariant; further, a classical time-scale decomposition assumption between mobile-AN association and channel scheduling cannot be easily justified.

From a channel (average) rate perspective, our setting is one where the BS-AN, AN-AN and the AN-user links have higher data-rates than the BS-user links. Again, this is a natural setting to consider because the ANs are expected to be mounted in more suitable locations, as well as have superior hardware in terms of the number of antennas, as compared to the mobile users. Moreover, the ANs that a user is associated with are typically much closer to the user than the central BS.

Formally, the system consists of a base-station and $M(n)$ ANs, where n is the number of users in the system. We assume that the ANs have two RF chains, one to communicate with the BS and the other to communicate with the users and other ANs. As recommended in [1], the BS-AN communication happens at a different spectrum than the BS-user and AN-user communication. To keep the notation simple, we assume that the number of orthogonal frequency channels for BS-AN communication and AN-user communication are n each. This setting was also considered in [24]. Our results can easily be extended for other linear scalings.

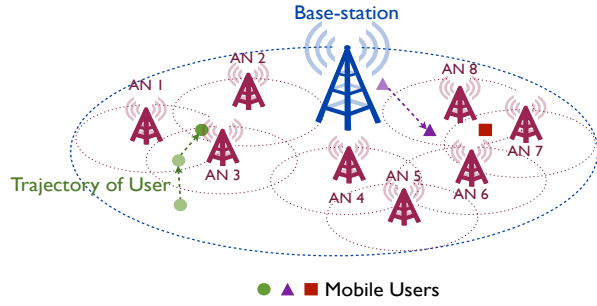


Fig. 1. A wireless network with a base-station, densely deployed ANs and mobile users. The users move in and out of the coverage area of the ANs due to mobility, but are always in the coverage area of the base-station. BS/AN image courtesy: <http://intersales.com.au/GPSNetwork.aspx>; see also [14].

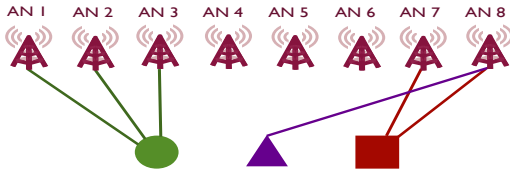


Fig. 2. Association graph between the ANs and mobile users in the network in Figure 1. Each AN is associated with all user that are currently in its coverage range, represented by an edge between the AN and the mobile user. AN image courtesy: <http://intersales.com.au/GPSNetwork.aspx>

A. User Mobility

We use a general notion of mobility which allows both fast moving users that move in every time-slot as well as users which move rarely. Formally, we assume that the probability that a user moves from its current position between two consecutive time-slots is $\Omega(1/\text{poly}(n))$ (at least of the order of $1/\text{poly}(n)$). This assumption allows the expected time spent at a location to be anything between one and a polynomial function of n . For example, the probability that a user moves between two consecutive time-slots can be a constant independent of n as is the case for the Levy-walk process, which is known to be a good model for human mobility in various outdoor settings including college campuses and theme parks [29]. Other popular models, for instance, (discretized versions of) the Random Waypoint Mobility model (RWM) [17] and its variants that have been shown to be more appropriate for user mobility in cellular networks [21], also satisfy this condition.

B. User-AN Connectivity

Since we consider a setting where the ANs are densely deployed, the user is very likely to be connected to multiple ANs. However, we also include the possibility that, in some time-slots, the system fails to obtain the location information of a user. This could happen for various reasons: (i) when a user goes out of the coverage area of the ANs, (ii) when the user is within the communication range of some ANs, but fails to communicate its position to those ANs, or (iii) when a user tracking/position learning algorithm fails.

Specifically, we assume that, at the beginning of each time-slot, the location of the user is known with probability at least $1 - \epsilon(n)$, i.i.d across users. When the user location is known, it is connected to at least $C(n)$ ANs. The density of the ANs in the network can be non-uniform, and therefore, $C(n)$ imposes a lower bound on it. In this paper we work in the setting where $C(n)$ is at least $O(\log n)$. We also assume that the base-station can always communicate with all users albeit at lower (average) rates than the ANs. Figure 2 illustrates the user-AN association graph for the system shown in Figure 1.

