

% In-Lecture Assignment #4 Related to Homework #6

% Consider performing an iterative *maximization* of
% $J(x) = 8 - x^2 + 6 \cos(6x)$
% via the steepest descent (*ascent*) algorithm (JSK equation (6.5) on page 116)
% with the sign on the update reversed from negative to positive so that
% the algorithm will *maximize* rather than minimize; i.e.
% $x[k+1] = x[k] + \mu \left. \frac{dJ(x)}{dx} \right|_{x=x[k]}$

% a. Visualize and analyze the shape of the objective function $J(x)$.

% 1) Plot $J(x)$ for $-5 < x < 5$. Give the Matlab code for your answer.

```
x = [-5 : 0.01 : 5];  
J = 8 - x.^2 + 6 * cos(6*x);  
plot(x, J); %% At end of document
```

% 2) Describe the plot.

% **Sum of concave down parabola and cosine creates many local maxima -OR-**

% **Headband-like rainbow shape composed in a parabolic wavy pattern -OR-**

% **Comic (graphical novel) sketch of a head with hair or a crown**

% 3) How many local maxima do you see?

% **11, which are the 9 peaks with valleys plus the two end points.**

% 4) Of these local maxima, how many are global maxima?

% **Only one, located at $x = 0$.**

% b. Derive the steepest descent (*ascent*) update equation

% $dJ(x)/dx = -2x - 36\sin(6x)$

% and modify the code below to include the derivative of $dJ(x)/dx$

```
% Code below modified from a solution by a Spring 2019 student  
% polyconverge.m find the maximum of  $J(x)=x$  via steepest descent  
N=50; % number of iterations  
mu=0.001; % algorithm stepsize  
x=zeros(1,N); % initialize sequence of x values to zero  
x(1)=0.7; % starting point x(1)  
for k=1:N-1  
    x(k+1)= x(k) + (-36*sin(6*x(k)) - 2*x(k))*mu; % update equation  
end  
figure();  
stem(x); % to visualize approximation of x  
x(N)
```

% c. Implement the steepest descent (*ascent*) algorithm in Matlab with $x[0] = 0.7$.

% 1) To what value does the steepest descent algorithm converge?

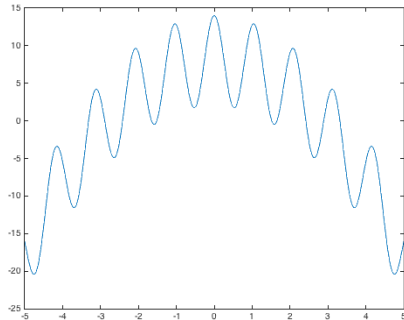
% **$x = 1.0376$**

% 2) Is the convergent value of x in the global maximum of $J(x)$? Why or why not?

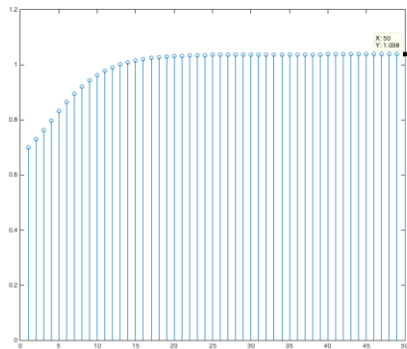
% **No. The only global maximum of $J(x)$ occurs at $x = 0$. The initial guess**

% of $x = 0.7$ is on an upward slope of $J(x)$ towards a local maximum at
 % $x = 1.0376$ and the iterates of $x[k]$ go towards the local maximum.

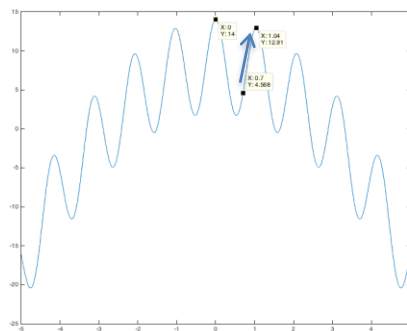
% The objective function $J(x)$ is plotted below vs. x



% The plot below shows the trajectory of $x[k]$ values vs. k



% Below, the objective function $J(x)$ is highlighted with the global maximum at $x = 0$,
 % the starting point of the steepest descent (ascent) algorithm at $x = 0.7$, and
 % the point where the steepest descent (ascent) algorithm converges at $x = 1.0376$.



% *Debugging hint:* What happens if one makes a mistake computing
 % the derivative? How I can tell that there's a mistake? The steepest
 % descent (ascent) will not correctly find the minimum (maximum).