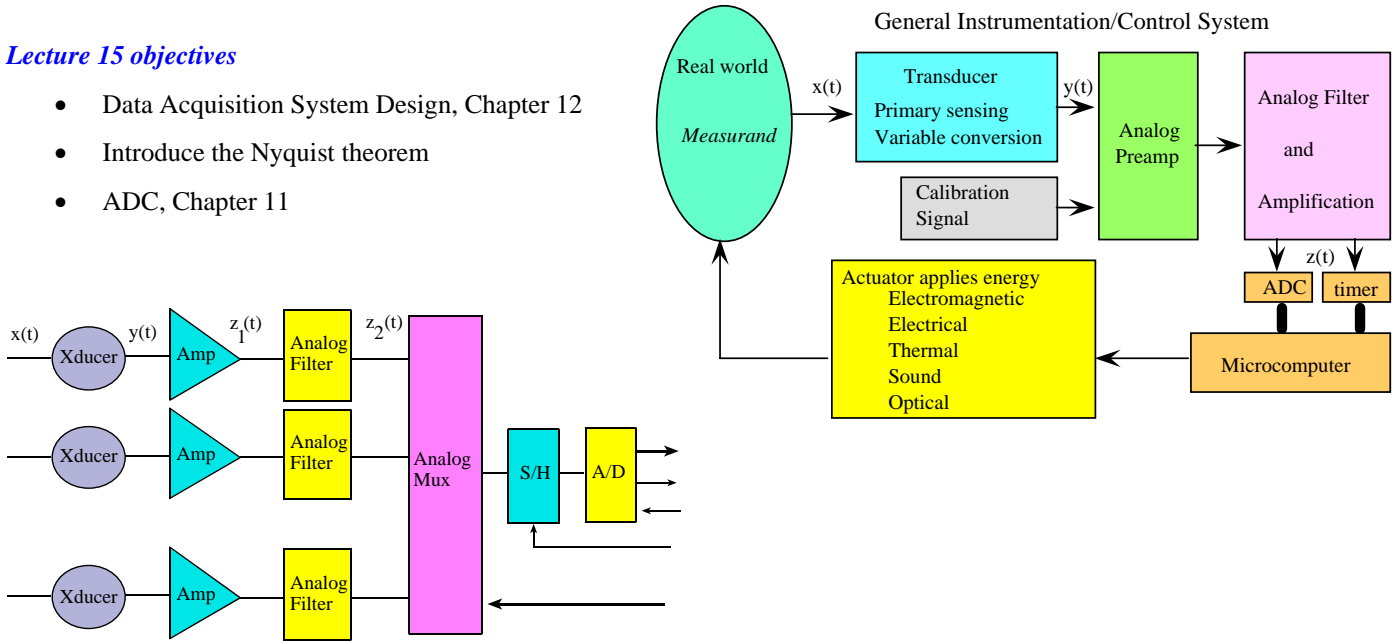


**Lecture 15 objectives**

- Data Acquisition System Design, Chapter 12
- Introduce the Nyquist theorem
- ADC, Chapter 11



**Quantitative DAS**

- range ( $r_x$ )
- resolution ( $\Delta x$ )
- precision ( $n_x$  in alternatives)
- frequencies of interest ( $f_{min}$  to  $f_{max}$ )
- repeatability ( $\sigma$  of repeated measurements, same conditions)
- reproducibility ( $\sigma$  of repeated measurements, different conditions)

**Qualitative DAS**

- “sounds good”
- “looks pretty”
- “feels right”

Other qualitative DAS's involve the detection of events.

- true positive (TP)
  - baby stops breathing and apnea monitor detects it*
- false positive (FP)
  - baby is breathing OK but apnea monitor alarms*
- false negative (FN)
  - baby stops breathing but monitor does not alarm*

Prevalence =  $(TP + FN) / (TP + TN + FP + FN)$   
 Sensitivity =  $TP / (TP + FN)$   
 Specificity =  $TN / (TN + FP)$   
 PPV =  $TP / (TP + FP)$   
 NPV =  $TN / (TN + FN)$

**Using Nyquist Theory to Determine Sampling Rate.**

**Voltage quantizing**

precision  $n_z = 2^n$

**Time quantizing**

**Nyquist theory** states that if the signal is sampled at  $f_s$ , then the digital samples only contain frequency components from 0 to  $\frac{1}{2}f_s$ .

Conversely, if the analog signal does contain frequency components larger than  $\frac{1}{2}f_s$ , then there will be an **aliasing** error. Aliasing is when the digital signal appears to have a different frequency than the original analog signal.

$$V(t) = A \sin(2\pi ft + \phi)$$

Nyquist theory says that if  $f_s$  is strictly greater than twice  $f$ , then one can determine  $A$ ,  $f$  and  $\phi$  from the digital samples.

But if  $f_s$  less than or equal to  $2 \cdot f$ , then the apparent frequency, as predicted by analyzing the digital samples, will be shifted to a frequency between 0 and  $\frac{1}{2}f_s$ .

