**Solution Set for Homework #1 *Version 4.0***

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**PROBLEM 1**

**Prologue:** This problem helps you to identify the points of interest in a sinusoidal signal and calculate the parameters of the waveform based on your observations. It relies on the definitions given in Sec. 2-3 of the *Signal Processing First* textbook.

**Solution**: To calculate *A*, consider the peak-to-peak amplitude (*Xpp*) of the waveform which is the difference between the maximum value and the minimum value:

*Xpp* = Maximum – Minimum = 4 – (–4) = 8. So*, A* = *Xpp* /2 = 4.

To estimate the frequency, we would first estimate the period and the invert of the period. We could estimate a period in the plot by measuring the amount of time it takes to go from a peak to the next peak, or from one valley to the next. The plot has valleys at three different instances:

* *t*0 = -2.4 ms,
* *t*1 = -0.9 ms,
* *t*2 = 0.6 ms.

One period has elapsed from *t*0 to *t*1 and two periods have elapsed from *t*0 to *t*2. The period (*T*) is about 1.5 ms long.

T = *t*1 – *t*0 = -0.9 - (-2.4) = -0.9 + 2.4 = 1.5ms **OR** T = = 1.5ms.

So, ω0 = 2π/T = 1333.33π = 4188.79 rad/s or *f*0 = 667 Hz. (Ground truth is 660 Hz.)

We are left with one unknown, φ. We can pick any point in time to give us one equation in one unknown: *x*(*t*) = *A* cos(ω0 *t* + φ)

* *Approach #1:* If we pick *t* = 0, then *x*(0) = 4 cos(φ) ≈ 3.25 and φ ≈ cos-1 (3.25/4) = 0.622 rad (1)

Recall that cos(-φ) = cos(φ), so φ could be +0.622 rad or -0.622 rad. Also, any addition of φ and a multiple of 2π would also be a valid answer.

Values used to generate the plot

*A* = 4

*f*0 = 660 Hz

φ = p/4 ≈ 0.7854 rad

Note: 660 Hz is a C in the 4th octave on Western scale.

To decide between the values, consider the slope at *t* = 0, which is negative. (2)

*x*(*t*) = *A* cos(ω0 *t* + φ) => x’(t) = -*A* ω0 sin(ω0 *t* + φ) => *x* ’(0) = -*A* ω0 sin(φ)

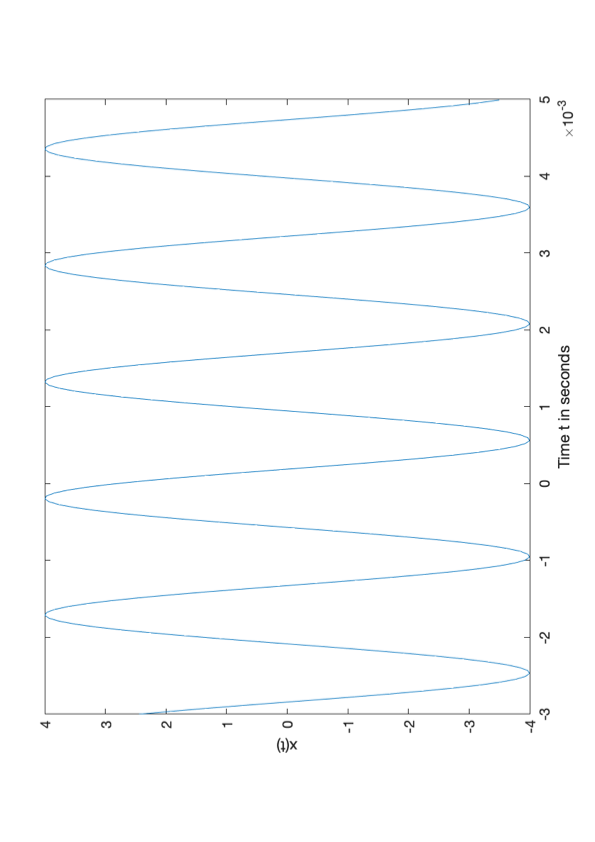
So, condition (2) implies that x’(0) < 0 => sin(φ) > 0.

