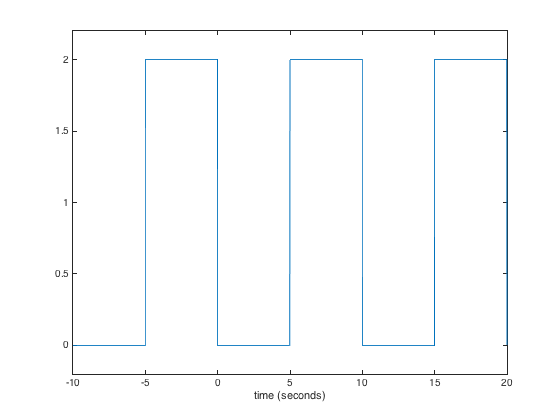
EE 313 Linear Signals & Systems (Fall 2018)

***Solution Set for Homework #3 on Fourier Series and Sampling***

Mr. Houshang Salimian & Prof. Brian L. Evans

1. **Prolog:** This problem relates to square waves, their Fourier series, and how adding a DC value affects Fourier series coefficients.

**Part (a):** This signal is periodic with a fundamental period of *T*o = 10 s, and hence the fundamental frequency is *f*0 = 1 / *T*0 = 0.1 Hz. From Section 3.6.1 of *Signal Processing First*, the square wave has an infinite number of harmonic frequencies, and their strength decays inversely proportionally to the harmonic index *k*. For plotting the time domain signal in MATLAB, we’ll want to pick a sampling rate *f*s that is a multiple of *f*0 and that can capture the sudden transitions in amplitude at -5s, 0s, 5s, 10s, 15s, and 20s. We’ll need *f*s to be a large multiple of 2 *f*0. For the plot below, we’ve chosen *f*s to be 10 kHz.



T0 = 10;

f0 = 1/T0;

fs = 10^6 \* f0;

Ts = 1 / fs;

t = -10 : Ts : 20;

% mod(t, T0) puts t in the fundamental period

% comparisons return 1 if true and 0 if false

x = 2 \* ( mod(t, T0) >= 5 );

plot(t, x);

ylim( [-0.2 2.2] );

xlabel( 'time (seconds)');

**Part (b):**

To calculate *a*o, the DC value, or signal average over a period should be calculated.

**Part (c):**



**Part (d):**  
By adding a constant value to a signal, only the DC value of that signal changes. Therefore, all Fourier series coefficient, except a0 remain unchanged.

b0 = a0 + 1 = 2



1. **Prolog:** This problem concerns continuous-time sinusoids and their fundamental periods, and discrete-time sinusoids and their fundamental periods.



In the continuous-time domain, the fundamental period is (2/11) seconds:









**Part (a):**



from sampling at *f*s = 10 Hz, which gives



This signal is under-sampled, because *f*0 > *f*s/2. The following shows aliasing due to folding that is caused by the under-sampling:



*A* = 7 , *φ* = π/2 rad.

**Part (b):**



from sampling at *f*s = 5 Hz, which gives



This signal is under-sampled, because *f*0 > *f*s/2. The following equation shows the effect of aliasing (but not related to folding) caused by the under-sampling:



*A* = 7, *φ* = -π/2 rad.

**Part (c):**



This signal is 15/11 times over sampled, because *f*0 < *f*s/2.



*A* = 7, *φ* = -π/2 rad

**Part (d):**

As shown at the beginning of this problem’s solution:

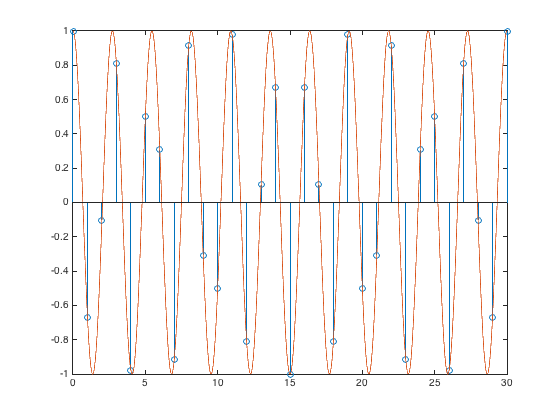




According to the hint that is provided for this solution, which comes from Handout D on Discrete-Time Periodicity, *x*[*n*] is periodic with discrete-time period of *N*0 if *x*[*n*] = *x*[*n*+*N*0]:



Because 11 and 30 are relatively prime, the smallest possible positive integer for *N*0 is 30 samples. Therefore, this discrete-time signal *x*[*n*] is periodic with a fundamental period of 30 samples. Those 30 samples contain 11 continuous-time periods, which corresponds to 2.67 samples in each continuous-time period.

Although not required, here’s a plot of *x*(*t*) and *x*[*n*] for part (d):

fs = 15;

Ts = 1/fs;

wHat = 2\*pi\*f0/fs;

N0 = 30;

n = 0 : N0;

yofn = cos(wHat\*n);

t = 0 : 0.01 : N0;

yoft = cos(wHat\*t);

figure;

stem(n, yofn);

hold;

plot(t, yoft);