% In-Lecture Assignment #1 on Feb. 2, 2026. Based on homework problem 1.2.

% Key takeaways: (1) Chirp signals are useful in localization, testing and training
% because they linearly sweep a range of frequencies, and (2) Spectrograms analyze
% a signal in the time and frequency domains simultaneously so that frequencies can
% be localized in time. Spectrogram trades off frequency resolution for time resolution.

% **Chirp Signals**: Please see slides 1-14 to 1-16 of CommonSignalsInMatlab.pptx.

% **Spectrograms**: Please see slides 1-17 to 1-20 of CommonSignalsInMatlab.pptx.

% **Introduction**: A chirp signal is a sinusoid whose principal frequency
% increases (or decreases) over time. A chirp signal has the form
% $c(t) = \cos(\theta(t))$ where $\theta(t) = 2\pi(f_0 + 0.5 f_{step} t)\, t = 2\pi f_0 t + \pi f_{step} t^2$
% The principal frequency in Hz is f_0 when $t = 0$ and then changes over time at a
% rate of f_{step} in units of Hz/s. The principal frequency of a sinusoid at a given
% point in time is called the *instantaneous frequency*, and it is defined as
% $d\theta(t) / dt$ in units of rad/s. $d\theta(t) / dt = 2\pi f_0 + 2\pi f_{step} t = 2\pi (f_0 + f_{step} t)$.
% We divide $d\theta(t) / dt$ by 2π to obtain instantaneous frequency in Hz of $f_0 + f_{step} t$.

% **(a) Generate a chirp** signal that lasts 10s with f_0 = 20 Hz and f_{step} = 420 Hz/s.
% Use sampling rate f_s of 44100 Hz. The chirp will sweep through the principal
% frequencies of the keys on an 88-key piano. Here's Matlab code to get started.

```
%%% Generate a chirp signal with frequency increasing
%%% from f0 to (f0 + fstep time) over time seconds
time = 10;
f0 = 20;
fstep = 420;
fs = 44100;
Ts = 1 / fs;
t = 0 : Ts : time;
%%% Add code here to define the chirp signal y = cos( angle(t) )
angle = 2*pi*f0*t + pi*fstep*t.^2;
y = cos(angle);
```

% **(b) Play the chirp signal** as an audio signal. Describe what you hear.
% *I hear a rising pitch over time. Sounds like a slide whistle or a tsunami warning siren*
% ([rb.gy/18exl](rb.gy/18exl)). *Note: Some laptop playback systems cannot play frequencies below 200 Hz.*

```
sound(y, fs);
pause(time+1);
```

% (c) Plot the spectrogram of the chirp signal and describe the visual representation.
% Spectrogram shows a yellow line that represents the principal frequency in the chirp
% signal. The line goes from 20 Hz at time 0s to 4220 Hz at time 10s. The spectrogram
% plot is on the next page. See Appendix A for explanation of spectrogram arguments.

```
figure;
blockSize = 256; overlap = 128;
spectrogram(y, hamming(blockSize), overlap, blockSize, fs, 'yaxis');
title('(c) Spectrogram with block size 256 and overlap 128');
```

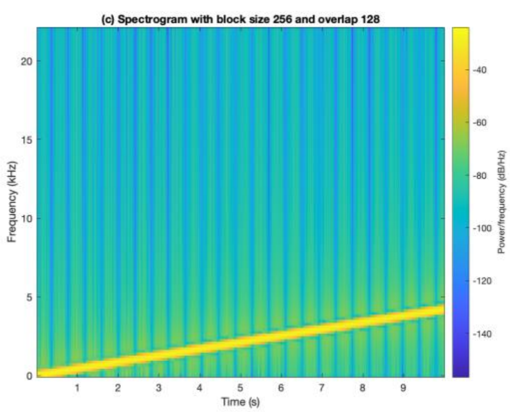
% **(d) Give the code** for the spectrogram that would improve the
% frequency resolution by a factor of two vs. part (c).
% *The frequency resolution is what is possible from observing a signal for a block of*
% *N samples which lasts for N Ts seconds. From homework problem 0.1, the frequency*
% *frequency resolution in Hz is the inverse of the observation time or 1 / (N Ts) = fs / N.*
% *Increase N to decrease (improve) frequency resolution.*
% *The yellow line in the spectrogram with N doubled is half the width vs. part (c).*
% *Please see the derivation of frequency resolution in Appendix B.*

```
figure;
blockSize = 2*256; overlap = 128;
spectrogram(y, hamming(blockSize), overlap, blockSize, fs, 'yaxis');
title('(d) Spectrogram with block size 512 and overlap 256');
```