Out of the two values in (1), only φ = 0.198p = 0.622 rad satisfies the condition.

* *Approach #2.* Pick *t* = 0.6 ms where there is a valley. Then,

x(0.6ms) = 4 cos(2π(0.6ms/1.5ms) + φ) = -4 => cos(4π/5 + φ) = -1 =>

4π/5 + φ = => φ = π/5 = 0.6283 rad.



t = 0.6

x (t) = -4

t = 0

x (t) = 3.25

***Epilogue***: There is nothing special about the time points considered for the solution. The points were chosen due to convenience, and the problem could have been done by taking other points as well. Additionally, the value of the phase φ is not unique. Using a ruler to measure points of interest will help give more accurate answers.

**PROBLEM 2**

***Prologue***: This problem helps you calculate the summation of several sinusoids having the same amplitude and frequency, but with different phases. By using the phasor addition rule, the answer can be derived.

**Part (a)**

Using the phasor addition rule from section 2-6.2 of the textbook:

Using Euler’s formula:



Therefore:

So, A = 231.82 & φ = 0.262 rad

See the next page for the plot.

**Part (b)**

MATLAB Code:

w0 = 120\*pi;

f0 = w0/(2\*pi); %fundamental period in Hz

fs = 40\*f0; %Sampling frequency in Hz

Ts = 1/fs; %Sampling time in seconds

t = -0.05:Ts:0.05;

x = 120\*cos(w0\*t)+120\*cos(w0\*t+pi/6);

plot(t,x);

xlabel('Time(s)');

ylabel('Amplitude');

figure;

xParta = 231.82 \* cos(w0\*t + 0.262);

plot(t, xParta);

T0 is the fundamental period of *x*(*t*)

ω0 =2π/T0 = 120π

T0 = (2π)/(120π) = 1/60s = 0.0167s

This range, e.g. –0.05 ≤ t ≤ 0.05, covers T1 = 0.05 – (-0.05) = 0.1s of the time scale  
m = T1 / T0 = 0.1 / 0.0167 = 6 , where m = 6 is the number of periods that are included in the plot.

The two plots are identical! See next page for the plot.

**Part (c)**

The answer is calculated in part a:

**Figure from Part (a)**

Chart, histogram

Description automatically generated

**Figure from Part (b)**

Chart, histogram

Description automatically generated

**PROBLEM 3:**

***Prologue***: This problem shows that a time shift leads to a phase shift. This principle is used in ranging systems that try to localize objects in an environment, such as ultrasound, sonar and radar. In those applications, the ranging system transmits a sinusoidal signal and then goes quiet. The signal will bounce off an object in the environment and return to the ranging system. The ranging system then computes the time delay between the transmitted and received signals to estimate the distance to the object by using the speed of propagation in the environment. The ranging system can compute the time delay in the time domain or the phase delay in the frequency domain. A common time domain method uses correlation, which is covered in EE 351K Probability and computed using convolution. We’ll see convolution after midterm #1.

Here’s the connection between time shift and phase shift:

Therefore,

T0 = 8s

**Part (a)**

t1 = -2s

Therefore, the phase is equal to:

**True**

**Part (b)**

t1 = 3s

Therefore, the phase is equal to:

**False**

**Part (c)**

t1 = 7s

Therefore, the phase is equal to:

Due to property cos(*x* + 2 π) = cos(*x*). Each multiple of 2π corresponds to picking a different peak.

**True**

**PROBLEM 4**

***Prologue***: This problem shows how wireless reception can vary with the location of the receiving equipment (e.g. smart phone). In some locations, the receiving equipment will experience very weak reception, and strong reception in other locations. For the basic model of one reflector in this problem, the received amplitude strength is periodic with location on the horizontal axis. A takeaway is that by moving the receiving equipment, e.g. walking while on a videocall on the smart phone will provide a good connection on average.