1) *Unpredictability of user-AN associations*: Since the users are mobile, the set of ANs a user is connected to can change between two consecutive time-slots. Let $M_u(t)$ be the set of ANs that user u is connected to in time-slot t . We assume that,

- Between two consecutive time-slots, the probability that a connected mobile user, u moves to a new location such that it is no longer associated with a previously connected AN, m is not negligible. Formally, for every t and $m \in M_u(t-1)$,

$$P(m \notin M_u(t)) \geq \mu_1(n),$$

where $\mu_1(n) = \Omega(1/\text{poly}(n))$.

- Further, the motion of the mobile user cannot be predicted with very high accuracy, i.e., for every t and $m \notin M_u(t-1)$,

$$P(m \in M_u(t)) \leq 1 - \mu_2(n),$$

where $\mu_2(n) = \Omega(1/\text{poly}(n))$.

These two conditions are fairly general and are satisfied both by users moving at a very fast time-scale (every time-slot) to users that move rarely ($\text{poly}(n)$ time-slots in expectation). These conditions are also satisfied by the Levy walk process and the RWM model.

2) *User-AN associations in consecutive time-slots*: We consider the setting where the mobility of users is such that there is some overlap between the ANs a user is connected to in two consecutive time-slots. This imposes a restriction on the maximum velocity of the mobile users. Formally, we assume that, for a user u connected to the ANs in time-slots t and $t+1$, $|M_u(t) \cap M_u(t+1)| = \Omega(\log n)$.

3) *Concentration of users around an AN*: In dense networks where each AN has a small footprint, it is unlikely that a large number of users will be connected to any one particular AN. Therefore, we can assume that, with high probability, not more than a constant fraction of the total number of users are connected to a particular AN at the same time. Specifically, if $U_m(t)$ is the set of users connected to AN m in time-slot t , then

$$P\left(\max_{1 \leq m \leq M(n)} |U_m(t)| > n^\nu\right) \leq e^{-bn},$$

for a positive constant $\nu < 1 - \beta$ and a constant $b > 0$. This condition is satisfied, for example, if the users are executing a lazy random walk on the network of ANs independent of other users in the system.

C. Communication between Access Nodes

We consider the setting where each AN can communicate with $O(\log n)$ other ANs located close to it. We assume that the set of ANs that a given AN m can communicate with is large enough so that even if a mobile user connected to AN m in time-slot t moves in the next two time-slots ($t+1$ and $t+2$), AN m can communicate with at least one AN in $M_u(t+2)$.

D. Interference between Access Nodes

Although the dense deployment of ANs enables multi-point connectivity, it can cause interference at the mobile user due to simultaneous transmissions on the same channel. Let I_m be the set of ANs which interfere with an AN m , i.e., no AN in I_m can successfully transmit on the same channel as m . We assume that the interference set for every AN satisfies the following: For all m ,

$$|I_m| \leq n^\beta,$$

for some constant $\beta < 1$. Note that this condition is quite general, and allows for interference sets that grow polynomially in the network size. As a point of reference, spatial stochastic models (where ANs are randomly scattered over the plane), and connectivity as suggested in the Gupta-Kumar model [13] have interference sets that scale only logarithmically in network size, and thus is allowed by our model.

E. Notation

We add to the notation previously used in [24] and [5], [6], [7] to incorporate mobility of users which leads to time-varying user-AN associations. There are n queues at the base-station and ANs (one per user). Our system evolves in discrete time $\{t = 0, 1, 2, \dots\}$, where arrivals happen at the beginning of time-slots, and queues are updated at the end of a time-slot.