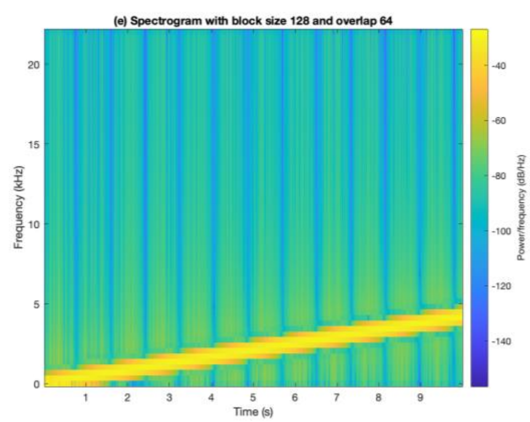
% **(e) Give the code** for the spectrogram that would improve the time resolution,
% *i.e. localizing frequency components in time,* by a factor of two *vs. part (c).*
% *The time resolution means the ability to identify when a frequency component occurs*
% *in time. In a block of N samples, we do not know when frequency components occur, and*
% *hence, our time resolution in seconds is N Ts. We improve time resolution by reducing N.*

```
figure;
blockSize = 256/2; overlap = blockSize/2;
spectrogram(y, hamming(blockSize), overlap, blockSize, fs, 'yaxis');
title('(e) Spectrogram with block size 128 and overlap 64');
```

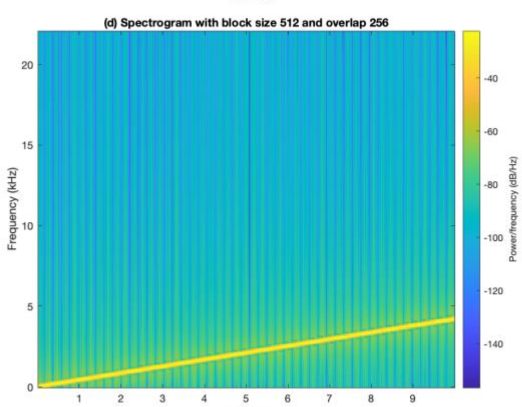
(c)



(e)



(d)



In all three spectrogram plots, the extent of the horizontal time axis is the same (from 0 to 10s) and the extent of the vertical frequency axis is the same (from 0 to ½ f_s where f_s = 44100 Hz). We have chosen f_s to satisfy the sampling theorem $f_s > 2 f_{max}$ where f_{max} is the maximum frequency of interest (4220 Hz) and to be a standard audio sampling rate.

Appendix A: Arguments to the MATLAB spectrogram function by Dan Jacobellis

In HW 1.2 and the in-lecture assignment, a spectrogram is used to visualize the chirp signal.

There are 10 possible input arguments for the spectrogram function in MATLAB which often leads to confusion.

Here are a few notes about using the spectrogram function in MATLAB.

1. If the output argument is saved, no plot will be generated.

   `s = spectrogram(...)` saves the complex-valued DFT coefficients to the variable `s` but does not create a plot.

   `figure; spectrogram(...)` creates a new window with the plot of the spectrogram.

2. The `window` parameter has two different uses

   If the `window` parameter is an integer, then MATLAB will construct a Hamming window of that length, and multiply each frame of data by the hamming window before taking the DFT. This is the suggested mode to use the function, i.e.