**Error in the textbook figure from the** [**book errata**](http://dspfirst.gatech.edu/spfirst/SPFirst-errata.pdf)

**Diagram

Description automatically generated**

**Part (a)**

**Part (b)**  
time delay at

Using the phasor addition rule:

Using Euler's formula:

**Part (c)**

when signal strength

since the denominator of and is equal to

**Part (d)**

x = -100:0.1:100;

c = 3e8; % speed of light in m/s

dr = 55;

dt = 1e6;

w0 = 300\*10^6\*pi; % carrier frequency (150 MHz)

t1 = sqrt(x.\*x+dt)/c;

t2 = (sqrt((x.\*x-2\*dr\*x+dr^2)+dt)+dr)/c; % expanded sqrt((x-dr)^2 + dt) term

s1 = cos(w0\*t1)+cos(w0\*t2);

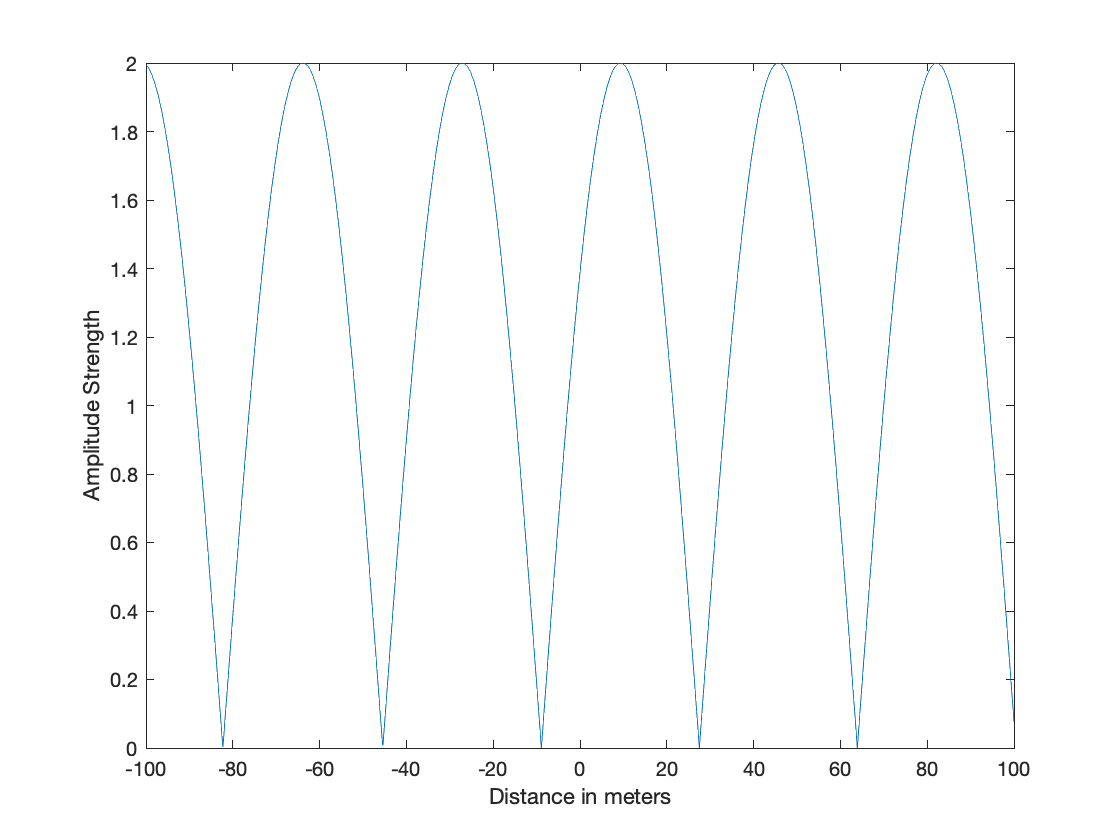
s2 = sin(w0\*t1)+sin(w0\*t2);

a = sqrt(s1.\*s1 + s2.\*s2);

plot(x, a);

xlabel('Distance in meters');

ylabel('Amplitude Strength');

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