- Q_i, R_{mi} = Queue of mobile user i at the BS and at AN m respectively.
- $Q_i(t)$ = BS queue-length of mobile user i at the end of time-slot t .
- $\mathbf{Q}(t) = \{Q_i(t) : 1 \leq i \leq n\}$: BS queue-length vector (across all mobile users)
- $A_i(t), A_i^m(t)$ = Number of packet arrivals for mobile user i at the BS and AN m respectively at the beginning of time-slot t .
- $\mathbf{A}(t) = \{A_i(t) : 1 \leq i \leq n\}$: Arrival vector (across all mobile users).
- $M_i(t)$ = The set of ANs connected to mobile user i in time-slot t .
- $C(n) = \min_{i: M_i(t) \neq \emptyset} |M_i(t)|$ is the minimum number of ANs connected to mobile users whose locations are known.
- $U_m(t)$ = The set of mobile users connected to AN m in time-slot t .
- $X_{i,j}(t)$ = Channel rate (number of packets) for the j -th channel from the BS to mobile user i .

- $X_j^{B,m}(t)$ = Channel rate for the j -th channel from the BS to AN m .
- $X_{i,j}^m(t)$ = Channel rate for the j -th channel from AN m to mobile user i .
- $X_j^{l,m}(t)$ = Channel rate for the j -th channel from AN l to AN m .

In each time-slot, channels are allocated to service appropriate queues; this is captured via the decision variables $Y_{i,j}^{B,m}(t)$, $Y_{i,j}^m(t)$, $Y_{i,j}(t)$ and $Y_j^{l,m}(t)$ for users $1 \leq i \leq n$, channels $1 \leq j \leq n$ and ANs $1 \leq m, l \leq M(n)$ (each of the variables takes the value ‘1’ if it corresponds to an allocation, and ‘0’ otherwise).

Finally, $T_{i,j}(t)$ corresponds to the number of packets transmitted from user i 's queue at the base-station on channel j .

III. MAIN RESULTS AND DISCUSSION

A. Algorithm: DIST

In a system implementing this algorithm, the ANs do not cache packets for more than a fixed number of time-slots (say L). Any packet which arrives to an AN at the beginning of time-slot t is deleted by the AN at the end of time-slot $t+L-1$. The base-station stores all packets which have not reached their destination (user).

For each packet $p \in A_i(t)$ we introduce an indicator variable Z_p which is 1 if the packet reaches the user i by the end of time-slot $t+L$. The queue-length evolution is now given by:

$$Q_i(t) = \left(Q_i(t-1) + F_i(t) - \sum_{j=1}^n X_{i,j}(t) Y_{i,j}(t) \right)^+,$$

where

$$F_i(t) = A_i(t-L-1) - \left| \sum_{p \in A_i(t-L-1)} Z_p \right|.$$

are the packets which arrived at the BS at the beginning of time-slot $t-L$, but, could not be sent to user i by the end of time-slot $t-1$.

We now describe the DIST algorithm (both at the BS and AN).

Base-Station Algorithm: The base-station algorithm proceeds in an iterative manner (see [6] for a detailed discussion of iterative algorithms), allocating one channel at a time. Queue-lengths are updated after each round of allocation. Channel k is allocated in iteration k .

1: Forward New Arrivals to ANs

$$\text{Find } \{i^*, m^*\} \in \underset{1 \leq i \leq n, m \in M_i(t)}{\operatorname{argmax}} A_i^{(k-1)}(t) X_k^{B,m}(t).$$

where $A_i^{(k-1)}(t)$ is the updated (accounting for packets scheduled for transmission on channels from 1 to $k-1$) number of arrivals to user i in time-slot t and $M_i(t)$ is the set of ANs that user i is currently connected to. Packets for user i^* are scheduled for transmission from the base-station to the AN m^* on channel k .

2: Direct Forwarding to Users

If channel k is not used by the base-station to forward new arrivals to the ANs, search for the queue index

$$i^* \in \operatorname{argmax}_{1 \leq i \leq n} Q_i^{(k-1)}(t-1)X_{i,k}(t),$$

breaking ties in the favor of the smaller user index. Allocate channel k to transmit $X_{i^*,k}(t)$ from the queue for user i^* at the base-station directly to user i^* .

3: Update Queue-lengths

Update all queue-lengths before allocating the next channel.

