Convolution of Two Rectangular Pulses

1. Continuous-Time Convolution

\[ y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\lambda) h(t-\lambda) \, d\lambda \]

There is no overlap when \( a+t < -2 \Rightarrow t < -4 \)
There is no overlap when \( a+t > 2 \Rightarrow t > 4 \)
For \( -4 \leq t \leq 0 \), there is overlap from \( \lambda = -2 + t \) to \( \lambda = 2 + t \)

\[ y(t) = \int_{-\infty}^{\infty} x(\lambda) h(t-\lambda) \, d\lambda = \int_{-2}^{a+t} x(\lambda) \, d\lambda = [\lambda]_{-2}^{a+t} = t + 4 \]
For $0 < t \leq 4$, there is overlap from $\lambda = -2 + t$.

So $\lambda = 2$

\[
y(t) = \int_\infty^{-\lambda} x(\lambda) h(t - \lambda) \, d\lambda = \int_{-2 + t}^{2 + t} 1 \, d\lambda = 2 \left[ \frac{\lambda}{2 + t} \right]_{-2 + t}^{2 + t} = 4 - t
\]

\[
y(t) = \begin{cases} 
0 & \text{for } t < -4 \\
4 + t & \text{for } -4 \leq t \leq 0 \\
4 - t & \text{for } 0 \leq t \leq 4 \\
0 & \text{for } t > 4
\end{cases}
\]

To check a convolution result, check the value of $y(t)$ at the endpoints of each interval — they should agree.

2. Discrete-Time Convolution

We will work a similar convolution in discrete time.

\[
x(n) \quad h(n)
\]

\[
y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k)
\]

where $k$ has the same role as $\lambda$. 

\[
h(n-k)
\]
There is no overlap when \( 2 + n < -2 \Rightarrow n < -4 \)
There is no overlap when \( 2 + n > 2 \Rightarrow n > 4 \)

For \(-4 \leq n \leq 0\), there is overlap from \(k = -2\) to \(k = 2 + n\)
\[y(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k) = \sum_{k=-2}^{2+n} 1 = n + 5\]

For \(1 \leq n \leq 4\), there is overlap from \(k = -2 + n\) to \(k = 2\)
\[y(n) = \sum_{k=-2+n}^{\infty} x(k) h(n-k) = \sum_{k=-2+n}^{2} 1 = 5 - n\]

\[y(n) = \begin{cases} 
0 & \text{for } n < -4 \\
5 + n & \text{for } -4 \leq n \leq 0 \\
5 - n & \text{for } 1 \leq n \leq 4 \\
0 & \text{for } n > 4 
\end{cases}\]