   `figure; spectrogram(x, 2^10...)`

3. The relationship between time and frequency resolutions is easiest to see when no overlap is used.

   Consider the following two spectrograms. Suppose the signal length is N = 2^{20} = 1048576

   Spectrogram 1:

   ```
   window = 2^10;
   noverlap = 0;
   nfft = 2^10;
   figure; spectrogram(x,window,noverlap,nfft)
   ```

   Spectrogram 2:

   ```
   window = 2^12;
   noverlap = 0;
   nfft = 2^12;
   figure; spectrogram(x,window,noverlap,nfft)
   ```

   The first spectrogram will have (2^{20} / 2^{10}) = 1024 divisions on the time axis and $2^{10}/2$ = 512 divisions on the frequency axis (the division by two is because the negative frequencies are discarded). It will result in an image that is 1024 x 512 pixels.

   The first spectrogram will have (2^{20} / 2^{12}) = 256 divisions on the time axis and $2^{12}/2$ = 2048 divisions on the frequency axis. It will result in an image that is 256 x 2048 pixels.

   Both images have the same number of pixels total, but there is a tradeoff in time and frequency resolution.

Appendix B: Derivation of Frequency Resolution

Frequency resolution of Δf Hz means two frequency components spaced Δf Hz apart can each be clearly identified by an algorithm, e.g. well separated in a plot of the frequency domain.

We'll illustrate the concept of frequency resolution by revisiting homework problem 0.1.

Homework 0.1 concerned a sine signal $c(t)$ lasting from 0s to 1s. The mathematical expression is a two-sided sine signal multiplied by a rectangular pulse that lasts from 0s to 1s:

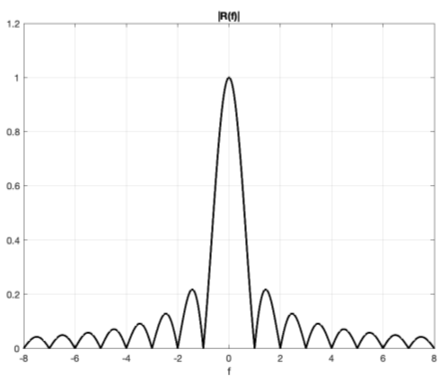
$$c(t) = \sin(2\pi f_c t)\, \mathrm{rect}(t - \tfrac{1}{2})$$

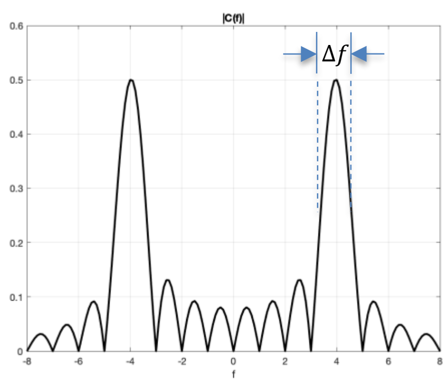
The continuous-time Fourier transform of $r(t) = \mathrm{rect}(t - \tfrac{1}{2})$ is a sinc function times a phase shift

$$R(f) = F\left\{ \mathrm{rect}\left(t - \frac{1}{2}\right) \right\} = \mathrm{sinc}(f)\, e^{-j\pi f} \text{ where } \mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x} \text{ and}$$

$$C(f) = \frac{j}{2} e^{-j\pi(f+f_c)} \mathrm{sinc}(f + f_c) - \frac{j}{2} e^{-j\pi(f-f_c)} \mathrm{sinc}(f - f_c) \text{ due to the modulation property.}$$

Below are the plots of $|R(f)|$ on the left and $|C(f)|$ for $f_c = 4$ Hz on the right:

