Course notes for EE394V
Restructured Electricity Markets:
Locational Marginal Pricing

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Hedging and day-ahead and real-time markets

(i) Volatility of energy prices,
(ii) Forward markets,
(iii) Day-ahead and real-time markets.
(iv) Contract for differences.
(v) Relationship to capital formation,
(vi) Transmission prices,
(vii) Financial transmission rights/Congestion revenue rights.
(viii) Revenue adequacy,
(ix) Hedging real-time prices.
11.1 Volatility of energy prices

- As discussed in Sections 8.12.9.1 and 10.9, randomness and uncertainty is pervasive in supply and demand of electricity.
- Sections 8.12.1.4, 9.9.3, and 10.9 discussed approaches to making dispatch robust to generator and transmission failures.
- Changing supply and demand conditions, together with generation and transmission failures, result in variation of LMPs over time (and space):
  - the prices are said to be **volatile**,
  - affecting all market participants, even for example, a “perfectly reliable” generator, or a constant load.
Volatility of energy prices, continued

• Consider a generator and demand that are co-located at the same bus.
• More precisely, consider a generator and demand that are exposed to the same LMPs.
• Each might prefer to be exposed to a less varying price over time.
• That is, they may want to **hedge** (or remove) their exposure to LMP volatility.
• A sequence of **forward markets** can be used to help cope with randomness and hedge volatility.
• Successive forward markets can compensate for and correct the forecast errors and other errors from previous markets.
11.2 Forward markets

- In the previous discussion of offer-based markets, we did not discuss in the detail the differences between forward markets and real-time markets, except briefly in Section 8.11.3.
- The real-time market matches physical supply and physical demand over short time intervals.
- The real-time prices on any given day will depend on the varying supply and demand conditions on that day:
  - prices will tend to fluctuate considerably, or be highly volatile.
- The day-ahead market is a forward market that provides a mechanism to reduce exposure to this volatility:
  - the day-ahead commitment and dispatch market sets up (financial) forward agreements a day in advance to generate based on forecasts or specifications of demand a day in advance.
  - the real-time market deals with the deviations of actual supply and demand from day-ahead specifications.
- Discuss day-ahead and real-time markets in Section 11.3.
Forward markets, continued

- Other forward markets provide enable generation and demand to agree to a price for a specific quantity of energy in advance of the day-ahead market.
- In the context of an offer-based economic dispatch market, the most natural forward contract is called a **contract for differences**:  
  - this is a forward financial commitment,  
  - other types of forward agreements are possible.
- Discuss contracts for differences in Section 11.4.
11.3 Day-ahead and real-time markets

• For each commodity there is, in principle, both a day-ahead price and a real-time price:
  – each price results from offer-based economic dispatch,
  – day-ahead prices are typically associated with day-ahead unit commitment as discussed in Chapter 10,
  – (in some markets such as ERCOT, there are only day-ahead prices currently for AS and not real-time AS prices).
11.3.1 Day-ahead market

- The day-ahead price applies to commodities bought and sold in the day-ahead market.
- The day-ahead market is technically a **short-term forward market**.
- Participation in the day-ahead market entails a **forward financial commitment** to produce or consume in real-time:
  - a financial commitment means that either the action is carried out physically, or the actual deviation from the financial commitment is sold to or purchased from the real-time market,
  - (in some cases, there is also some expectation of physical performance, such as in provision of AS in ERCOT, and in the day-ahead commitment of generators).
11.3.2 Real-time market

- Payment in the real-time market applies to the deviations from day-ahead positions:
  - Generally, a positive deviation in dispatched generation coincides with the real-time price being higher than the day-ahead price, while
  - a negative deviation in dispatched generation coincides with the real-time price being lower than the day-ahead price.

- Unlike in the day-ahead market, generators are required to follow real-time dispatch signals in their physical dispatch, as mentioned in Section 10.4.5:
  - deviation penalties are charged for deviating too far from real-time dispatch instructions.
11.3.3 Example

- Suppose that the ISO day-ahead market yields a day-ahead price of $50/MWh in a particular hour and generation of 40 MW by a particular generator.
- The payment from the day-ahead market is:
  \[(\text{DA quantity})(\text{DA price}) = 40 \times 50,\]
  \[= 2000/\text{h}.\]
- In real-time, if the generator actually produces 40 MW, then it has physically fulfilled its forward commitment and receives no further payment or charge:
  - total payment is equal to payment from day-ahead market,
  - deviation from forward position is zero.
**Example, continued**

- In the real-time market, if the real-time price is $60/MWh and the generator actually produces 45 MW, then its payment from DA and RT is:

\[
(DA \text{ quantity})(DA \text{ price}) + (RT \text{ quantity} - DA \text{ quantity})(RT \text{ price}),
\]

\[
= 40 \times 50 + (45 - 40) \times 60,
\]

\[
= $2300/h.
\]

- Note that the condition that the generator is dispatched to a higher level in real-time than its financial commitment in the day-ahead is consistent with the real-time price being higher than the day-ahead price.

