The University of Texas at Austin  
Dept. of Electrical and Computer Engineering
Midterm #1

Date: October 18, 2013  
Course: EE 445S Evans

Name:   Bollywood, Sally
Last, First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. Please disable all wireless connections on your computer system(s).
- Please turn off all cell phones.
- No headphones allowed.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote text from a source, please give the quote, page number and source citation.

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Discrete-Time Filter Analysis  
Discrete-Time Filter Design  
System Identification  
Modulation and Demodulation
Problem 1.1 Discrete-Time Filter Analysis. 29 points.

A causal stable discrete-time linear time-invariant filter with input \( x[n] \) and output \( y[n] \) is governed by the following block diagram:

Constants \( a_1, b_0 \) and \( b_1 \) are real-valued, and \( |a_1| < 1 \).

(a) From the block diagram, derive the difference equation relating input \( x[n] \) and output \( y[n] \). Your final answer should not include \( v[n] \). 6 points.

Working backwards from transfer function in part (c) below,

\[
\frac{\hat{y}(z)}{\hat{x}(z)} = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}} \quad \Rightarrow \quad (1 - a_1 z^{-1}) \hat{y}(z) = (b_0 + b_1 z^{-1}) \hat{x}(z)
\]

Applying the inverse \( z \)-transform to both sides,

\[
y[n] - a_1 y[n-1] = b_0 x[n] + b_1 x[n-1]
\]

(b) What are the initial condition(s)? What value(s) should they be assigned and why? 4 points.

\( v[-1] = 0 \) for the system to be causal, linear and time-invariant. Equivalently, \( x[-1] = 0 \) and \( y[-1] = 0 \).

(c) What is the transfer function in the \( z \)-domain? What is the region of convergence? 5 points.

\[
H(z) = \frac{\hat{y}(z)}{\hat{x}(z)} = \frac{\hat{V}(z) + \hat{V}(z)}{\hat{X}(z)} = \frac{1}{1 - a_1 z^{-1}} \frac{(b_0 + b_1 z^{-1})}{b_0 + b_1 z^{-1}} \text{ for } |z| > |a_1|
\]

(d) Find the equation for the frequency response of the filter. Justify your approach. 6 points.

Because \( |a_1| < 1 \), the region of convergence \( |z| > |a_1| \) includes the unit circle. \( H_{freq}(\omega) = H(z) \bigg|_{z = e^{j\omega}} = \frac{b_0 + b_1 e^{-j\omega}}{1 - a_1 e^{-j\omega}} \)

(e) For \( a_1 = -0.9, b_0 = 1 \), and \( b_1 = -1 \), draw the pole-zero diagram. What is the best description of the frequency selectivity: lowpass, highpass, bandstop, bandpass, allpass or notch? 7 points.

Passband is centered at \( \omega = \pi \) due to pole at \( z = -0.9 \).

Stopband is centered at \( \omega = 0 \) due to zero at \( z = 1 \). Highpass filter.
Problem 1.2 Discrete-Time Filter Design. 24 points.  Configurable / programmable notch filter

Consider a causal second-order discrete-time infinite impulse response (IIR) filter with transfer function $H(z)$.

The filter is a bounded-input bounded-output stable, linear, and time-invariant system.

Input $x[n]$ and output $y[n]$ are real-valued.

The feedback and feedforward coefficients are real-valued. \( \Rightarrow \) Poles are conjugate symmetric.  

Zeros are conjugate symmetric.  

You will be asked to design and implement a notch filter:

- $f_0$ is the frequency in Hz to be eliminated, and
- $f_s$ is the sampling rate in Hz where $f_s > 2f_0$

Assume that the gain of the biquad is 1. \( \Rightarrow \) $C = 1$

(a) Give a formula for the discrete-time frequency $\omega_0$ in rad/sample to be eliminated. 3 points.

\[ \omega_0 = 2\pi \frac{f_0}{f_s} \]

(b) Give formulas for the two poles and the two zeros as functions of $\omega_0$. 6 points.

\[ \text{Poles: } p_0 = 0.9 e^{j\omega_0} \quad \text{and} \quad p_1 = 0.9 e^{-j\omega_0} \]

\[ \text{Zeros: } z_0 = e^{j\omega_0} \quad \text{and} \quad z_1 = e^{-j\omega_0} \]

(c) Give formulas for the three feedforward and two feedback coefficients. Simplify the formulas to show that all of these coefficients are real-valued. 9 points.

\[ H(z) = \frac{(1 - z_0 z^{-1})(1 - z_1 z^{-1})}{(1 - p_0 z^{-1})(1 - p_1 z^{-1})} = \frac{1 - (z_0 + z_1)z^{-1} + z_0 z_1 z^{-2}}{1 - (p_0 + p_1)z^{-1} + p_0 p_1 z^{-2}} \]