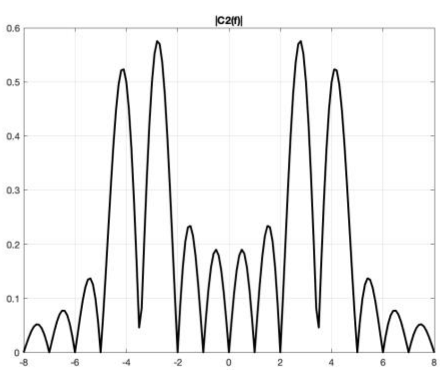




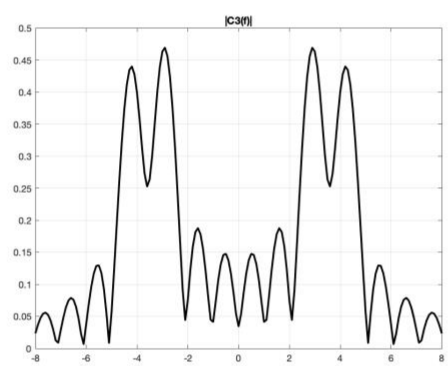
For a signal lasting 0s to 1s and containing sinusoids at frequencies 3 Hz and 4 Hz,

$$c_2(t) = \sin(2\pi f_0 t)\, \mathrm{rect}(t - \tfrac{1}{2}) + \sin(2\pi f_1 t)\, \mathrm{rect}(t - \tfrac{1}{2})$$

let's see if we can resolve the two frequencies. We're looking for two peaks in the frequency domain plot that are well separated at 3 Hz and 4 Hz. Between the peaks, the magnitude response should not be higher than the "sidelobes" at frequencies higher than 1 Hz in $|R(f)|$.



Clean separation of 3 Hz and 4 Hz frequency components



Difficulty separating 3.2 Hz and 4 Hz frequency components

More generally, for a rectangular pulse of duration T seconds, the frequency resolution is $1/T$. The value of $1/T$ is also the null bandwidth.

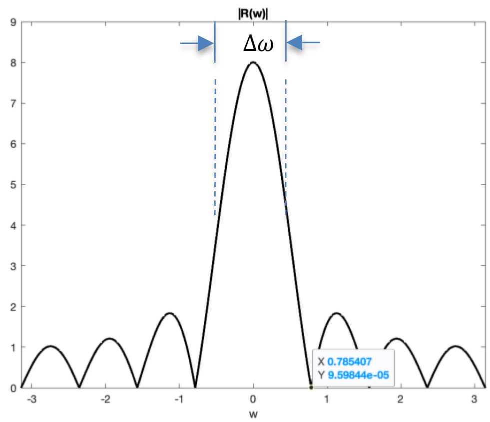
In the course of computing the spectrogram, we apply a rectangular pulse to the discrete-time signal to extract a block of samples to compute their Fourier series coefficients using the fast Fourier transform. Consider a discrete-time signal that is a two-sided sine signal and the first N samples are kept:

$$c[n] = \sin(\omega_c\, n)\ \mathrm{rect}((n - N/2)/N)$$

Here, $r[n] = \mathrm{rect}((n - N/2)/N)$ which has amplitude 1 for $n \in \{0, 1, \ldots, N-1\}$ and 0 elsewhere. We can also write $r[n] = u[n] - u[n-N]$ where $u[n]$ is the unit step function. The discrete-time Fourier transform of $r[n]$ is a periodic sinc function times a phase shift :

$$R(\omega) = \underbrace{\frac{\sin\left(\frac{N\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}}_{\substack{periodic\ sinc \\ function}} e^{-j\omega(N-1)/2}$$

where ω is in units of rad/sample. The periodic sinc function is periodic in ω with period 2π. Here's one period of $|R(\omega)|$ for $N = 8$:




```
w = -pi : 0.001 : pi;
N = 8;
Rw = sin(N*w/2) ./ sin(w/2);
figure;
plot(w, abs(Rw), 'k','LineWidth',2 );
title( '|R(w)|' );
xlabel( 'w' );
ylim( [0, 9] );
```

This is the magnitude response of an averaging filter with 8 coefficients. Please see the Designing Averaging Filters handout.

The first zero for the magnitude response in positive frequencies occurs at 2π / N. This is the null bandwidth and also the frequency resolution $\Delta\omega$.

Let's connect the frequency resolution in the discrete-time frequency domain to the continuous-time frequency domain:

$$\Delta\omega = \frac{2\pi}{N} = 2\pi\frac{\Delta f}{f_s} \text{ means that } \Delta f = \frac{f_s}{N}$$