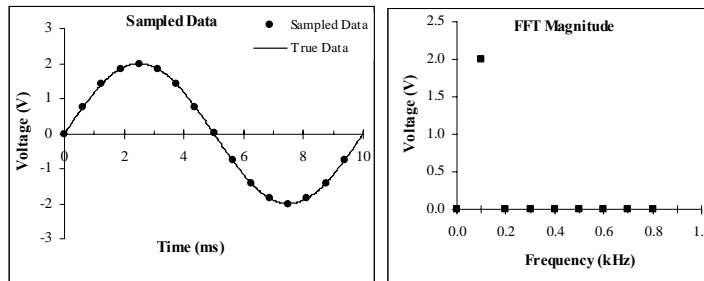
**Valvano Postulate:** If  $f_{max}$  is the largest frequency component of the analog signal, then you must sample more than ten times  $f_{max}$  in order for the reconstructed digital samples to look like the original signal when plotted on a voltage versus time graph.

The choice of **sampling rate**,  $f_s$ , is determined by the maximum useful frequency contained in the signal.

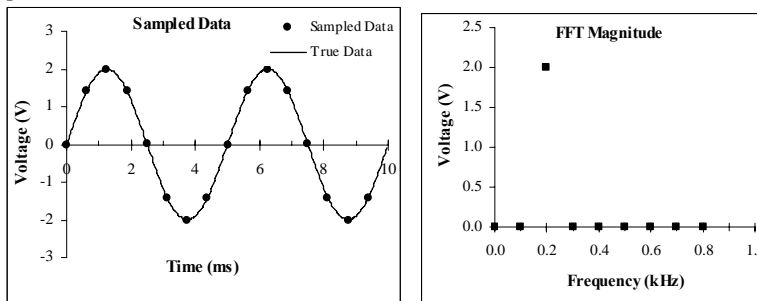
$$f_s > 2 f_{max}$$

A low pass analog filter may be required to remove frequency components above  $0.5f_s$ . A digital filter can not be used to remove aliasing.

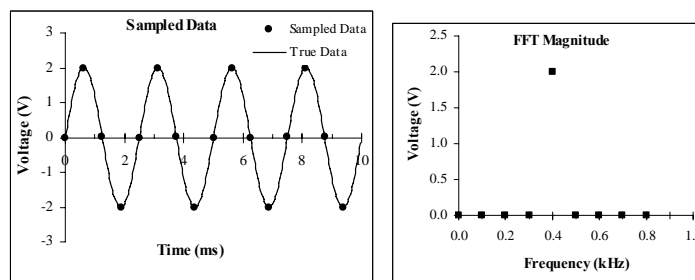
\*\*\*\*\*the following plots were created with FFT16.XLS\*\*\*\*\*



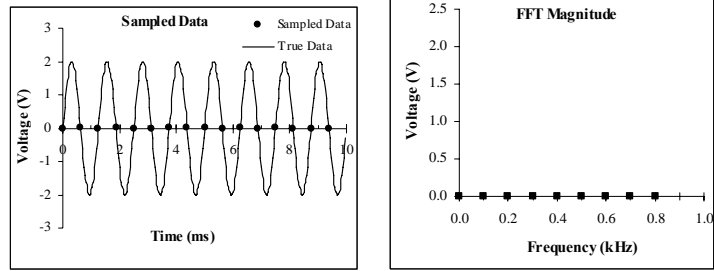
A 100 Hz sine wave is sampled at 1600 Hz.



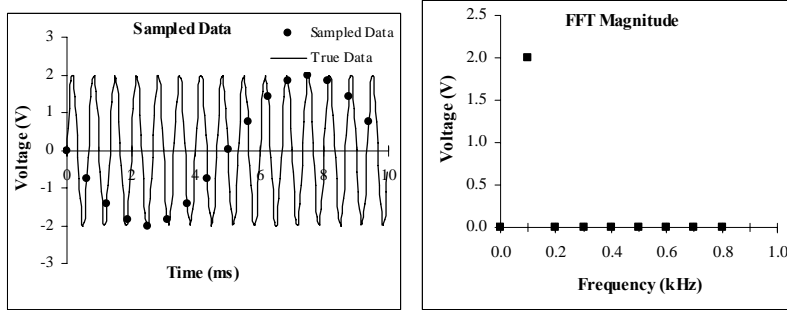
A 200 Hz sine wave is sampled at 1600 Hz.



A 400 Hz sine wave is sampled at 1600 Hz.



An 800 Hz sine wave is sampled at 1600 Hz.

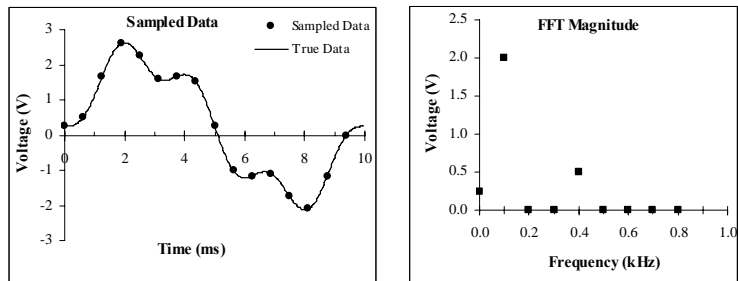


A 1500 Hz sine wave is sampled at 1600 Hz.

