Comparing Massive MIMO at Sub-6 GHz and Millimeter Wave Using Stochastic Geometry

Robert W. Heath Jr., PhD, PE

Wireless Networking and Communications Group
Department of Electrical and Computer Engineering
The University of Texas at Austin
http://www.profheath.org
Joint work with Tianyang Bai

Funded by the NSF under Grant No. NSF-CCF-1218338 and a gift from Huawei
Going massive in 5G

Massive MIMO at sub 6 GHz

> 64 antennas

10 to 30 users sharing same resources

Massive MIMO at mmWave with small cells

Fewer than 4 users sharing same resources

Massive MIMO and small cells are a competing or complimentary technology depending on the carrier frequency

Outline

- Features of massive MIMO at sub-6 GHz and mmWave
- Framework for comparison
- Analytical results with infinite & finite #s of antennas
- Visualizing the gains of going massive

Some results are described here:


Other results are in various submitted papers
Massive MIMO at sub-6 GHz
Features of massive MIMO & implications

Large antenna arrays serve more users to increase cell throughput
[Compare sum rate as performance metric]

Fading and noise become minor with large arrays
[Ignore noise in sub-6GHz analysis]

Out-of-cell interference reduced due to asymptotic orthogonality of channels
[Show SIR convergence]

TDD (time-division multiplexing) avoids downlink training overhead
[Include pilot contamination]

Simple signal processing becomes near-optimal, with large arrays
[Assume matched filter beamforming]
Massive MIMO at millimeter wave
Why millimeter wave?

- Huge amount of spectrum possibly available in mmWave bands
- Technology advances make mmWave possible for low cost consumer devices
- MmWave research is as old as wireless itself, e.g. Bose 1895 and Lebedow 1895
Why large arrays at mmWave?

Millimeter wave band possible bands used for cellular:

- 1.3 GHz
- 2.1 GHz
- 7 GHz (unlic)
- 10 GHz
- 28 GHz
- 37 / 42 GHz
- 60 GHz
- E-Band
- ... to 300 GHz

Spatial multiplexing & beamforming:

- Just beamforming

MIMO is a key feature of 5G mmWave systems.

Array gain:

- Isotropic radiator
- MMWave aperture
- TX
- RX
- Sub-6GHz aperture

Multiple data streams:

Spectral efficiency:

Features of mmWave massive MIMO & implications

- Increase cell throughput with large bandwidth at mmWave [Compare with sub-6 GHz w/ different bandwidth]
- Exploit channel sparsity to reduce training overhead [Apply compressed sensing channel estimation (future work)]
- 256 antennas or more @ BS
- MmWave requires directivity gain from large arrays to overcome high path loss and noise [Model directional beamforming]
- Out-of-cell interference reduced due to directional transmission and blockage [Incorporate blockages]

Need common framework to make a fair comparison
Differentiating features between sub-6 GHz & mmWave included in the analysis
<table>
<thead>
<tr>
<th></th>
<th>sub-6 GHz</th>
<th>mmWave</th>
</tr>
</thead>
<tbody>
<tr>
<td>bandwidth</td>
<td>~100 MHz</td>
<td>500 GHz @28 GHz</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 GHz @E-Band</td>
</tr>
<tr>
<td>small-scale fading</td>
<td>correlated with high rank</td>
<td>correlated with low rank, varies</td>
</tr>
<tr>
<td></td>
<td></td>
<td>with LOS or NLOS</td>
</tr>
<tr>
<td>large-scale fading</td>
<td>distant dependent pathloss</td>
<td>distant dependent with random</td>
</tr>
<tr>
<td></td>
<td></td>
<td>blockage model and total outage</td>
</tr>
<tr>
<td>network deployment</td>
<td>low BS density</td>
<td>high BS density</td>
</tr>
<tr>
<td>UE array configuration</td>
<td>single antenna</td>
<td>directional antenna with gain</td>
</tr>
<tr>
<td># users served</td>
<td>higher (10 or more)</td>
<td>1 to 4 users (limited by hardware)</td>
</tr>
</tbody>
</table>
Comparisons built around a stochastic geometry framework
Stochastic geometry in cellular systems

Shows reasonable fits with real BS distributions

Analyzes the system performance in large networks (in closed form for certain cases)

