Lecture 9 Discrete-Time Frequency Response — Fall 2018

Prof. Brian L. Evans — EE 313 Linear Systems and Signals

Notes by Mr. Houshang Salimian — The University of Texas at Austin

Consider random variables x, y and z:

Connections to EE 351K Probability

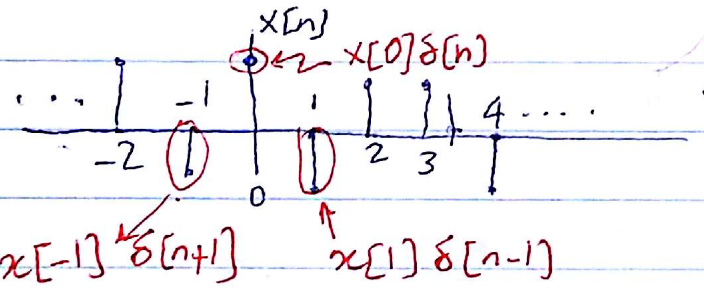
$$z = x + y$$

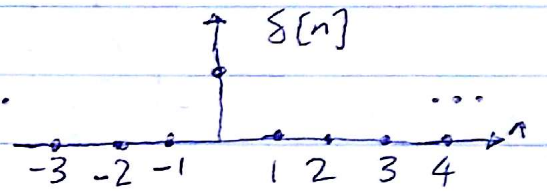
$$\underbrace{f_z(z)} = f_x(z) * f_y(z)$$

probability density function for random variable z

$$x[n] \longrightarrow \boxed{\text{LTI}} \longrightarrow y[n]$$

$$\delta[n] \qquad\qquad\qquad h[n]$$





Discrete-Time Impulse

Kronecker Delta

Additivity: $x_1[n] + x_2[n] \xrightarrow[\text{System}]{\text{LTI}} y_1[n] + y_2[n]$

Input: $\sum_{m=-\infty}^{\infty} x[m]\,\delta[n-m]$

Output: $\sum_{m=-\infty}^{\infty} T\{x[m]\,\delta[n-m]\}$

$$y[n] = \sum_{m=-\infty}^{\infty} x[m]\,h[n-m]$$

Let $k = n - m$

$m \to \infty : \; k \to -\infty$

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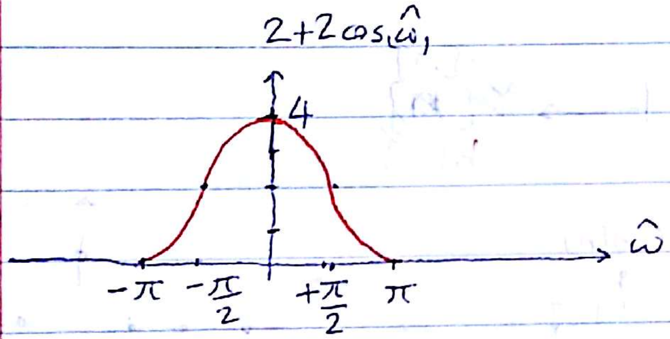
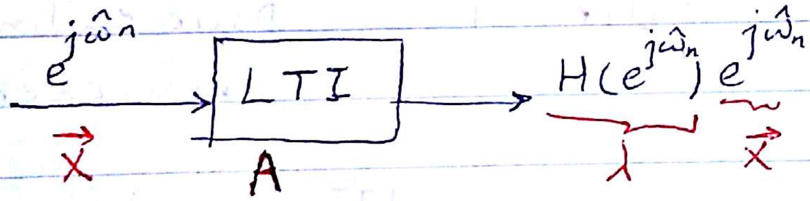
$k = n - m \Rightarrow m = n - k$

$$y[n] = \sum_{k=-\infty}^{\infty} x[n-k]\,h[k] = \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{2} h[k]\,e^{-j\hat{\omega}k} = \underbrace{h[0]\,e^{-j\hat{\omega}0}}_{k=0} + \underbrace{h[1]\,e^{-j\hat{\omega}1}}_{k=1} + \underbrace{h[2]\,e^{-j\hat{\omega}2}}_{k=2}$$



Connections to matrices



$$A\vec{X} = \lambda\,\vec{X}$$

complex scalar

A complex sinusoid is an eigenfunction of an LTI system; i.e., an eigenfunction is analogous to an eigenvector for a matrix “system” A, and the frequency response evaluated at a particular frequency is analogous to an eigenvalue (both are complex scalars).