

Using Symbolic Canceling to Improve Diagnosis from Compacted Response

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Abstract

This paper addresses the problem of performing diagnosis using production test results in a test compression environment. For linear response compactors, such as multiple-input shift register (MISRs), diagnosis must be performed from signatures. The key idea in this work is to use symbolic canceling in MISR signatures to extract information from both the MISR signature bits with errors as well as those that are error-free to provide more precise diagnostic information. A fundamentally new technique for precisely identifying error locations for propagation cones reaching fewer scan cells than the size of the MISR is described. The proposed approach does not require any additional hardware or extra data to be collected. It uses off-line software-based processing (symbolic simulation combined with Gaussian elimination) to extract information from signatures to deduce error locations even when there are a large number of errors. Unlike existing techniques for diagnosis from signatures, the proposed diagnosis approach can be used even when there are unknown (X) values in the output response. Experimental results demonstrate the reductions in suspect set size that can be obtained with the proposed techniques.

1. Introduction

An important task for high volume manufacturing is performing diagnosis using production test results for yield analysis. With uncompact output response, the exact location of all captured errors is known. However, when test compression is employed, the output response is returned to the tester in compacted form, thus error locations are not directly available. The more highly compacted the output response is, the more difficult the task of trying to perform diagnosis.

The problem addressed in this paper is diagnosis in a test compression environment where the compacted response is typically for individual scan vectors. This differs from diagnosis in a built-in self-test (BIST) environment where the compacted response may be across the whole test set. The proposed approach can be applied for any linear response compactor where each compacted response bit is an XOR combination of scan

cells (i.e., XOR of the response values captured in the scan cells).

This includes both combinational and sequential linear decompressors. Most notably it includes multiple-input shift registers (MISRs) which are widely used and present a significant challenge for diagnosis. Without loss of generality, the paper will focus on diagnosis from MISR signatures, but the same principles apply for any linear compactor.

The key idea in this work is to exploit the fact that MISR signature bits can be XORed together to generate new linear combinations of scan cells. By doing the same XOR operations on both the faulty and fault-free MISR signature, one can determine whether an odd number of errors were captured in different combinations of scan cells. By carefully selecting which MISR bits to XOR together using Gaussian elimination, the effects of selected scan cells can be canceled. This can be used to vindicate suspects during diagnosis. Moreover, by doing this in a systematic manner taking structural information into consideration, precise error locations can be determined.

Techniques for increasing the amount of diagnostic information that is extracted from signatures are presented along with a fundamentally new technique for identifying error locations. *The proposed approach does not require any additional hardware or extra data to be collected.* It uses off-line software-based processing (symbolic simulation combined with Gaussian elimination) to extract information from signatures to deduce error locations even when there are a large number of errors. Unlike existing techniques for diagnosis from signatures, the proposed diagnosis approach can be used even when there are unknown (X) values in the output response.

The paper is organized as follows: Sec. 2 describes related work in diagnosis from compacted response. Sec. 3 describes how symbolic simulation can be used to obtain the scan cell dependence of each MISR signature bit. Sec. 4 explains how canceling can be used to reduce a suspect set. Sec. 5 describes how precise error locations can be found for propagation cones sufficiently smaller than the size of the MISR. Sec. 6 puts everything together to give the overall proposed diagnosis procedure. Sec. 7 shows the experimental results that have been obtained. Sec. 8 is a conclusion.

2. Related Work

A number of techniques have been proposed for extracting the error locations from compacted output responses for different types of compactors. In [Stanojevic 05], it was shown that for X-compact [Mitra 04], where a combinational XOR network is used for output compaction, if only one error occurs in a particular scan slice, it can be uniquely located. However, if multiple errors occur in a particular scan slice, a heuristic statistical procedure is proposed to find the most likely locations of the errors, but the exact locations may not be determined. If one or more X's is present in a particular scan slice, then the ambiguity of which locations captured errors is increased. A method to add additional outputs to improve diagnosis at the cost of reduced compression is described in [Stanojevic 05]. In [Mrugalski 07], a diagnosis technique for convolutional compactors [Rajski 05] is described. It uses a heuristic branch-and-bound algorithm to try to find the most likely error locations, but the exact locations are not guaranteed to be found. For extreme space compaction where a single bit is generated per scan slice, diagnosis procedures are described in [Holst 09] and [Ye 11].