Extends to many applications:
Heterogeneous, offloading, mmWave …

Modeling base stations locations as Poisson point process

Apply stochastic geometry to compare massive MIMO @ sub-6 GHz and mmWave


& many more…
Challenges of analyzing massive MIMO using SG

Most prior SG cellular models

<table>
<thead>
<tr>
<th>Single user per cell</th>
<th>Multiple user per cell</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single base station antenna</td>
<td>Massive base station antennas</td>
</tr>
<tr>
<td>Rayleigh fading</td>
<td>Correlated fading MIMO channel</td>
</tr>
<tr>
<td>No channel estimation</td>
<td>Pilot contamination</td>
</tr>
<tr>
<td>Mainly focus on downlink</td>
<td>Analyze both uplink and downlink</td>
</tr>
</tbody>
</table>
Sub-6 GHz massive MIMO: system model
System model

- Base station w/ M antennas
- 1st scheduled user
- 2nd scheduled user

Presence of a “red” user in one cell prevents those of the other red

Base stations distributed as a PPP

Users PPP w/ high density
BS randomly schedules K users

Scheduled users do not form a PPP (# of scheduled users fixed)

Use certain hardcore Matérn process

Channel model

Channel vector from BS $l$ to user $k$ in cell $n$

$$h_{ln}^{(k)} = \left( \beta_{ln}^{(k)} \right)^{1/2} \Phi_{ln}^{(k)/2} w_{ln}^{(k)}$$

- **Bounded path loss model**
  - Path loss of a link with length $R$
    $$C \max(\delta, R)^{-\alpha}$$
  - Address near-field effects in path loss

- **IID Gaussian vector for fading**
  - Covariance matrix for correlated fading

- **Mean square of eigenvalues uniformly bounded**

  $$\lim_{M \to \infty} \sup_{M} \frac{1}{M} \sum_{m=1}^{M} \lambda_{ln}^{(k)}[m] \leq \gamma$$

- Reasonable for rich scattering channel
Uplink channel estimation

Assume perfect time synchronization & full pilot reuse in the network

Channel estimate of $\ell$-th BS to its $k$-th user

$$\tilde{h}_{\ell \ell}^{(k)} = h_{\ell \ell}^{(k)} + \sum_{\ell' \neq \ell} h_{\ell \ell'}^{(k)}$$

Error from pilot contamination

Need to incorporate pilot contamination in system analysis
Uplink data transmission

BSs perform maximum ratio combining based on channel estimates

$$\text{SIR}_U = \frac{|\tilde{h}_{00}^{(1)} * h_{00}^{(1)}|^2}{\sum_{\ell > 0} |\tilde{h}_{00}^{(1)} * h_{0\ell}^{(1)}|^2 + \sum_{k > 1} \sum_{\ell \geq 0} |\tilde{h}_{00}^{(k)} * h_{0\ell}^{(k)}|^2}$$

As $M$ grows large

Out-of-cell interference with different pilots disappears from expression
Downlink data transmission

BSs perform match-filtering beamforming based on channel estimates

\[
SIR_D = \frac{|h_{00}^{(1)}f_0^{(1)}|^2}{\sum_{\ell \neq 1} |h_{\ell 0}^{(1)}f_\ell^{(1)}|^2 + \sum_{k=2}^{K} \sum_{\ell > 0} |h_{\ell 0}^{(k)}f_\ell^{(k)}|^2}
\]

As M grows large, out-of-cell interference with different pilots disappears from expression.
Sub-6 GHz massive MIMO: asymptotic performance analysis when # of BS antennas goes to infinity
Prior results assuming IID fading & finite # BSs

UL received signal

\[ \hat{s}_{0}^{(1)} = \frac{\| h_{00}^{(1)} \|^2 s_{0}^{(1)}}{M} + \sum_{\ell > 1} \frac{\| h_{0\ell}^{(1)} \|^2 s_{\ell}^{(1)}}{M} + \sum_{k=2}^{K} \frac{h_{0\ell}^{(1)} h_{0\ell}^{(k)} s_{\ell}^{(k)}}{M} \]

By LLN for IID variables

\[ \lim_{M \to \infty} \frac{h_{st}^{(k)} h_{st}^{(k)}}{M} \xrightarrow{p.} \beta_{st}^{(k)} \]

\[ \lim_{M \to \infty} \hat{s}_{0}^{(1)} \xrightarrow{p.} \beta_{00}^{(1)} s_{0}^{(1)} + \sum_{\ell > 0} \beta_{0\ell}^{(1)} s_{\ell}^{(1)} + 0 \]

SIR_{UL} \xrightarrow{p.} \frac{(\beta_{00}^{(1)})^2}{\sum_{\ell \neq 0} (\beta_{0\ell}^{(1)})^2}

What about spatial correlation and infinite number of BSs??