Feedforward coefficients

\[ b_0 = 1 \]
\[ b_1 = -(z_0 + z_1) = -(e^{j\omega_0} + e^{-j\omega_0}) = -2 \cos(\omega_0) \]
\[ b_2 = z_0 z_1 = e^{j\omega_0} e^{-j\omega_0} = 1 \]

Feedback coefficients

\[ a_1 = p_0 + p_1 = 1.8 \cos(\omega_0) \]
\[ a_2 = -p_0 p_1 = -0.81 \]

(d) How many multiplication-accumulation operations are needed to compute one output sample given one input sample? 3 points.

\[ y[n] = a_1 y[n-1] + a_2 y[n-2] + x[n] + b_1 x[n-1] + b_2 x[n-2] \]

3 multiplications and 4 additions \( \Rightarrow \) 4 multiply-accumulate

(e) How many instruction cycles on the TI TMS3206748 digital signal processor used in lab will take to compute one output sample given one input sample? 3 points.

\[ N = 5 \text{ coefficients} \]
\[ N + 28 = 33 \text{ instruction cycles from Appendix N in reader.} \]
Alternate Solution:

\[ H(z) = \frac{Y(z)}{X(z)} = \frac{1 + z^{-1}}{1 - z^{-1}} = (1 + z^{-1})(1 - z^{-1}) = 1 - 2z^{-2} \Rightarrow h[n] = 8[n] - 8[n-2] \]

**Problem 1.3 System Identification. 24 points.** Solution uses de-convolution.

Consider a causal discrete-time finite impulse response (FIR) filter with impulse response \( h[n] \).

The filter is a bounded-input bounded-output stable, linear, and time-invariant system.

For input \( x[n] = u[n] \), the output is \( y[n] = \delta[n] + \delta[n-1] \). Let \( h[n] \) have \( M+1 \) coefficients.

(a) Determine the impulse response \( h[n] \). 18 points.

\[
y[n] = x[n] * h[n] = \sum_{m=0}^{M} h[m] \times [n-m]
\]

\[
1 = y[0] = h[0] \times [0] \quad \Rightarrow \quad 1 = h[0] \quad \Rightarrow \quad h[0] = 1
\]

\[
1 = y[1] = h[0] \times [1] + h[1] \times [0]
\]

\[
1 = h[0] + h[1] \quad \Rightarrow \quad h[1] = 0
\]

\[
\]

\[
0 = h[0] + h[1] + h[2] \quad \Rightarrow \quad h[2] = -h[0] \quad \Rightarrow \quad h[2] = -1
\]

\[
\]

\[
\]

Check: \( h[n] \times u[n] \neq \delta[n] + \delta[n-1] \quad \text{[Yes]} \)

(b) Compute the group delay through the filter as a function of frequency. 6 points.

\[
H_{\text{freq}}(\omega) = 1 - e^{-j2\omega}
\]

\[
e^{-j\omega} (e^{j\omega} - e^{-j\omega})
\]

\[
= 2 \sin(\omega) \quad \text{[Amplitude term]}
\]

\[
\quad \quad \quad \quad \quad \text{[Phase term]}
\]

\[
\text{Delay} (\omega) = -\frac{d}{d\omega} \Delta H_{\text{freq}}(\omega) = 1
\]

Except for two points of discontinuity, \( \Delta H_{\text{freq}}(\omega) = -\omega + \frac{\pi}{2} \)
Problem 1.4. Modulation and Demodulation. 24 points.

A mixer can be used to realize sinusoidal amplitude modulation $y(t) = x(t) \cos(2 \pi f_c t)$ for baseband signal $x(t)$.

\[ m(t) \xrightarrow{\text{Lowpass Filter}} x(t) \xrightarrow{\text{Sampler at sampling rate of } f_s} v(t) \xrightarrow{\text{Bandpass Filter}} y(t) \]

Assume that $x(t)$ is a ideal bandpass signal whose magnitude spectrum is zero for $|f| > f_{\text{max}}$.

Assume that $f_c > 2f_{\text{max}}$ and $f_c = m f_s$ where $m$ is a positive integer.

(a) Draw the magnitude spectrum of $x(t)$. 6 points.

(b) Draw the magnitude spectrum of $v(t)$. 6 points.

Spectrum of $X(f)$ is replicated at offsets in frequency equal to multiples of $f_s$.

(c) Draw the magnitude spectrum of $y(t)$. 6 points.

(d) Using only a lowpass filter, bandpass filter, and a sampler, give a block diagram for demodulation. 6 points.

\[ f_c = m f_s \]

\[ f_{\text{pass band}} = f_{\text{max}} \]

\[ f_{s+pb} = f_s - f_{\text{max}} \]