- If, on the other hand, the generator suffers a forced outage so that real-time generation is zero, then it is obligated to buy back the forward energy from the market and its “payment” from DA and RT is:

\[
(DA \text{ quantity})(DA \text{ price}) + (RT \text{ quantity} - DA \text{ quantity})(RT \text{ price}),
\]

\[
= 40 \times 50 + (0 - 40) \times 60,
\]

\[
= $ - 400/h.
\]
Example, continued

• In the real-time market, if the real-time price is $30/MWh and the generator actually produces 35 MW, then its payment from DA and RT is:

\[(\text{DA quantity})(\text{DA price}) + (\text{RT quantity} - \text{DA quantity})(\text{RT price})\],

\[= 40 \times 50 + (35 - 40) \times 30,\]

\[= $1850/h.\]

• Note that the condition that the generator is dispatched to a lower level in real-time than its financial commitment in the day-ahead is consistent with the real-time price being lower than the day-ahead price.
11.3.4 Consistency between day-ahead and real-time markets

- It is desirable that each market represents similar system constraints:
  - Otherwise there would be price differences between the two markets even in the absence of randomness.

- Conceptual difficulties:
  - Ancillary services, such as reserves, acquired in the day-ahead market are commitments to be available in real-time.
    - What happens when reserves are actually deployed?
    - In markets with real-time trading of ancillary services, deployment effectively means that generator must “buy back” the ancillary services and sell capacity instead as energy,
    - Requires representation of ancillary services in real-time market.
  - What happens if a generator receives make-whole payments in the day-ahead but does not actually commit?
    - Commitment not fully financial in that make-whole only paid if generator actually commits.
11.4 Contract for differences

- Forward contracting reduces exposure to variability of prices.
- If (representatives of) a generator and (representatives of) demand are exposed to the same LMP, then a **contract for differences** removes both their exposures to variation in that LMP for a given contract quantity over a given contract period.
- A contract for differences is an agreement between a generator and demand that entails a payment by the demand to the generator equal to:

  \[(\text{contract quantity}) \times (\text{(contract price)} - \text{LMP})\],

- for each pricing interval in the contract period.
- The contract quantity is in MW, while the contract price is in $/MWh.
- The LMP could, in principle, be either day-ahead or real-time, but we will first consider day-ahead:
  - hedging real-time prices will be discussed in Section 11.7.5.
Contract for differences, continued

- As well as agreeing to the contract for differences, the generator and demand can still participate in the day-ahead market.
- Figure 11.1 shows the revenue streams under a contract for differences and offer-based economic dispatch:
  - (representatives of) demand pay the ISO for the energy consumed,
  - (representatives of) demand pay the (representatives of) generators under the contract for differences, and
  - the ISO pays the (representatives of) generators for the energy produced.

Fig. 11.1. Revenue streams in offer-based economic dispatch with contract for differences.
Contract for differences, continued

- Since the demand will also be required to pay the ISO for its energy based on the LMP then, if it purchases power in the day-ahead market equal to the CfD contract quantity in each pricing interval, its total payment from CfD and market is:

\[
(\text{contract quantity}) \times (\text{contract price} - \text{LMP}) + (\text{contract quantity}) \times \text{LMP} = (\text{contract quantity}) \times (\text{contract price}).
\]

- Similarly, the net payment from CfD and market to a generator that sells into the day-ahead market at a level equal to its CfD contract quantity is:

\[
(\text{contract quantity}) \times (\text{contract price} - \text{LMP}) + (\text{contract quantity}) \times \text{LMP} = (\text{contract quantity}) \times (\text{contract price}).
\]
Contract for differences, continued

- If the demand purchases more or less than the contract quantity, then it will pay for or (effectively) be refunded for the deviation at the day-ahead LMP:
  - analogous to the case of the deviation of real-time quantity from day-ahead quantity.
- If the generator sells more or less than the contract quantity, then it will be paid paid or (effectively) pay back the deviation at the day-ahead LMP:
  - analogous to the case of the deviation of real-time quantity from day-ahead quantity.
- This means that the previous results about incentives will still apply even in the presence of forward contracts:
  - first-order necessary conditions for profit maximization of consumption and production decision will be based on the LMPs, not the contract price.
11.5 Relationship to capital formation

- **Bilateral contracts**, such as contracts for differences, between generators and demand have an important role in the financing of new generation.
- If a developer builds a generator, and does not have a bilateral contract, then it is taking the risk that LMPs are high enough to cover its capital and operating costs.
- With a bilateral contract, a developer can assure a less volatile revenue stream.
- Bilateral contracts typically allows for lower cost financing of construction compared to speculative development.
11.6 Transmission prices

- If the generator and demand are not co-located, they are effectively exposed to the *difference* in LMPs between their buses:
  - the price difference is the short-term **transmission price**.