Remarks. *The salient features of DIST (at the Base-station level) are:*

- i. Step 1 – Local Information + Greedy: *Unlike the BackPressure algorithm, the DIST algorithm does not use differential backlogs to make its routing decisions and therefore does not try to balance the load at the ANs. Instead, the algorithm tries to push packets to the ANs in a greedy manner whenever it sees high channel rates.*
- ii. Step 2 – Direct Forwarding over Free Channels: *Unused channels (i.e., unused by BS-to-AN transmissions) are used by the base-station to route packets which are queued at the base-station directly to the users. These packets may have previously been successfully received by one or more ANs, but which failed to forward it to the intended user. The base-station transmits these packets directly to the users.*

Access Node Algorithm: We now describe how each AN carries out the task of channel allocation.

For each AN m , we define two sets:

- $V_m :=$ the set of ANs that AN m can communicate with.
- $D_m(t) := \{u : m \in M_u(t-1) \setminus M_u(t)\}$ be the set of users which were connected to AN m in the previous time-slot, but are not connected to AN m in this time-slot.

Remarks. *Before we formally describe the algorithm, the key features of DIST (at the AN level) are:*

- i. Local Information: *Each AN makes its decisions using local queue-length and channel information (channel rates to from m to ANs in V_m and users connected to AN m).*
- ii. Forwarding Strategy: *For users that are connected to the AN, the AN forwards packets directly to the users. Packets for users that were connected to the AN in the previous time-slot, but are no longer connected to the AN in the current time-slot (users in the set $D_m(t)$), are forwarded to neighboring ANs (ANs in the set V_m).*
- iii. Channel Randomization: *Each AN chooses the channel it transmits on uniformly at random from the set of channels which have the highest channel rate. This can lead to collisions, but, since we work in the large scale multi-channel setting, the expected number of collisions are a vanishing fraction of the supportable load.*

Formally, each AN implements the following steps:

1: Initialize $J = \{1, 2, \dots, n\}$ and $B_u^{l(0)}(t) = A_u^m(t)$ for $u \in D_m(t)$ and $l \in V_m$.

2: *Forward Packets to Connected Users*

If $\max_{i \in U_m(t)} A_i^{m(k-1)} = 0$, $k = 1$ and goto step 4. Else,

$$\{i^*, j^*\} \in \operatorname{argmax}_{i \in U_m(t), j \in J} A_i^{m(k-1)} X_{i,j}^m(t),$$

breaking ties uniformly at random. Allocate channel j^* to serve the queue for user i^* and update $J = J \setminus j^*$.

3: $A_{i^*}^{m(k)} = (A_{i^*}^{m(k-1)} - X_{i^*,j^*}^m(t))^+$, $k = k + 1$, and goto Step 2.

4: *Forward Packets to Neighboring ANs*

$$\{l^*, u^*, j^*\} \in \operatorname{argmax}_{l \in V_m, u \in D_m(t), j \in J} B_u^{l(k-1)} X_j^{m,l}(t),$$

breaking ties uniformly at random. Allocate channel j^* to forward packets for user $u^* \in D_m(t)$ to AN l^* and update $J = J \setminus j^*$.

5: $B_{u^*}^{l^*(k-1)} = (B_{u^*}^{l^*(k-1)} - X_{j^*}^{m,l^*}(t))^+$, $k = k + 1$, and goto Step 4.

B. Stability

The DIST algorithm allows the base-station to retransmit packets which have already been received by one or more ANs. Retransmission can lead to the instability of queues in the system, but we show that under some reasonable assumptions on the channel and arrival processes, the DIST algorithm stabilizes the queues in the system. These assumptions are analogous to those in [24] (see also [9], [6]), with the natural additions to account for user mobility.

Assumption (1.1). *(Bounded Channel Processes)*

- *The channel processes are i.i.d. across time-slots (and independent of the arrival process).*
- $X_{i,j}^{B,m}(t) \leq C_{max} < \infty$.
- $X_{i,j}^m(t) \leq C_{max} < \infty$.
- $X^{m,l} \leq C_{max} < \infty$.
- $X_{i,j}(t) \leq C_{max}^d < C_{max} < \infty$.
- *For every j and user i in time-slot t ,*

