The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1 ***Version 2.0***

Date: March 11, 2026 Course: EE 445S Evans

Name: ____
Last, First

- **Exam duration**. The exam is scheduled to last 75 minutes.
- **Materials allowed**. You may use books, notes, your laptop/tablet, and a calculator.
- **Disable all networks**. Please disable all network connections on all computer systems. You may not access the Internet or other networks during the exam.
- **No AI tools allowed**. As mentioned on the course syllabus, you may not use GPT or other AI tools during the exam.
- **Electronics.** Power down phones. No headphones. Mute your computer systems.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the content in the source for your justification. You could reference homework solutions, test solutions, etc.
- **Matlab**. No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Put all work on the test**. All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Academic integrity.** By submitting this exam, you affirm that you have not received help directly or indirectly on this test from another human except the proctor for the test, and that you did not provide help, directly or indirectly, to another student taking this exam.

Problem	*Point Value*	*Your score*	*Topic*
1	24		IIR Filter Analysis
2	24		Real-Time Audio
3	28		Sinusoidal Amplitude Modulation
4	24		Potpourri
Total	100		

Problem 1.1 *IIR Filter Analysis*. 24 points.

A causal linear time-invariant (LTI) bounded-input bounded-output (BIBO) stable discrete-time infinite impulse response (IIR) filter with input $x[n]$ and output $y[n]$ observed for $n \geq 0$ described by

$$y[n] = a_1\, y[n-1] + b_0\, x[n] + b_1\, x[n-1]$$

where the coefficents a_1, b_0 and b_1 are real-valued and non-zero.

(a) What are the initial condition(s) and their value(s)? Why? *3 points*.

(b) Draw a block diagram. Be sure to use arrows to indicate the order of operations. *3 points*.

(c) Compute the transfer function in the z-domain including the region of convergence. 6 *points*.

(d) Derive a formula for the discrete-time frequency response of the filter. *3 points*.

(e) Give numeric values for the coefficents a_1, b_0 and b_1 that would allow the discrete-time IIR filter to meet the following frequency domain specification: *9 points*.

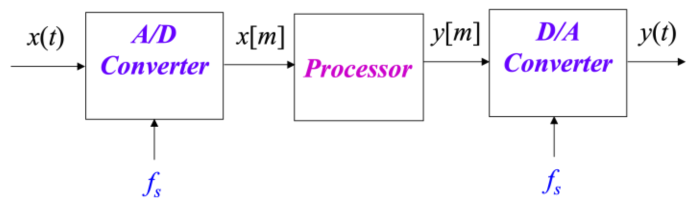
- Completely removes the discrete-time frequency of π rad/sample.
- Passes as many other frequencies as possible with a constant gain.
- Has a frequency response of 1 at a discrete-time frequency of 0 rad/sample.

Problem 1.2 *Real-Time Audio.* 24 points.

Consider a real-time single-channel audio system.

Sampling rate is $f_s = 48$ kHz.

The processor implements a finite impulse response (FIR) to extract 200-2000 Hz woofer frequencies.



The linear phase bandpass FIR filter has 1000 coefficients.

(a) Computational requirements. *6 points.*

1. How many multiplications per second are required to process the signal in real time?

2. If each multiply-add operation takes 1 CPU cycle, what is the minimum processor clock frequency required to sustain the number of multiplications required?

(b) Latency for sample-by-sample filtering. *6 points.*

1. What is the group delay (in samples) of the FIR filter? Why?

2. Convert this group delay into milliseconds for the given sampling rate.

(c) Block-based processing. A/D samples are put in a buffer of 64 samples before filtering any of them, and the filter output samples are placed in a buffer of 64 samples before sending any of them through the D/A Converter. *6 points*

1. Does the latency increase, decrease, or stay the same? Why?

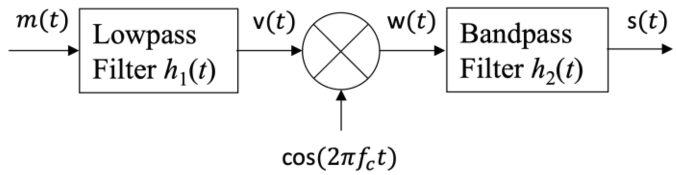
2. If you said increase or decrease, please say by how many samples.

(d) Tradeoffs. If the processor becomes overloaded due to other tasks, propose two ways to reduce the computational load while still extracting 200-2000 Hz woofer frequencies. *6 points.*

ChatGPT was used to create the initial version of this question.

