$$\int_{-\infty}^{\infty} x(at)\,dt = ?$$

For $a > 0$, let $u = at \Rightarrow t = \frac{1}{a}u$

$$dt = \frac{1}{a}du$$

limits $t \to \infty \Rightarrow u \to \infty$

$t \to -\infty \Rightarrow u \to -\infty$

$$\int_{-\infty}^{\infty} x(u)\frac{1}{a}du = \frac{1}{a}\int_{-\infty}^{\infty} x(u)\,du$$

For $a < 0$, let $u = at \Rightarrow t = \frac{1}{a}u$

$$dt = \frac{1}{a}du$$

limits $t \to \infty \Rightarrow u \to -\infty$

$t \to -\infty \Rightarrow u \to \infty$

$$\int_{\infty}^{-\infty} x(u)\frac{1}{a}du = -\frac{1}{a}\int_{-\infty}^{\infty} x(u)\,du$$

Hence, $\int_{-\infty}^{\infty} x(at)\,dt = \frac{1}{|a|}\int_{-\infty}^{\infty} x(u)\,du$ for $a \neq 0$

$$\boxed{u(t)\Big|_{t=nT_s} = u(nT_s) = \frac{1}{T_s}u[n]}$$