Schemes for diagnosis from MISR signatures are described in [Keller 05] where signature data is sent to the tester in every scan shift cycle, and [Cheng 06] where signature data is sent once per scan out. Note that the method in [Cheng 06] utilizes the direct diagnosis strategy outlined in [Cheng 04] and [Leininger 05]. In direct diagnosis, conventional logic diagnosis methods are used by considering the compactor as part of the design itself. In [Benware 10], an orthogonal response compaction method is proposed. This method generates two signatures, one combinational and one sequential, and is able to provide better accuracy when more scan cells capture errors, but at the cost of less compression. Moreover, it still relies on a heuristic procedure that is not guaranteed to find the exact locations, and also requires that no X's reach the compactor. A methodology for diagnosing multiple faults using superposition is described in [Cook 14].

The ideas proposed in this paper can be used together with most of the previous techniques for diagnosis from MISR signatures to improve the overall precision as will be discussed.

3. Symbolic Simulation

The proposed approach for extracting error locations from signatures involves using symbolic canceling (as used in [Touba 07] for X-canceling). The value in each scan cell is represented with a symbol. An example is shown in Fig. 1. Once the output response has been shifted in to the MISR using symbolic simulation, the

final MISR signature bits can be expressed in terms of the symbols. Each MISR bit is represented by a linear equation of the scan cell symbols. Fig. 1 illustrates this symbolic representation. The final value of the top bit of the MISR is $c_1 \oplus c_3 \oplus c_8 \oplus c_{13}$, where c_i denotes the value of the i -th scan cell. To keep the example small, Fig. 1 shows only three scan slices being compacted in one MISR signature, but note that any number of scan slices could be compacted.

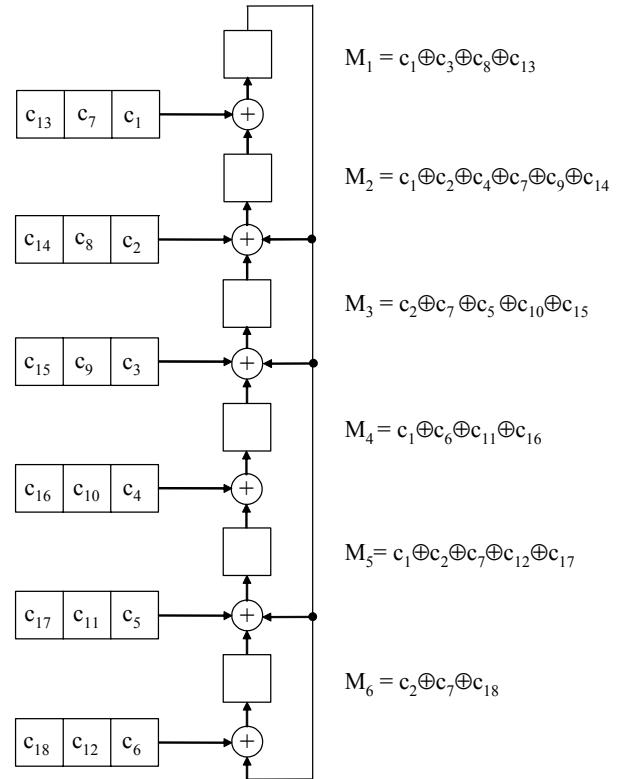


Figure 1. Example of Symbolic Simulation to obtain MISR Linear Equations

c_1	c_4	c_7			c_1	c_4	c_7	
1	0	0	M_1		1	0	0	M_1
1	1	1	M_2		0	1	0	$M_1 \oplus M_2 \oplus M_3$
0	0	1	M_3		0	0	1	M_3
1	0	0	M_4		0	0	0	$M_3 \oplus M_6$
1	0	1	M_5		0	0	0	$M_1 \oplus M_3 \oplus M_5$
0	0	1	M_6		0	0	0	$M_1 \oplus M_4$