- Although they could hedge exposure to either the generator LMP or to the demand LMP with a contract for differences, they will nevertheless be exposed to variation in the price difference between the two LMPs:
  - for example, with a contract for differences based on the demand LMP, the generator will be paid the contract for difference price minus the transmission price.

- In ERCOT and some other markets, retailers are charged for their customer consumption not at the LMP, but at a zonal demand-weighted average of the LMPs:
  - in this case, even a generator and a load that was physically co-located would be exposed to different prices.
11.7 Financial transmission rights

• A **financial transmission right (FTR)** or **congestion revenue right (CRR)** is the right to receive the product of:
  
  \((\text{contract quantity}) \times (\text{the difference in LMPs between a notional point of withdrawal and a point of injection})\),

• for each pricing interval in the contract period.

• The CRR hedges the exposure to volatile LMP differences:
  
  – if a generator and a demand have signed a contract for differences based on the LMP at the demand, and the generator also possesses a CRR then the net payment by the demand to the generator is:
    
    \((\text{contract quantity}) \times (\text{contract price})\),

  – as in the case of the generator and demand being exposed to the same LMP.
11.7.1 Issuing CRRs

- Since prices are typically higher at demand than generation, CRRs will typically pay out positive amounts of money in expectation.
- That is, the expected payout over a contract duration will be positive.
- It can therefore be expected that a typical CRR will cost money to acquire!
- A speculator might offer to sell a CRR based on the expected payout over the contract duration (plus a risk premium for taking on the risk).
- Is there any other source of money to fund CRRs not involving speculation?
11.7.2 CRRs funded out of congestion rent

- Congestion rent accrues to the ISO.
- It can be used to fund (non-speculative) CRRs.
- The ISO can sell CRRs.
- How “much” CRRs can be sold?
- Consider a collection of CRRs that have been sold that are claims on payment of LMPs in the day-ahead market.

Model each injection as a generation and each withdrawal as a demand, with power level equal to the CRR contract quantity.
Let the total vectors of implied dispatch be \( P' \) and \( D' \), respectively.

- If \( \pi_P^{LMP} \) is the vector of LMPs in the day-ahead market in a particular hour then the total payment to CRR holders for that hour specified by the implied dispatch \( P' \) and \( D' \) is:

\[
\left[ \pi_P^{LMP} \right]^\dagger (D' - P').
\]
CRRs funded out of congestion rent, continued

• If the corresponding vectors of optimal generation and demand for that hour in the day-ahead market are $P^*$ and $D$, then the congestion rent is equal to:

$$[\pi_{P}^{\text{LMP}}]^{\dagger} (D - P^*)$$.

• Is the congestion rent sufficient to cover the payments to the CRR holders?

• That is, do we have that:

$$[\pi_{P}^{\text{LMP}}]^{\dagger} (D' - P') \leq [\pi_{P}^{\text{LMP}}]^{\dagger} (D - P^*)$$.

• The following Corollary to Theorem 9.2 provides the required result.
CRRs funded out of congestion rent, continued

Corollary 11.1

• Suppose locational marginal prices $\pi_P^{LMP}$ were determined from the offer-based optimal power flow solution, corresponding to the demand $\overline{D}$ and generation $P^*$, and let $\hat{\lambda}$ and $\hat{\mu}$ be the Lagrange multipliers on system constraints in the shift factors formulation.

• Let $D'$ and $P'$ be any vectors of demand and generation, respectively, that satisfy the system constraints, so that:

$$-1^\top P' = -1^\top D',$$

$$\hat{C}(P' - D') \leq d.$$  

• The values $D'$ and $P'$ may differ from the demand $\overline{D}$ and generation $P^*$ in offer-based optimal power flow.

• Then:

$$[\pi_P^{LMP}]^\top (D' - P') \leq [\pi_P^{LMP}]^\top (\overline{D} - P^*). \quad (11.1)$$
CRRs funded out of congestion rent, continued

