**EE 313 Linear Signals & Systems (Fall 2018)**

***Solution Set for Homework #10 on Laplace Transforms***

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**Problem 1.**

1. 

From lecture and 

, for all s

This is a finite-amplitude finite-duration signal, and hence the region of convergence is the entire s plane.

At first glance, it would seem that the region of convergence should have been Re{s} > 0 because the signal is causal and the denominator goes to zero when *s* goes to zero. However, when *s* goes to zero, the numerator also goes to zero. We can use L’Hôpital’s rule by letting *s* go to zero. The derivative of the numerator with respect to s is exp(-*s*) and the derivative of the denominator with respect to s is 1. The limit of *X*(*s*) as *s* goes to zero is 1.

1. 

From and region of convergence is Re{*s*} > -Re{*a*}.





And the region of convergence is Re{*s*} > -3.

Writing the exp(-2*s*) separately from the rest of the expression can help highlight the term, which corresponds in the time domain to a delay by 2 seconds.

1. 

This part is similar to part b.



The region of convergence is Re{*s*} > -3.

1. 

From and region of convergence is Re{*s*} > 0.



And the region of convergence is Re{*s*} > 0.

**Problem 2.**



From for Re{s} >0.



The region of convergence is Re{s} > 0.

MATLAB code

t = -1:1/10000:1;

unitstep = zeros(size(t));

unitstep (t>= 0) = 1;

x = cos(20\*pi\*t).\*unitstep;

plot(t,x)

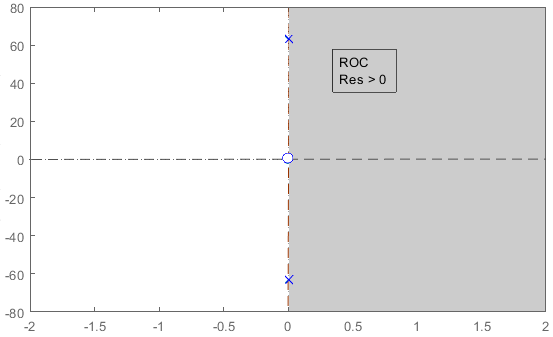
xlabel('Time(s)')

ylabel('x')



Zeros are roots of nominator and poles are roots of denominator.

,  In the figure legend, Res means Re{s}.





From for Re{s} > -Re{a}.



The region of convergence is Re{s} > -8.

t = -1:1/10000:1;

unitstep = zeros(size(t));

unitstep (t>= 0) = 1;

x = exp(-8\*t).\*unitstep;

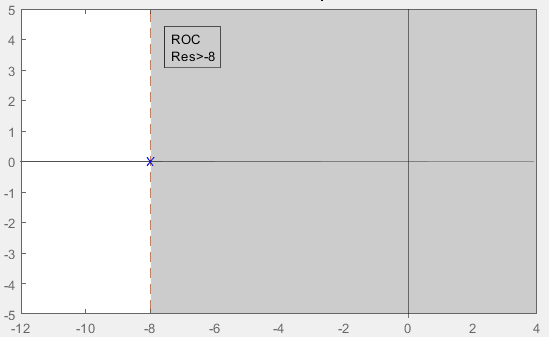
plot(t,x)

xlabel('Time(s)')

ylabel('x')



 In the figure legend, Res means Re{s}.







From for Re{s} > -Re{a}.



The region of convergence is Re{s} > 0.

t = -1:1/10000:1;

unitstep = zeros(size(t));

unitstep (t>= 0) = 1;

x = (1-exp(-8\*t)).\*unitstep;

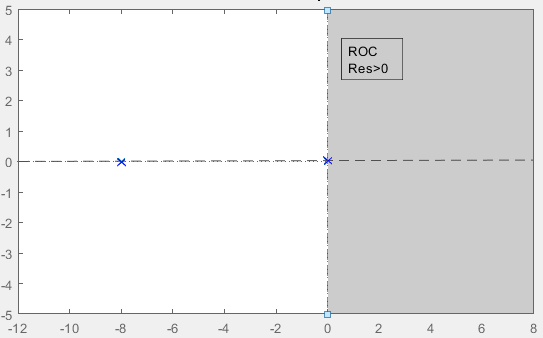
plot(t,x)

xlabel('Time(s)')

ylabel('x')

****

 In the figure legend, Res means Re{s}.



**Problem 3.**

Using the property: for zero initial conditions, we get:

Because the system is causal, the region of convergence is Re{*s*} > -2.

1. Using the Laplace transform pair for Re{*s*} > -Re{a}, for all s, we obtain





1. by substituting into *H*(*s*) above. This substitution is valid because the imaginary axis lies within the region of convergence of Re{*s*} > -2.

w = -10:1/10000:10;

H= j\*w./(j\*w+2);

Hmag=abs(H) ;

Hphase=angle(H);

plot(w,Hmag)

title('Magnitude Response');

figure

plot(w,Hphase)

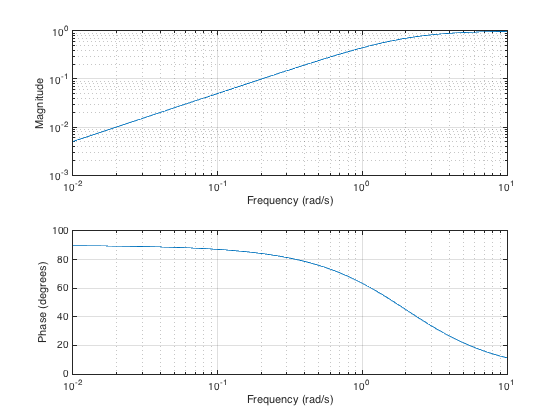
title('Phase Response');



According to magnitude response, the filter notches out zero frequency. Hence, it is a notch filter. It could also be called a highpass filter, but a DC notch filter would be more descriptive and a better answer.

Here’s the plot of the magnitude and phase using the freqs command in Matlab, which will plot the frequency responses on a log scale in frequency. The magnitude will also be on a log scale.

freqs( [1 0], [1 2] );



We see a highpass response over the frequencies plotted.

Please note that freqs( [1], [1 2] ) would mean for the transfer function instead of.

1. for Re{*s*} > 0. We could have also obtained the transform by using and substituting *a* = 0.





Using the Laplace transform pair , for we get

**Problem 4.**

1. Using the Laplace transform pair , we get





From 



1. Using convolution:





For a discrete-time version of this problem, please see Handout F Convolution of Exponential Sequences at <http://users.ece.utexas.edu/~bevans/courses/signals/handouts/Appendix%20F%20Convolution%20Exp%20Sequences.pdf>



From 



Poles are the roots of the denominator:



Thus, *Y*(*s*) has a double pole at *s* = -2.