Figure 2. Example of Using Gauss-Jordan Elimination to Find Linearly Dependent Combinations of MISR bits with respect to c_1 , c_4 , and c_7

If a particular MISR bit has an error (differs from the value in the fault-free MISR signature), then an odd number of scan cells that it depends on have captured errors (since an even number of errors would cancel out when XORed together). Existing methods for performing diagnosis from signatures use this fact to deduce suspect fault locations. This is done by performing critical path tracing starting from each scan cell that the MISR bit depends on. The set of suspect fault locations is equal to the union of all gates on all critical paths for all scan cells that a MISR bit with an error depends on. Assuming the same fault caused all MISR bits with errors, the suspect fault locations for each MISR bit with an error can be intersected to produce the final set of suspect fault locations. While this approach produces a valid set of suspect fault locations, it may not fully utilize all the information encoded in the MISR signature as will be shown here. In particular, it does not take advantage of the information that can be obtained from the fault-free MISR signature bits. The set of suspect fault locations can be reduced by making use of this information as well which can be harnessed through symbolic canceling.

As a simple illustration of how the fault-free MISR signature bits can be used to help narrow down the set of suspect fault locations, consider the example in Fig. 1. Suppose after compaction, MISR bit M_3 has an error, but M_6 is fault-free. Since $M_3 = c_2 \oplus c_7 \oplus c_5 \oplus c_{10} \oplus c_{15}$ has an error, it is known that at least one of $c_2, c_7, c_5, c_{10},$ and c_{15} has captured an error, so the union of all gates on their critical paths can be taken to form the set of suspects as existing methods would do. However, since M_6 is fault-free, M_3 and M_6 can be XORed together to create a new equation, $M_3 \oplus M_6 = c_5 \oplus c_{10} \oplus c_{15} \oplus c_{18}$ which also will have an error. Thus, it is known that at least one of $c_5, c_{10}, c_{15},$ and c_{18} has captured an error. The union of all gates on their critical paths can be taken and then intersected with the previous set of suspects obtained earlier by considering only M_3 . This will help to further narrow the set of suspect fault locations. This process could be repeated for all fault-free MISR bits and in fact for all linear combinations of MISR bits. Given this ability to generate many linear combinations of MISR bits that have either captured errors or are fault-free, the question is how to maximally exploit this information to minimize the final set of suspect fault locations. The idea proposed here for doing this uses concepts developed in earlier work on X-canceling MISRs [Touba 07].

4. Using Canceling to Reduce Suspect Set

As shown in earlier work on X-canceling MISRs [Touba 07], it is possible to cancel out the effects of a set of scan cells by finding linearly dependent combinations of MISR bits with respect to those scan cells. This is illustrated in the example in Fig. 2 where the effects of

scan cells $c_1, c_4,$ and c_7 are canceled out using Gauss-Jordan elimination [Cullen 97]. The last three rows of the matrix after Gaussian elimination now have all 0's in the columns corresponding to scan cells $c_1, c_4,$ and c_7 and hence correspond to linearly dependent combinations. Thus the last three rows correspond to combinations of MISR bits that when XORed together cancel out all the effects of the first three scan cells, but still depend on the remaining scan cells. Now consider the case where only scan cells $c_1, c_4,$ and c_7 capture errors, but all the other scan cells do not capture errors. Then the last three rows of the matrix in Fig. 2 correspond to combinations of MISR bits that cannot have errors. In other words, if the actual signature containing errors obtained in the MISR is used and M_3 and M_6 are XORed together, $M_3 \oplus M_6$, the errors will cancel out, and the resulting value will match the fault-free signature with $M_3 \oplus M_6$ XORed together. The same is true for $M_1 \oplus M_3 \oplus M_5$ and $M_1 \oplus M_4$. If all three of these linearly dependent MISR bit combinations are equal to the fault-free value, then it has been shown that errors in some combination of scan cells $c_1, c_4,$ and c_7 can explain all observed MISR bit errors. However, that doesn't mean there cannot be other combinations of scan cell errors that could also explain all observed MISR bit errors. On the other hand, if one or more of the three linearly dependent MISR bit combinations does not equal the fault-free value, then it is known for sure that all observed errors cannot be explained by any combination of errors in $c_1, c_4,$ and c_7 . This is very helpful information for diagnosis because it proves that any fault whose forward propagation cone is limited to scan cells $c_1, c_4,$ and c_7 cannot explain the observed MISR signature. Such a fault could only cause error combinations in those three scan cells, but when those three scan cells are canceled out of the signature, errors are still present in the signature.