The data plotted in the next figure demonstrate the result that occurs when we sample an input with multiple frequency components. This original signal has three frequencies 0, 100 and 400 Hz. In particular,

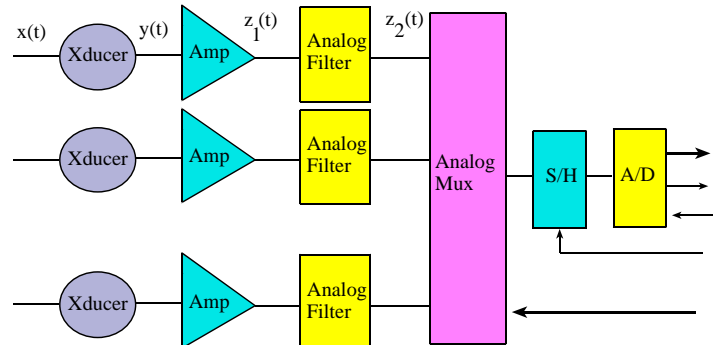
$$V(t) = 0.25 + 2 \sin(2\pi 100t) + 0.5 \sin(2\pi 400t)$$

Notice in the FFT that the information from all three components is properly retained. This will be true for any input that has its highest frequency component less than  $\frac{1}{2}f_s$ .



A signal with DC, 100 Hz and 400 Hz components is sampled at 1600 Hz.

**How Many Bits Does One Need for the ADC?**



Let the following describe the nonlinear system.

$$z = f(x)$$

The required ADC precision,  $n_z$ ,

$$\Delta_x = \frac{r_x}{n_x}$$

$$\Delta_z = \min \{f(x+\Delta_x)-f(x)\} \text{ for all } x \text{ in } r_x$$

$$\Delta_z = \frac{\partial f}{\partial x} \cdot \Delta_x \quad \text{for all } x \text{ in } r_x$$

$$n_z = r_z / \Delta_z$$

If  $z=f(x)$  is linear, then  $n_z = n_x$

If  $z=f(x)$  is nonlinear, then  $n_z > n_x$

**Specifications for the Analog Signal Processing**

The analog signal processing is linear then

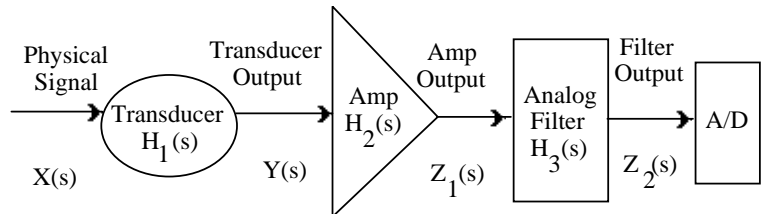
$$z = Gy + b$$

Choose so the full scale range of the input signal x, maps into the full scale range of the ADC.

**Another factor to consider is the electrical noise.**

Let  $e_z$  be the electrical noise referred to the ADC, any ADC  $\Delta_z < e_z$  would be wasteful

E.g., range is 0 to +8V, noise at the ADC is 1mV then any ADC bits beyond 13 would be wasteful.

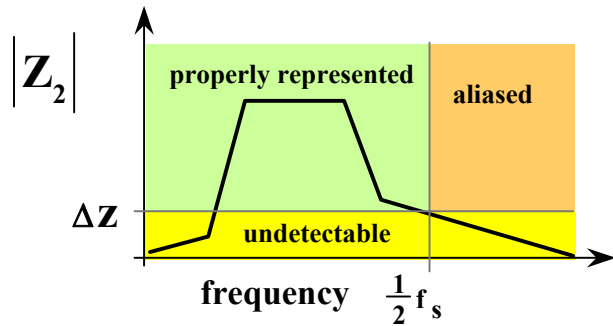
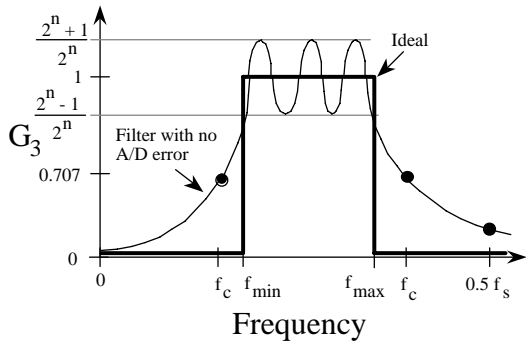


$$Z_2(s) = H_3(s)Z_1(s) = H_3(s)H_2(s)Y(s) = H_3(s)H_2(s)H_1(s) X(s)$$

**Analog Filter**

Let the gain of the analog filter be  $G_3 = |H_3(s)|$ .

Then the system should pass, with little error as seen by the ADC, for signal frequencies between  $f_{min}$  and  $f_{max}$ .



*Ideal and practical filter responses.*

*To prevent aliasing => no measurable signal above 0.5fs.*