$$P(X_{i,j}(t) = C_{max}^d) \geq q_1^{(C_{max}^d)} > 0.$$

- *For every j , t and user i connected to AN m in time-slot t ,*

$$P(X_{i,j}^m(t) = C_{max}) \geq q_2^{(C_{max})} > 0.$$

- *For every j , t and every AN l which can communicate with AN m ,*

$$P(X_j^{m,l}(t) = C_{max}) \geq q_3^{(C_{max})} > 0.$$

Assumption (1.2). *(Arrival Process)*

- *We assume that $\mathbf{A}(t)$ (arrival vector per time-slot) is an aperiodic, irreducible, finite state Discrete Time Markov Chain.*
- $A_i(t) \leq \kappa(n)$ such that $\kappa(n)\epsilon(n) = o(1)$ and $\kappa(n)n^\nu n^\beta = o(n^\alpha)$ for some $\alpha < 1$.

- We define the load $\lambda = \frac{1}{n} E \left[\sum_{i=1}^n \left[\frac{A_i(0)}{C_{max}} \right] \right]$. Then,

$$P \left(\sum_{i=1}^n \left[\frac{A_i(0)}{C_{max}} \right] = n(\lambda + \delta) \right) = o \left(\frac{1}{n} \right),$$

for any $\delta > 0$.

Recall that $\epsilon(n)$ is the probability that a user cannot be located by the ANs in a time-slot; thus requiring the BS to use a (lower rate) channel to directly transmit packets to the mobile. Clearly, as $\epsilon(n)$ increases, $\kappa(n)$ has to decrease to maintain stability of the system. The assumption $\kappa(n)\epsilon(n) = o(1)$ quantitatively captures this effect. For example if $\epsilon(n) = 1/\sqrt{n}$, users can have up to $o(\sqrt{n})$ arrivals in a time-slot.

Recall that w.h.p., the number of users connected to an AN in a time-slot is less than n^ν and the size of the interference set for each AN is at most n^β . Therefore, $\kappa(n)$ has to be small enough to ensure that using n channels, it is possible for each AN to forward all incoming packets to the corresponding users or other ANs without coordinating with other ANs, yet the number of collisions in each time-slot is a vanishing fraction of the total load on the system.

Assumption (1.3). (*Base-station to AN Channel Process*)

Consider the event F_1 that for channel j , $X_j^{B,m} < C_{max}$ for all ANs user i is connected to in time-slot t . This is equivalent to saying that in time-slot t , channel j cannot be used at rate C_{max} by the base-station to forward packets for user i to the ANs. Then,

$$P(F_1) = o \left(\frac{1}{n^2} \right).$$

Assumption (1.4). (*AN to Users Channel Process*)

For an AN m and user i connected to AN m , consider the event F_2 that there exist at least $n \frac{q_2^{(C_{max})}}{2}$ channels such that $X_{i,j}^m(t) = C_{max}$ for each channel. Then,

$$P(F_2^c) = o \left(\frac{1}{n^3} \right).$$

Assumption (1.5). (*AN to AN Channel Process*)

For an AN m which can communicate with AN l , consider the event F_3 that there exist at least $n \frac{q_3^{(C_{max})}}{2}$ channels such that $X_j^{m,l}(t) = C_{max}$ for each channel. Then,

$$P(F_3^c) = o \left(\frac{1}{n^3} \right).$$

Assumption (1.6). (*Base-station to Users Channel Process*)

- Let I be a set of users such that that $|I| \geq kn$, for some constant $k < 1$. Consider the event F_4 that for a channel j and for every user $i \in I$, $X_{i,j}(t) < 1$, $\forall i$. Then,

$$P(F_4) = o \left(\frac{1}{n^3} \right).$$

- Let I be a set of user such that that $|I| = kn$ for some constant $k < 1$ and let J be a set of channels such that $|J| = \frac{2kn}{q_1^{(C_{max})}}$, where

Consider the event F_5 that for every relay in I there exist kn channels in J such that $X_{i,j}(t) = 1$. Then,

$$P(F_5^c) = o \left(\frac{1}{n^3} \right).$$

For instance, these assumptions (1.3 – 1.6) are satisfied by i.i.d. Bernoulli(q) channels, or more generally, by correlated (across users) channels that have a spatial correlation decay property (modeled via the α -mixing condition [4]).