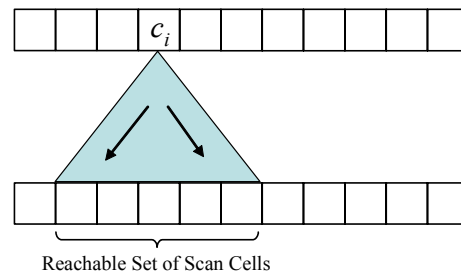


Figure 3. Example of Propagation Cone Originating from Scan Cell c_i

Using this concept, a fast and accurate diagnostic procedure can be constructed. First, the maximum size propagation cones, namely those constructed from each input (either primary input or pseudo primary input, i.e.,

scan cell output), are used as the starting point. This is illustrated in Fig. 3. All single location fault propagation cones are contained in these largest cones. If the size of the MISR is m bits, assume for the moment that no propagation cone reaches more than say $m-16$ scan cells (e.g., if a 128 bit MISR is used, no propagation cone reaches more than 112 scan cells), the case where this is not true is discussed later. For each of these propagation cones, the scan cells that are reachable are canceled from the MISR signature, and if any errors remain in the MISR signature after canceling, then all faults contained in the propagation cone are vindicated and can be dropped from the suspect list. Only the faults in the propagation cones for which canceling all reachable scan cells eliminated all errors from the MISR signature remain in the suspect list. However, even more precise information can be obtained as will be explained in the next section.

The reason why the number of scan cells needs to be less than m is because it guarantees that there will be linearly dependent combinations of MISR signature bits when Gaussian elimination is performed. It ensures that the number of rows in the matrix illustrated in Fig. 2 is more than the number of columns (i.e., more equations than variables). If the number of reachable scan cells is $m-x$, then there will be at least x linearly dependent MISR bit combinations after performing Gauss-Jordan reduction. This is explained in greater detail in [Touba 07].

5. Determining Precise Error Locations

When compacting n scan cells into an m MISR signature bits, the number of possible combinations of scan cell errors that can occur is 2^n-1 (all possible combinations of n scan cells minus the fault-free case). Assuming the MISR is properly designed with a primitive polynomial and phase shifter, the probability of each possible MISR signature occurring is roughly equal. So the number of scan cell error combinations that will give the same signature is:

$$\frac{\text{error combinations}}{\text{MISR signatures}} = \frac{2^n - 1}{2^m}$$

In the general case where output response is being compressed, n will be much larger than m such that there are many possible error combinations that will produce the same faulty signature. So it is ambiguous as to which error combination is the one actually occurring. However, an interesting observation can be made. Suppose a fault propagation cones reaches fewer than m scan cells, i.e., $m-x$ scan cells. If a fault in that cone is causing the faulty signature, the probability that another error combination in that propagation cone other than the

actual one will produce the same faulty signature is equal to:

$$1 - \left[\frac{(2^m - 1)}{2^m} \right]^{2^{m-x}} \approx 1 - \left[\frac{1}{e} \right]^{2^{-x}}$$

Consider the case where $x=16$, then the aliasing probability is equal to:

$$1 - \left[\frac{(2^m - 1)}{2^m} \right]^{2^{m-16}} \approx 1 - \left[\frac{1}{e} \right]^{2^{-16}} = 1.5 \times 10^{-5}$$

This aliasing probability is negligibly small, and it can be assumed that there is only one subset of scan cells that a propagation cone reaches that can explain the erroneous signature. This subset can be found very quickly with a linear search. This is done by starting with the full set of scan cells that the propagation cone reaches as the initial set of scan cells that capture errors and removing one candidate scan cell at a time and seeing if the resulting linearly dependent MISR bit combinations still have no errors. If an error now appears, then the candidate scan cell must be added back to the set, otherwise if no errors appear, it can be safely removed from the set since it is not capturing an error.