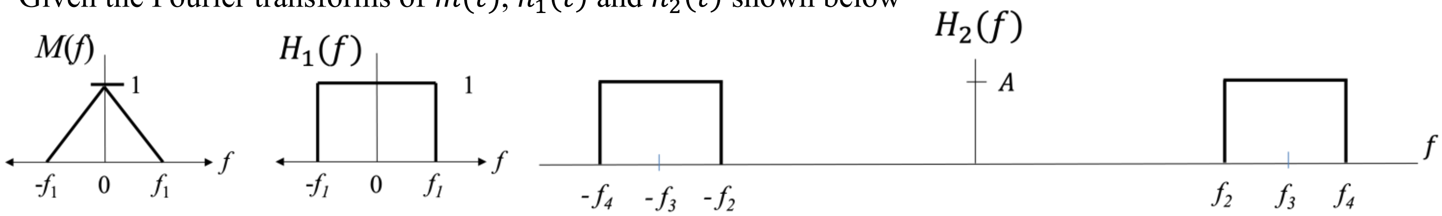
Problem 1.3. *Sinusoidal Amplitude Modulation*. 28 points.

Sinusoidal amplitude modulation can be used to convey a baseband message signal $m(t)$ wirelessly as a bandpass RF signal $s(t)$ that can propagate further. Here's a block diagram representation:



Assume that we can observe the system for $-\infty < t < \infty$ and f_c is much greater than f_1.

Given the Fourier transforms of $m(t)$, $h_1(t)$ and $h_2(t)$ shown below



(a) Draw $V(f)$ and $W(f)$. *6 points*.

(b) Draw $S(f)$. What is the value of A so that the maximum value of $S(f)$ is 1? *3 points*.

(c) Give equations for f_2, f_3 and f_4 in terms of f_1 and f_c. *6 points*.

(d) To implement the entire modulation system in discrete time, give a formula in terms of f_1 and f_c for the choice of sampling rate f_s for the entire system that would avoid aliasing. All continuous-time signals will be sampled by the same sampling rate f_s. *6 points*.

(e) Give two poles and two zeros for the discrete-time bandpass filter. Both zeros should be on the unit circle. *7 points*

Problem 1.4. *Potpourri.* 24 points.

(a) Consider a continuous-time signal that is a sum of two cosine signals observed for $-\infty < t < \infty$

$$x(t) = \cos(2\,\pi\, f_0\, t) + \cos(2\,\pi\, f_1\, t)$$

where $f_1 = 10\, f_0$. After sampling at a sampling rate of $f_s > 2\, f_0$, the discrete-time signal is

$$x[n] = \cos(\omega_0\, n) + 1$$

where $\omega_0 = 2\,\pi\, \frac{f_0}{f_s}$ and frequency f_0 does not alias; however, the frequency f_1 aliases.

What are all the possible values of f_s ? Give expressions for f_s in terms of f_0 . *12 points*.

(b) Consider a continuous-time cosine signal of a single frequency observed for $-\infty < t < \infty$

$$x(t) = \cos(2\,\pi\, f_2\, t)$$

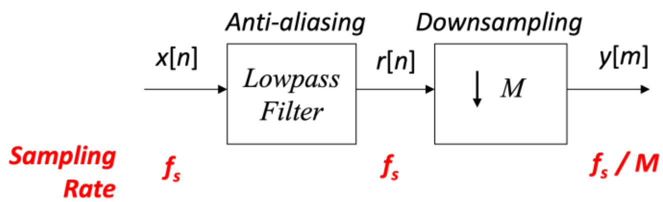
After sampling at a sampling rate of $f_s = 10\, f_2$, the discrete-time signal is

$$x[n] = \cos(\omega_2\, n)$$

where $\omega_2 = 2\,\pi\, \frac{f_2}{f_s}$ and frequency f_2 does not alias.

The signal $x[n]$ is decimated as shown on the right.

Assume that the maximum passband frequency of the discrete-time lowpass filter is π / *M*.



1. Assuming an ideal lowpass filter, what is the maximum integer value of *M* that can be used and still allow us to reconstruct $x(t)$ from $y[m]$? Why? *6 points.*

2. Assuming a practical lowpass filter, what is the maximum integer value of *M* that can be used and still allow us to reconstruct $x(t)$ from $y[m]$? Why? *6 points.*