Dealing with infinite interferers

Difficulty: cannot swap limit and infinite sum directly, with infinite BSs

\[ \sum_{\ell > 0} \left( \frac{||\mathbf{h}_{0\ell}^{(1)}||^2}{M} - \beta_{0\ell}^{(1)} \right) = \sum_{\ell: ||Y_{\ell}^{(1)} - X_0|| \leq R_0} \left( \frac{||\mathbf{h}_{0\ell}^{(1)}||^2}{M} - \beta_{0\ell}^{(1)} \right) + \sum_{\ell: ||Y_{\ell}^{(1)} - X_0|| > R_0} \left( \frac{||\mathbf{h}_{0\ell}^{(1)}||^2}{M} - \beta_{0\ell}^{(1)} \right) \]

Solution: use SG in proof

Use stochastic geometry to prove convergence of infinite sum
Asymptotic SINR results

<table>
<thead>
<tr>
<th>Compared with SISO: Path loss exponent doubles</th>
<th>Asymptotic SINR expression</th>
<th>CCDF of SINR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uplink</td>
<td>( \frac{(\beta_{00}^{(1)})^2}{\sum_{\ell \neq 0} (\beta_{0\ell}^{(1)})^2} )</td>
<td>( 1 - e^{-\left(\frac{a_{\ell}}{T}\right)^{1/\alpha}} )</td>
</tr>
<tr>
<td>Downlink</td>
<td>( \frac{\beta_{00}^{(1)} / a_0^{(1)}}{\sum_{\ell \neq 0} \beta_{\ell 0}^{(1)} / a_{\ell}^{(1)}} )</td>
<td>( \min \left( 1, \frac{\alpha \sin(\pi/\alpha)}{\pi T^{1/\alpha}} \right) )</td>
</tr>
</tbody>
</table>

\[ a_{\ell}^{(k)} = \sum_{\ell'} \beta_{\ell' \ell'}^{(k)} \] due to power normalization in DL

Stochastic geometry allows simple expressions for coverage under the bounded spatial correlation model the following hold:

DL and UL SIR distribution are different.
Asymptotic uplink SIR plots

Convergence to asymptotic SIR
(IID fading, $K=10, \alpha=4$)

Asymptotic better than SISO

Require >10,000 antennas to approach asymptotic curves
Comparing UL and DL distribution

Much different SIR distribution observed in DL and UL

Indicate decoupled system design for DL and UL
MmWave massive MIMO
MmWave massive MIMO network model

Sected beamforming pattern model @ UE
Back lobe gain
Main lobe array gain
Main lobe beamwidth

Modeling blockage effects of buildings

Use LOS probability function of the link length to determine LOS/ NLOS/ outage

<table>
<thead>
<tr>
<th></th>
<th>LOS</th>
<th>NLOS</th>
<th>Total outage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path loss</td>
<td>LOS path loss</td>
<td>NLOS path loss</td>
<td>No signal</td>
</tr>
<tr>
<td>Fading</td>
<td>Deterministic (no fading)</td>
<td>Sub-6 GHz fading (w/ more correlations)</td>
<td>NA</td>
</tr>
</tbody>
</table>

### MmWave asymptotic SINR results

<table>
<thead>
<tr>
<th></th>
<th>Asymptotic SINR expression</th>
<th>CCDF of SINR</th>
</tr>
</thead>
</table>
| **Asymptotic mmWave uplink** | \[
\frac{Q^2 \beta_{00}^{(1)^2}}{\sum_{\ell \neq 0} D_{0\ell}^{(1)^2} \beta_{0\ell}^{(1)^2}}
\] |
| Directivity gain from      |                             |
| **Asymptotic mmWave downlink** | \[
\frac{Q^2 \beta_{00}^{(1)^2}}{\sum_{\ell \neq 0} D_{0\ell}^{(1)^2} \beta_{0\ell}^{(1)^2} / a_0^{(1)}}
\] |
| **UE beamforming**        |                             |

**Can be computed through numerical integration**

LOS/ NLOS effects make expressions complicated
**MmWave SINR sensitive to BS densities**