At the end of this procedure, which requires $O(m)$ Gauss-Jordan eliminations for an m -bit MISR, the exact minimum suspect set of scan cells (assuming no signature aliasing) that capture errors for a propagation cone for which canceling all reachable scan cells eliminated all errors from the MISR signature has been obtained. A simple check can even be made to verify that there are no additional aliasing subsets by canceling all scan cells in the propagation cone (no error will appear), and then not canceling one of the suspect error capturing scan cells at a time and verifying that an error appears in the linearly dependent MISR bit combinations each time one of the suspect error capturing scan cells are not canceled.

Once the error capturing scan cells for a propagation cone that can explain the faulty signature have been determined, conventional logic diagnosis tools (for uncompact outputs) can be used to identify all suspect fault locations.

Note that all of the processing in terms of selecting which scan cells to cancel out and actually computing the XOR of different combinations of MISR bits can be done off-line during diagnosis and does not require any on-chip hardware support. Another advantage of symbolic canceling based diagnosis is that it can easily handle X's in the output response as well. Any scan cell that captures an X can simply be included in the set of scan cells that

are canceled when performing the diagnosis. This eliminates the impact of the X on the diagnosis procedure.

The runtime complexity for the procedure is $O(m^4)$ where m is the size of the MISR. This comes from the fact that the matrix has $O(m)$ columns and $O(m)$ rows, so Gauss-Jordan Elimination runs in $O(m^3)$, and the number of times this needs to be done is $O(m)$ for a propagation cone reaching fewer than m scan cells. Thus, the run-time complexity scales only with the size of the MISR which is a nice property.

Of course the procedure described in this section for finding precise error locations is only applicable for propagation cones that reach say $m-16$ scan cells or fewer. As the size of the propagation cone gets closer to m or exceeds m , the probability of having more than one set of error locations that can explain the faulty MISR signature increases and is no longer negligible. In the next section, the issue of how to handle larger propagation cones will be discussed.

6. Overall Diagnosis Procedure

In this section, the overall diagnosis procedure is described. Given a design, the first step is to perform symbolic simulation to find the linear equations for the MISR signature bits in terms of the scan cells. The next step is to find the set of reachable scan cells for the maximum sized propagation cones which originate from each primary input and pseudo-primary input (i.e., outputs of scan cells). For the propagation cones that reach $m-16$ scan cells or less, Gaussian elimination is performed to find linearly dependent combinations of MISR signature bits which cancel out all the reachable scan cells. For propagation cones originating from a primary input or pseudo-primary input that reach more than $m-16$ scan cells, the outputs of the first-level of logic gates reached can be used as starting points for propagation cones and each traversed gate is marked. For any of these propagation cones that reach more than $m-16$ scan cells, this process can be recursively performed for each unmarked gate. Note that by marking the gates, the number of propagation cones considered cannot exceed the number of gates in the circuit. At the end of this process, a set of propagation cones and their corresponding linearly dependent combinations of MISR signature bits has been determined. Note that faults whose propagation cone reaches more than $m-16$ scan cells will not reside in any of these propagation cones. All the steps mentioned so far are done one time for the design, and the results are saved and reused each time diagnosis is performed. The remainder of the diagnosis procedure is done each time diagnosis is performed.