**Proof** We have:

\[
\begin{align*}
[\pi_P^{\text{LMP}}]^{\dagger} (D' - P') &= \left[ 1\hat{\lambda}^* - [\hat{C}]^{\dagger} \hat{\mu}^* \right]^{\dagger} (D' - P'), \text{ by (9.4)}, \\
&= [\hat{\lambda}^*]^{\dagger} (1^{\dagger} D' - 1^{\dagger} P') - [\hat{\mu}^*]^{\dagger} \hat{C} (D' - P'), \\
&= - [\hat{\mu}^*]^{\dagger} \hat{C} (D' - P'), \text{ since } 1^{\dagger} D' - 1^{\dagger} P' = 0, \\
&= [\hat{\mu}^*]^{\dagger} \hat{C} (P' - D'), \\
&= \sum_{\hat{\mu}^*_{(\ell k)} = 0} \hat{\mu}^*_{(\ell k)} \hat{C}_{(\ell k)} (P' - D') + \sum_{\hat{\mu}^*_{(\ell k)} \neq 0} \hat{\mu}^*_{(\ell k)} \hat{C}_{(\ell k)} (P' - D'), \\
&= 0 + \sum_{\hat{\mu}^*_{(\ell k)} \neq 0} \hat{\mu}^*_{(\ell k)} \hat{C}_{(\ell k)} (P' - D'), \\
&\leq \sum_{\hat{\mu}^*_{(\ell k)} \neq 0} \hat{\mu}^*_{(\ell k)} \overline{p}_{(\ell k)}, \text{ since } \hat{C}(P' - D') \leq d \text{ and } \hat{\mu}^*_{(\ell k)} \geq 0, \forall \ell, k, \\
&= [\pi_P^{\text{LMP}}]^{\dagger} (\overline{D} - P^*), \text{ from the proof of Theorem 9.2}.
\end{align*}
\]
11.7.3 Revenue adequacy

- To summarize, by (11.1) in Corollary 11.1, if $P'$ and $D'$ are feasible for the transmission system (so that the implied dispatch due to all CRRs is simultaneously feasible) then we have:

$$[\pi^\text{LMP}_P]^{\dagger} (D' - P') \leq [\pi^\text{LMP}_P]^{\dagger} (\overline{D} - P^*)$$

- where $\pi^\text{LMP}_P$ are the LMPs corresponding to the actual dispatch $P^*$ and $\overline{D}$.

- Note that:
  - the term on the left is payout from the CRRs, while
  - the term on the right is the congestion rent.

- That is, we have shown that the revenue to the ISO from the congestion rent is revenue adequate to fund the CRRs so long as the implied dispatch is simultaneously feasible.

- In issuing CRRs to market participants, the ISO only needs to make sure that the implied dispatch of the CRRs is simultaneously feasible.
11.7.4 Test system

- CRRs are typically issued for extended periods such as a month or longer.
- A **test system** is used for testing that the CRRs are simultaneously feasible.
- If the system used in the actual market has smaller line capacities (or there is a constraint represented in the market that was not in the test system) then the revenue adequacy result no longer applies.
- A **derating policy** is needed for the case of revenue shortfall.
11.7.5 Hedging real-time prices

- Most of the discussion has concerned hedging day-ahead prices, assuming CfDs and CRRs.
- Several markets, including ERCOT, allow the purchase in the day-ahead market of CRRs based on real-time prices.
- Approach is similar to CRR auction, except that injection and withdrawal is modeled in day-market along with offers and bids:
  - Model each injection as a generation and each withdrawal as a demand, with power level equal to the real-time CRR contract quantity.
- Allows hedging of real-time market positions.
11.8 Summary

- In this chapter we have considered volatility of energy and transmission prices and hedging.
- We considered day-ahead and real-time markets, and contracts for differences.
- We defined FTRs/CRRs and considered conditions for revenue adequacy.
References


11.1 Suppose that a generator and demand are exposed to the same LMP and that they have agreed to a contract for differences with contract price $30/MWh and contract quantity 10MW. Further suppose that the generator has capacity 10MW, which it offers into the market, and that the demand always has a demand of 10MW. For each of the combinations of generator offer prices and LMPs in the table, evaluate:

(i) the dispatch level for the generator in MW,
(ii) the demand payment to the ISO in $/h,
(iii) the demand payment to the generator under the contract for differences in $/h,
(iv) the total payment by demand in $/h,
(v) the ISO payment to the generator in $/h, and
(vi) the total payment to the generator in $/h.

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11.2 Suppose that a generator and demand are exposed to the different LMPs and that they have agreed to a contract for differences with contract price $30/MWh and contract quantity 10MW, based on the demand LMP. Also suppose that the generator possesses a CRR for 10MW for injection at its bus and withdrawal at demand. Further suppose that the generator has capacity 10MW, which it offers into the market, and that the demand always has a demand of 10MW. For each of the combinations of generator offer prices and LMPs in the table, evaluate:

(i) the dispatch level for the generator in MW,
(ii) the demand payment to the ISO in $/h,
(iii) the demand payment to the generator under the contract for differences in $/h,
(iv) the total payment by demand in $/h,
(v) the ISO payment to the generator under the CRR in $/h,
(vi) the ISO payment to the generator in $/h, and
(vii) the total payment to the generator in $/h.

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<td>Demand LMP ($/MWh)</td>
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