Carrier frequency: 28 GHz  
Bandwidth: 500 MHz  
BS: ULA of M antennas  
UE: Omni-directional  
Blockage parameter: NYU model in [1]  
(Avg. LOS 70 m, no signal > 200 m)  
TX power:  
UL: 20 dBm  
DL: 30 dBm

- **Good coverage achieved with dense BSs**
- **Converges fast to asymptotic when BSs dense**
- **Converges slow to asymptotic due to high noise power relative to NLOS signals**
- **Sparse network subject to severe outage**

MmWave massive MIMO needs dense BS deployment
Rate comparison
Comparing sub-6 GHz and mmWave massive MIMO

<table>
<thead>
<tr>
<th></th>
<th>2 GHz</th>
<th>28 GHz</th>
<th>73 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier freq.</td>
<td>2 GHz</td>
<td>28 GHz</td>
<td>73 GHz</td>
</tr>
<tr>
<td>bandwidth</td>
<td>100 MHz</td>
<td>Varies</td>
<td>Varies</td>
</tr>
<tr>
<td># of scheduled user per cell</td>
<td>10</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td># of base station antennas</td>
<td>8X8</td>
<td>16X16</td>
<td>40X40</td>
</tr>
<tr>
<td># of UE antennas</td>
<td>1</td>
<td>2X2</td>
<td>5X5</td>
</tr>
<tr>
<td>TX power (DL/ UL)</td>
<td>46/ 20 dBm</td>
<td>30/ 20 dBm</td>
<td>30/ 20 dBm</td>
</tr>
</tbody>
</table>

1. We vary the bandwidth of mmWave systems in the simulations
2. We assume the same amount of overhead for all systems
3. Use the parameters in the blockage model from [1] based on NYU measurements

Comparison of average cell throughput

28 GHz Cell throughput
Large gain with dense BSs deployment
“Black” in heatmap indicates same cell throughput in mmWave and 2 GHz

Gain for mmWave

73 GHz Cell throughput
Poor cell throughput due to severe outage in sparse mmWave network

Gain for sub-6 GHz

2GHz setup: bandwidth fixed as 100 MHz, while ISD varies

Gain over 2 GHz in cell throughput

MmWave benefits more from network densifications

100 m in ISD = 128 BS/ km²
200 m in ISD = 32 BS/ km²
Comparing massive MIMO w/ small cells

<table>
<thead>
<tr>
<th></th>
<th>Sub-6 GHz massive MIMO</th>
<th>28 GHz massive MIMO</th>
<th>73 GHz massive MIMO</th>
<th>Sub-6 GHz Small cell MIMO</th>
</tr>
</thead>
<tbody>
<tr>
<td># user/ cell</td>
<td>Varies</td>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td># BS antenna</td>
<td>8x8</td>
<td>16 x 16</td>
<td>40 x 40</td>
<td>2</td>
</tr>
<tr>
<td># User antenna</td>
<td>1</td>
<td>2x2</td>
<td>5x5</td>
<td>2</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>100 MHz</td>
<td>varies</td>
<td>varies</td>
<td>100 MHz</td>
</tr>
</tbody>
</table>

1. Small cell serves its user by 2x2 spatial multiplexing or SISO
2. Assume perfect channel knowledge for small cell case
3. Assume user density 40x macro massive MIMO BS density

Sub-6 GHz massive MIMO vs. Small cell

1. **Higher area throughput in massive MIMO by serving multiple users.**

2. **Gain for massive MIMO:**
   - 8 dB

3. **Gain for small cell:**
   - 0 dB

4. **Ratio of small cell density to massive MIMO:**
   - Small cell using SISO
   - Small cell using 2x2 SM

5. **Sub 6-GHz massive MIMO achieves comparable area throughput using sparser BS deployment.**

6. **Higher area throughput in small cell due to higher BS density.**
MmWave massive MIMO vs. Small cell

28 GHz massive MIMO outperforms small cell with same density

28 GHz vs 2x2 SM small cell

Gain for massive MIMO

15 dB

Gain for small cell

0 dB

MmWave provides large gain in area throughput in small-cell regime

73 GHz vs 2x2 SM small cell

Same performance

Inter-site distance in meters

Inter-site distance in meters
Conclusions

go massive @ mmWave w/ small cells

go massive @ sub-6 GHz w/ large cells