Abstract

This project will involve the exploration of a directional extension of multidimensional wavelet transforms, called “contourlets”, to perform pattern recognition. First, the general concept of a directional extension vs. a regular multidimensional wavelet transform will be discussed and explain the reasoning behind the directional extension. Then, an objective comparison will be done using sample images between the wavelet transform and other edge detection methods for feature detection.
Traditionally, feature detection was done with a variety of methods, such as Laplacian operators, gradient operators, the Laplacian of Gaussians, difference of Gaussians, Canny detectors, or anisotropic diffusion. However, wavelet transforms have come into light as a means of feature detection. [1]

Wavelets are classified as a linear transform that is capable of displaying the transformed output at multiple resolutions depending on the point of time/space and at the desired frequency. In contrast to the short-time Fourier transform (STFT), the resolution changes depending on the frequency that is to be examined and at what time or spatial area is to be examined. [2]

In the 1-D case, wavelets are used for signal processing by the virtue that wavelets can store more frequency information with less coefficients and reconstruction is only limited by the coefficients that are available. Wavelets can be naively extended to the 2-D case by means of separable functions, but there is limited directional information stored in a regular 2-D wavelet transform. Because of the seperability limitations, only a horizontal, vertical, and a 45 degree component can be easily determined. Incidentally, edges can be seen easily, but directional information about the edge is not known. Because of this, it takes more coefficients to do a proper reconstruction of the edges. Figure 1 shows an example of reconstruction of the Peppers image with varying scale factors. [3]

Typically, a separable 2-D wavelet transform provides:

- multiresolution, which is the ability to visualize the transform with varying resolution from coarse to fine
• localization, which is the ability of the basis elements to be localized in both the spacial and frequency domains

• critical sampling, which is the ability for the basis elements to have little redundancy.

However, it is not capable of providing:

• directionality, which is having basis elements defined in a variety of directions

• anisotropy, which is having basis elements defined in various aspect ratios and shapes. [4]
Figure 1: Reconstruction of the Peppers image with varying scale factors from 2-D wavelet transform. [5]

There are many other directional extensions of the 2-D wavelet transform that could be potentially examined that also possess directionality and anisotropy. However, most of the extensions are adaptive transforms. One of the first fixed transforms to possess directionality and anisotropy is the
curvelet transform. Conceptually, the curvelet transform captures curves instead of points as in the regular 2-D wavelet transform. Figure 2 shows the general concept of capturing curves. The transform involves splitting an image into subbands, windowing each of the subbands, and then applying the ridgelet transform to each windowed subband. The combination provides directionality through the ridgelet transform, and the anisotropy through the windowing of the subband. Figure 3 shows a diagram of how the curvelet transform is calculated. [6]

![Figure 2: Conceptual visualization of curvelets/contourlets.](image)
Contourlets are an extension of curvelets, which are defined on the continuous domain, which can be approximated in the discrete domain. Contourlets, however, are defined and derived in the discrete domain from the beginning. They both allow for directionality and anisotropy.

Contourlets are implemented by using a filter bank that decouples the multiscale and the directional decompositions. In Figure 4, Do and Vetterli show a conceptual filter bank setup that shows this decoupling. We can see that a multiscale decomposition is done by a Laplacian pyramid, then
a directional decomposition is done using a directional filter bank. The improvement with reconstruction and edge detection can be seen in Figure 5. This transform is suitable for applications involving edge detection with a high curve content, plus potential uses in compression. [4]

![Figure 4: Filter bank for contourlet transform.](image)

This project will involve the exploration of directional extension of 2-D wavelet transforms for feature detection and comparisons against each other. It is the hope of this project that we can demonstrate how contourlets can be used for feature detection and their advantages over the traditional 2-D wavelet transform and other standard edge detection techniques. Source code for contourlets, wavelets, and most of the other basic edge detection techniques are readily available for MATLAB. [7]
Figure 5: Reconstruction of the Peppers image with varying scale factors from contourlet transform. [5]

References


