Motivation

- Transistors are mainly used as amplifiers.
  - One of the main functions of a transistor is the amplifier.
  - Biased transistors are not linear ($I_e \propto V_{be}$, $I_d \propto (V_{as}-V_t)^2$).
  - Transistor gain cannot be controlled well.

Ex: $g_{m}$ depends on the biasing conditions.
$g_{m}$ can vary by >2x over process corners and Temp.

Q: How can we build reliable amplifiers despite these drawbacks?
A: We build circuits that do not rely on the amplifier.

The Operational Amplifier

For 2nd case:

\[
\begin{align*}
I_{in} &= \frac{V_{in}}{R} \\
I_o &= -I_{in} \\
V_{out} &= \frac{I_o}{j\omega C}
\end{align*}
\]

$V_{out} = \frac{-V_{in}}{R}$

$V_{in} = \frac{1}{j\omega RC}$

The integrator does not depend on the amplifier gain but on more stable, more linear passive components.
Q: How can we build amplifiers with high gain?

The simple transistor in common source configuration does not have enough gain.

\[ \text{Vout} = \text{Vin} \left( \frac{gm}{gds} \right) \]

\[ \frac{Vout}{Vin} : \tau = \frac{g_{m1}}{g_{ds1}} \left( - \frac{g_{m2}}{g_{ds2}} \right) = \frac{g_{m1} g_{m2}}{g_{ds1} g_{ds2}} \]

A1: We boost the gain. Ex. we cascade stages.

A2: We lower the \( g_{ds} \) (boost output impedance).

\[ \frac{V_{out}}{V_{in}} = \left( \frac{g_{m1}}{g_{ds1}} \right) \left( - \frac{g_{m2}}{g_{ds2}} \right) \]

\[ \frac{V_{out}}{V_{in}} = \frac{1}{\tau} \frac{g_{m1} g_{m2}}{g_{ds1} g_{ds2}} \]

The source degeneration technique.

Output Impedance: \( \frac{V_{out}}{I_{out}} \)

\[ r_0 = \frac{1}{g_{ds}} \]

\[ \frac{V_{out}}{I_{out}} = \frac{-V_{in} g_{m} + V_{i} - V_{out} g_{m}}{r_0} \]

\[ -V_{in} g_{m} + V_{i} - V_{out} g_{m} = 0 \]

\[ V_{out} \left( \frac{1}{r_0} + g_{m} \right) = \frac{V_{out}}{r_0} \]

\[ r_{out} = r_0 + R + 2 \cdot g_{m} r_0 \]

Unfortunately, the stage Transconductance is: \( g_{m} = \frac{g_{m} r_0}{r_0 + \frac{R}{2} + g_{m} r_0} \)

and the total gain is: \( g_{m} \cdot r_0 = \frac{g_{m} r_0}{r_0 + \frac{R}{2} + g_{m} r_0} \)

unchanged.
The cascode transistor

- Besides the unchanged gain, the source degeneration technique has the disadvantage that the max R is limited by large signal considerations.

- Substitute the degeneration resistance with a transistor in saturation region:

\[ V_{b0} - [M1] \]
\[ V_{in} - [M2] \]

Output Impedance

The small signal circuit is the same as the previous one therefore:

\[ V_0' = \frac{V_{out}}{I_{out}} = R_{o1} + R_{o2} + R_{o2} g_{m1} r_{on} \]

The effective \( g_{m}' \) is different:

\[ V_{in} g_m + \frac{V_{1}}{r_{o2}} + \frac{V_{1}}{r_{o1}} + V_{1} g_m = 0 \]

\[ V_{in} g_m + \frac{V_{1}}{r_{o2} r_{o1}} + g_m = -V_{in} g_m \]

\[ V_{in} g_m + \frac{1}{r_{o2} r_{o1}} \]

\[ \frac{V_{out}}{V_{in}} = \frac{g_m}{g_m} \frac{1}{r_{o1} + \frac{1}{g_m}} \times g_m \]

Therefore the gain is:

\[ A = g_m \cdot (r_{o1} r_{o2} + r_{o2} g_{m1} r_{o1}) \times g_m r_{o2} g_{m1} r_{on} \]

Intuitive explanation

- When \( V_{out} \) changes, \( V_{1} \) tends to follow it over the \( r_{o1} r_{o2} \) voltage divider. However, \( V_{1} \) activates Transistor M1 that provides additional current that flows through \( r_{o1} \) therefore most of the voltage drop goes on \( r_{o1} \) and \( r_{o2} \) is kept stable reducing the net current and hence increasing the impedance.
Derivatives of the basic cascode

1) The double cascode
   - Recursive use of the cascode idea

   $g_m = g_{m1}$
   $r_o = r_{01} \cdot g_{m2} r_{o2}$

2) The regulated cascode
   - $g_{m2}$ is boosted by an operational amplifier

   $g_m \approx g_{m1}$
   $r_o \approx r_{01} \cdot A g_{m2} \cdot r_{o2}$

   Boosted $g_m$

3) The folded cascode
   - PMOS input transistor and NMOS cascode transistor give more freedom in the choice of the common mode voltages

   $g_m = g_{m1}$
   $r_o = r_{01} \cdot g_{m2} r_{o2}$

(assuming $M3$ is an ideal current sink)
Noise contribution of the cascode transistor

\[ V_{\text{out}}^2 = V_{n2}^2 \cdot (g_m \cdot r_{o2})^2 + V_{n1}^2 \left( \frac{g_{m2} \cdot r_{o2} \cdot g_{m1} \cdot r_{o1}}{g_{m2} \cdot r_{o2} \cdot g_{m1} \cdot r_{o1}} \right)^2 \]

Input referred noise:

\[ V_{\text{in}}^2 = \frac{V_{\text{out}}^2}{(g_{m2} \cdot r_{o2} \cdot g_{m1} \cdot r_{o1})^2} \]

\[ = V_{n2}^2 \left( g_m \cdot r_{o2} \right)^2 + V_{n1}^2 \left( \frac{g_{m2} \cdot r_{o2} \cdot g_{m1} \cdot r_{o1}}{g_{m2} \cdot r_{o2} \cdot g_{m1} \cdot r_{o1}} \right)^2 \]

\[ = V_{n2}^2 \left( \frac{1}{g_{m1} \cdot r_{o1}} \right)^2 + V_{n1}^2 \]

⚠️ The noise contribution of M2 is scaled by the square of the gain of M1.

\[ \Rightarrow \text{M2 is usually a negligible source of noise.} \]
1) The cascode circuit is a very common and useful topology in CMOS analog circuits.

2) The $g_m$ of the circuit is the same as the one of bottom transistor. The output impedance is the $r_o$ of the bottom transistor boosted by the gain of the upper transistor.

3) The upper transistor does usually not contribute to the total noise.
Notation

NMOS Transistor: $I_s, I_d$

PMOS Transistor: $I_s, I_d$

current source: $I$

voltage source: $V$

voltage controlled current source ($g_m$): $g_m V_1$

noise source

gnd

Vdd