Homework: BDDs

1. For the following functions:

\[ F = ab + c \]
\[ G = bc' + d \]
\[ H = c + d' \]

(a) Draw BDDs \( F \), \( G \), and \( H \) using the variable ordering \( a < b < c < d \).

(b) Compute \( \text{ite}(F, G, H) \), and draw the corresponding BDD.

(c) Compute \( \text{ite}_\text{constant}(F, G, H) \), using the algorithm presented in class. Show all steps.

25 marks

2. Let \( f(x_1, x_2, x_3, x_4) = x_1x_2' + x_3x_4' + x_2x_4 \), and \( g(x_1, x_2, x_3) = x_1 + x_2'x_3 \).

(a) Draw the BDDs for \( f \) and \( g \) for the variable ordering \( x_1 < x_3 < x_4 < x_2 \).

(b) Compute the BDD for \( f(x_1, x_2, x_3, g(x_1, x_2, x_3)) \) using the \( \text{bdd-compose} \) algorithm given in class.

25 marks

3. Suppose you were given a BDD for a function \( f(x_1, x_2, x_3, y_1, y_2, y_3) \). How would you check if there existed functions \( g \) and \( h \) whose supports were \( x_1, x_2, x_3 \) and \( y_1, y_2, y_3 \) respectively such that

(a) \( f = g \cdot h \)
(b) \( f = g \oplus h \)

25 marks

4. Let \( F \) be a BDD for a Boolean function \( f : B^n \rightarrow B \). Suppose the cost of a minterm \((\alpha_1, \ldots, \alpha_n)\) in \( B^n \) was defined to be \( \sum_{i=1}^{n} c_i \cdot \alpha_i \), where the \( c_i \)'s are real-valued constants.

Given an efficient procedure (which operates on \( F \)) for finding the cheapest minterm in the onset of \( f \).

20 marks