Theorem (1). *If the load $\lambda > 1$, no algorithm can stabilize the queues (i.e., render the queue to be positive recurrent).*

Theorem (2). *Under Assumption 1, for a given load $\lambda < 1$, there exists $n_0(\lambda)$ such that for all $n > n_0(\lambda)$, the Markov Chain corresponding to the queue-lengths at the base-station and access nodes is positive recurrent.*

Thus, for n large enough, the DIST algorithm stabilizes the system for all loads $\lambda < 1$. Further, this is tight in the sense that beyond $\lambda = 1$, we cannot stabilize the queues by any means. This result is interesting because user mobility and collisions at the second hop (AN-user links) lead to retransmissions of those packets by the base-station and yet in the large-scale setting, the DIST algorithm keeps the system stable.

The proof leverages the fact that as the system scale increases, even if a user moves, at least 1 AN that the user is currently connected to has a copy of all the packets which arrived at the base-station less than L time-slots before the currently time-slot. Therefore, even if the user changes its position, it can receive packets from the ANs it is currently connected to. Moreover, as the number of channels increases, there are sufficient degrees of freedom in the system to ensure that the number of collisions is a vanishing fraction of the supportable load. Therefore, for a given load, as the system scale increases, there is sufficient additional capacity in the system to retransmit packets which are lost due to collisions and directly forward packets from the base-station to those users whose location information is not known. Therefore, we conclude that, in large scale systems, the benefits of multi-point connectivity can be achieved without the overhead of coordination.

C. Performance

1) *Single Transmission Algorithms:* We first characterize the performance of a class which we refer to as Single Transmission algorithms. An algorithm belongs to this class if it satisfies the following two conditions:

- Each packet is transmitted successfully by the base-station at most once i.e. once the intended receiver (AN/user) of a packet receives it successfully, the base-station deletes that packet from its queue.
- Each AN forwards a received packet only to the corresponding user.

This class of algorithms includes the BackPressure algorithm [32] which is known to be throughput optimal for multihop systems. Iterative versions of the BackPressure algorithm and the MaxWeight algorithm were proposed for multi-channel systems in [24] and were shown to have good buffer-usage or delay performance for system in which users are not mobile. These algorithms too belong to the ST class of algorithms. The next theorem characterizes the performance of algorithms belonging to the ST class for mobile users.

Theorem (3). *For a mobile user in a system implementing an ST algorithm, the delay for a packet that is routed to an AN by the base-station is such that*

$$d := \limsup_{n \rightarrow \infty} \frac{-1}{n} \log P(\text{Delay} > b) = 0,$$

for any $b < \infty$.

We thus conclude that traditional algorithms like BackPressure/MaxWeight [32] do not have good delay performance for mobile users.

2) *DIST*: We study the buffer overflow probability for the largest queue at the base-station:

$$r := \liminf_{n \rightarrow \infty} \frac{1}{b+1} \frac{-1}{n} \log P\left(\max_{1 \leq i \leq n} Q_i(0) > b\right).$$

This value of r is a bound on the rate of decay of the longest queue (large deviations rate function). Note that the queues at the access nodes delete packets within a small number of time-slots; thus stability or performance of these access node queues is not the focus here.

If an algorithm results in a positive value of r , then we have that (neglecting constants outside the exponent)

$$P\left(\max_{1 \leq i \leq n} Q_i(0) > b\right) \approx e^{-rn}.$$

Therefore, the probability that the system has any backlogged packets goes to zero very quickly which means that all packets that enter the system as served almost immediately, thus leading to low delay.

We analyze the performance of *DIST* for a restricted set of arrival and channel processes.

Assumption (2). *(Multi-level Bounded Arrivals and Channels)*

- $A_i(t) = k$ w.p. p_k for $0 \leq k \leq K$ and 0 otherwise
- $X_{i,j}^{B,m}(t) = c$ w.p. $q_1^{(c)}$ for $0 \leq c \leq C_{max}$ and 0 otherwise.
- $X_{i,j}^m(t) = c$ w.p. $q_2^{(c)}$ for $0 \leq c \leq C_{max}$ if user i is connected to AN m and 0 otherwise.
- $X_j^{m,l}(t) = c$ w.p. $q_3^{(c)}$ for $0 \leq c \leq C_{max}$ if AM m can communicate with AN l and 0 otherwise.
- $X_{i,j}(t) = 1$ w.p. q_4 and 0 otherwise.
- $\epsilon(n) = o(1)$.

