MODELING OF TEMPERATURE PROBES IN CONVECTIVE MEDIA

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This paper discusses the dynamic behavior of probes embedded in convective media during temperature measurements. In certain conditions the temperature measured by a probe can be written as the convolution of the true temperature with the impulse response of the probe. We present a general method to find the natural response of any kind of probe, and then we present results for a more realistic 1-D model for the thermistor probe in a thermodilution catheter. The results of these analyzes can be applied to enhance the dynamic response of temperature measurements made by probes in convective media.

We begin with a methodology for developing a convolutional model for temperature probes in a medium with a time-independent convection coefficient, even though it can be non-uniform along the boundary. The requirement of a constant, non-uniform convection coefficient will make it possible to write a linear time-invariant relationship between the true and measured temperatures. The bulk temperature of the fluid will be the magnitude to be measured. The step response measured by the finite size probe can be written:

$$T_{meas}(t) = \sum_{n=1}^{\infty} d_n e^{-\lambda_n^2 t} \qquad (1)$$

where the coefficients d_n and λ_n are a function of the physical and thermal properties of the probe. Equation (1) illustrates the fact that, for a probe with a generic geometry, the solution can always be written as a summation of exponentials.

The next step is to use Duhamel's equation to write the convolutional response to a generic input $T_{in}(t)$.

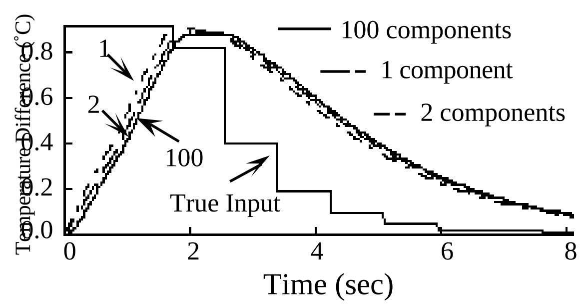
$$T_{out}(t) = \int_{\tau=0}^{t} T_{in}(\tau) \frac{\partial T_{meas}(t-\tau)}{\partial t} d\tau \qquad (2)$$

In other words, $T_{in}(t)$ is the true fluid temperature and $T_{out}(t)$ is the probe response. The hard problem here is to solve the eigenvalue problem for a composite body, to find the coefficients of the exponentials. The only way to do this in complex geometries, is by using numerical methods.

Next, to illustrate the general method presented, we develop a model for the temperature probe in a thermodilution catheter. We created a model for the thermistor in a standard Swan-Ganz thermodilution catheter. The thermistor probe is located a few centimeters from the catheter tip. Some simplifications are adopted. In the real catheter, the thermistor would not have the shape of a parallelepiped, but the shape of an oblate spheroid. Also the coating would not have uniform thickness. However, the model will provide a good qualitative understanding of the behavior of the probe. Another simplifying assumption will be that the catheter body is adiabatic. The derivative in Equation (2) is the impulse response of the probe. This response has an initial lag. Physically, the lag is caused by the presence of the protective shell. Mathematically, the lag is caused by the presence of one or more exponentials with negative coefficients, which cancel the positive exponentials for small values of time.

The catheter probe has a response that can be written as a summation of exponential responses. The most significant components are the first three components. The first harmonic is about 12 times the second, and 25 times the third. It is interesting to notice that the coefficients in Equation (2) define completely the response of the probe. If one is capable of measuring these values, then one will have the complete description of the behavior of the probe for any convection coefficient.

A good example of this kind of temperature measurement is the thermodilution curve. The catheter is inserted such that thermistor is located in the pulmonary artery, and a cold bolus of saline is injected in the right atrium. The resulting fluid temperature in the pulmonary artery, $T_{in}(t)$, is shown as the solid line. The dotted lines are the thermistor responses approximated by Eq. (2).



Armed with this model, we can now attempt the significant problem of signal enhancement. This model describes the smearing of the temperature signal because of the finite probe size. We believe it is possible to use this model to correct for this smearing, thereby recovering the true fluid temperature.