For each faulty signature, the linearly dependent combinations of MISR signature bits for each propagation cone are evaluated (XORed together) for both the faulty

signature and fault-free signature. If the faulty and fault-free computed values match, then a fault in that propagation cone can completely explain all observed errors, so the propagation cone is marked as a candidate for containing a fault. The procedure in Sec. 5 for extracting the error locations for each candidate propagation cone is then performed. These error locations can then be passed to a conventional logic diagnosis tool to obtain the set of suspects. For faults whose propagation cone reaches too many scan cells, existing conventional diagnosis procedures can be used to obtain the set of suspects while using the technique described in Sec. 3 to narrow down the suspect set by taking advantage of information from the fault-free MISR bits.

At the end, the suspect sets computed for each faulty signature are intersected to form the final suspect set.

Note that it is very easy to reduce runtime if needed. The number of propagation cones that are processed can be pruned by greedily selecting them for processing in order of the largest number of unmarked gates that they contain. The more propagation cones that are processed, the larger the potential improvement that can be achieved over conventional diagnosis methods. The results can never be worse.

7. Experimental Results

Experiments were performed to see how much the proposed canceling techniques can reduce the size of the initial suspect set in comparison to conventional approaches. Single stuck-at faults were inserted and simulated to obtain the failing signatures in a 128-bit MISR over a test set giving 100% fault coverage. Results were generated for many faults. Table 1 shows the average number of suspects that were obtained for four different diagnosis cases is shown. The first is for the case where no MISR is used for compaction, so the exact location of all errors is known. This is a lowerbound on the number of suspects. Next is the number of suspects that is obtained for the conventional approach where for each MISR signature bit in error, critical path tracing is performed from all scan cells that the MISR signature depends on. Next is for randomly XORing 500 different combinations of MISR bits and using that to generate additional intersections to further prune the suspects as was described in Sec. 3. Lastly, the number of suspects that are obtained using the proposed canceling approach for error location as was described in Sec. 6. These numbers show the general improvement that can be obtained with the proposed approach. However, the averages tend to get skewed by the faults with large suspect sets. To get a better feel for the types of results for individual faults, Table 2 shows a representative

Table 1. Average Suspect Set Size using Conventional, Linear Combinations, and Proposed Canceling Approaches

Circuit	Scan Cells	Gates	Average Number of Suspect Faults			
			Uncompacted (Lowerbound)	Conventional Approach	Using Random Combinations	Proposed Using Canceling
A	5,172	32K	11.3	57.2	51.3	19.8
B	3,214	21K	9.7	38.3	32.3	16.9

Table 2. Sample of Suspect Set using Conventional, Linear Combinations, and Proposed Canceling Approaches

Fault	Propagation Cone Size	Number of Suspect Faults			
		Uncompacted (Lowerbound)	Conventional Approach	Using Random Combinations	Proposed Using Canceling
1	6	2	5	3	2
2	126	5	28	27	27
3	5	7	23	18	7
4	17	8	22	14	8
5	8	9	9	9	9
6	51	13	17	16	13
7	37	14	35	31	14
8	24	15	52	37	15
9	28	18	64	27	18
10	101	38	78	55	38

sample of the kinds of results that were obtained. For each fault, the size of the propagation cone (number of scan cells the fault site has structural paths to) is shown followed by the number of suspect faults obtained for the four different diagnosis cases.

As can be seen, when the propagation cone size is smaller than the MISR size minus 16, the proposed approach was able to find the exact error locations and hence match the results for the uncompacted case. However, for fault 2, the propagation cone size was too large, so in that case, the proposed approach could only use random combinations to improve the results compared with the conventional approach. In most cases, using random combinations provided a considerable reduction in the number of suspects. In some cases, the conventional approach is very effective and not much improvement is seen (Fault 6 is an example of this).

8. Conclusions

This paper has shown that more diagnostic information can be extracted from faulty signatures through canceling which can exploit information from the error-free signature bits as well. Furthermore, when the propagation cone is sufficiently smaller than the size of the MISR, the exact error locations can be obtained. Note that the proposed technique can also be used in conjunction with the ideas in [Benware 10] for orthogonal

response compaction and in [Cook 14] for diagnosing multiple faults.

Acknowledgements

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