The arrival and channel processes are i.i.d. across users, ANs and time-slots. In addition we assume that $C(n) \geq 2 \log n$.

Theorem (4). *Under Assumption 2, for the *DIST* algorithm, for any integer $b \geq 0$,*

$$r = \liminf_{n \rightarrow \infty} \frac{-1}{n} \log P\left(\max_{1 \leq i \leq n} Q_i(0) > b\right) > 0.$$

From this theorem we conclude that under Assumption 2, using multi-point connectivity, good buffer-usage performance can be achieved without the overhead of multi-point coordination.

Like the proof of Theorem 2, this proof too leverages the fact that as the system scale increases, multi-point connectivity and the large number of channels ensure that the number of collisions is small and direct retransmission of packets from the base-station to users ensures that no packets stays in the system for too long.

IV. PROOF OUTLINES

In this section, we provide proof outline for Theorem 2. For the detailed proof of Theorem 2 and other proofs, please refer to [23].

A. Stability of *DIST* (Theorem 2)

Stability of multihop systems has been studied in literature in numerous settings, [24] being closest to the setting in the paper. In [24], stability of a static multihop system (no user mobility) for an iterative version of the MaxWeight algorithm was proved in a sequential manner by first showing the stability of base-station queues followed by showing that the relay queues are also stable. The reason why such a decoupling is possible in [24] is that the MaxWeight algorithm is an ST algorithm and therefore, once a packet is forwarded by the base-station to a relay/user, it is deleted from the queue at the base-station. The queue process at the base-station is therefore independent of the packet transmissions at the second hop (relay-user links). However, for the *DIST* algorithm, every packet in the system which has not reached its final destination (user) is queued the base-station even if it has been forwarded to the ANs. This couples the queue processes at the base-station with the channel allocation at the second hop (AN-user links).

Therefore, unlike [24], where stability was proved in a sequential manner, we have to analyze the entire system at once which requires a different proof structure. Moreover in our setting, since all packets which have not reached their destination (user) are queued at the base-station, it suffices to show that the base-station queues are stable in order to show stability of the system.

Apart from this key difference, the analysis of *DIST* has three other new aspects.

- 1) *Dealing with missing user location information*: Unlike settings considered previously, we deal with users whose location is sometimes unknown. We show that there is sufficient unused capacity for *DIST* to directly forward packets to such users (see also (3) below).
- 2) *Decentralized nature of *DIST**: The ANs forward packets received to connected users and scheduling decisions are made in a distributed manner. This can

lead to two bad events: (i) there are packets which no AN forwards to a user, and (ii) due to collisions, packets are not received successfully by the users. We show that for the DIST algorithm, the number of such bad events in the second hop (AN-user links) is $o(n)$ with probability $\geq 1 - o(e^{-n})$.

- 3) *Splitting packets at the BS into new and old packets:* The base-station forwards new arrivals to the ANs and old packets (packets that arrived more than L time-slots before the current time-slot) directly to the users. We show that all new arrivals for users whose location is known are forwarded to the ANs by the BS in a given time-slot with probability ($\geq 1 - o(1/n)$). We then show that there is sufficient additional capacity in the system (channels unused by the BS-AN links) to ensure that all packets that arrived L slots before the current time-slot t , and which could not be forwarded by the ANs either due to collisions due to the decentralized nature of DIST or because the location of those users was not known can be sent directly from the base-station to the users in time-slot t .

Using these properties of the DIST algorithm, we show that on average, the base-station queues can serve more packets than they receive in a time-slot (accounting for both new arrivals and old packets that re-enter the base-station queues because the ANs fail to forward them to the users). We then use the standard Foster's Lyapunov technique for Markov Chains (with a quadratic Lyapunov function) to show stability. In other words, for a given load λ , there exists a constant n_0 such that the DIST algorithm ensures that the base-station queues in a system with $n > n_0$ channels are positive recurrent.

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