Homework: 2-level minimization

1. Using the URP-based tautology procedure, check whether the following cover is irredundant: $bd + be'f + a'b'ce + ab'd' + acf + c'd'$.

   15 marks

2. In this problem we apply the unate recursive paradigm to the problem of deriving a cover for the complement of a function which is also given as a cover.

   (a) Prove that $\overline{f} = x\overline{(f_x)} + \overline{x}\overline{(f_{\overline{x}})}$. How can this result be exploited to obtain a merging process for complementation using the unate recursive paradigm?

   (b) Use the unate recursive paradigm to complement $f = xy'z' + wxy' + wy'z + wx'z + wyz + xyz + w'xy'z$ using the merging process inferred from *(a)*. Show results at each step, i.e., "personality matrix" B, all minimal column covers, etc. Also complement f by applying DeMorgan's Law.

   (c) Use ESPRESSO to complement this cover. (Hint — express the function in PLA format in terms of its offset, and run the command *espresso -epos -Dexact infile*.)

   30 marks

3. Let $f = w'y'z' + xy'z + wxy$ with the don't-care set $d = wy'z' + w'x'y'z + w'xyz + wx'y$. Use the Quine-McCluskey procedure to minimize f. Show results at each step, i.e., list the primes, give the matrix, and show how the minimum cover is found.

   20 marks

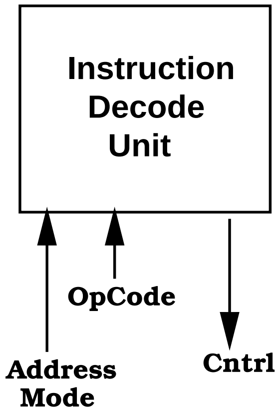
4. A circuit may malfunction due to exactly one of 8 possible faults $f_1, f_2, \ldots, f_8$. A designer has come up with six tests which when applied detect some of the faults. Specifically,

   - t_1 tests for the presence of f_1, f_3, f_5
   - t_2 tests for the presence of f_2, f_4, f_6
   - t_3 tests for the presence of f_1, f_2, f_4, f_7
   - t_4 tests for the presence of f_3, f_5, f_8
   - t_5 tests for the presence of f_2, f_6, f_8
   - t_6 tests for the presence of f_3, f_4, f_7

   Find a minimum subset of tests that will detect any of the faults.

   Suggestion—formulate the problem as a unate covering problem.

   20 marks



Adrs Mode	OpCode	Cntrl
INDEX	AND	Cnta
INDEX	OR	Cnta
INDEX	JMP	Cnta
INDEX	ADD	Cnta
DIR	AND	Cntb
DIR	OR	Cntb
DIR	JMP	Cntc
DIR	ADD	Cntc
IND	AND	Cntb
IND	OR	Cntd
IND	JMP	Cntd
IND	ADD	Cntc

Figure 1: Instruction Decode Unit.

5. Let A be a collection of cubes over variables $x_1, x_2, \ldots, x_n$. Iteratively perform the following operation until no new cubes can be added to A: take any pair of cubes of the form $x_i \cdot c_1$ and $x_i{}' \cdot c_2$, and add $c_1 \cdot c_2$ to A. Call the resulting set of cubes A^*.

   Now remove from A^* cubes which are contained in other cubes in A^*. Call the resulting set P. Show that P is the set of all primes of A.

   Compute the primes of $ab'c'd + a'bcd' + bc' + abcd$ in this fashion.

20 marks

6. In this problem you are to implement control logic using a PLA. The design is given in Figure 1; it is derived from Example 7.5 in DeMicheli's book. Use the following encoding:

Address Mode		Op Code		Control	
index	00	AND	00	*Cnta*	00
dir	01	OR	01	*Cntb*	01
ind	10	JMP	10	*Cntc*	10
		ADD	11	*Cntd*	11

   (a) Express the controller in `.pla` format. (Use don't cares from the unused codes.)

   (b) Minimize it using ESPRESSO.

   (c) Can you derive a better encoding?

40 marks

7. Consider a communication system consisting of a transmitter, a receiver, and a noisy link connecting the two. Let A be a set of symbols that can be transmitted by the transmitter, e.g., a set of voltage levels.

   Because of noise, it may be the case that one symbol of A may be mistaken for another symbol of A. For example, A be the symbols $\{a, b, c, d, e\}$, and the link may distort a to the extent that it cannot be differentiated from b and e, although it cannot distort a so much that it cannot be differentiated from c, and d.

   We can model the effect of the channel by a graph whose vertex set is A—we add an edge between vertices if the corresponding symbols can be mistaken for each other.

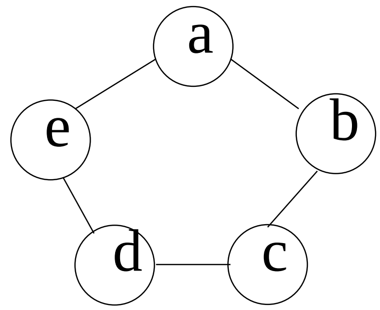


Figure 2: A symbol graph

Consider the scenario shown in Figure 2. For such a channel, the transmitter is restricted to send only two symbols, since any subset of 3 vertices has a pair connected by an edge. As a result, it would appear only one bit can transmitted per cycle (rather than $\log_2 5$, which is what it would be in the absence of noise).

However, by selecting pairs of symbols appropriately it may be possible to do better that 1 bit/cycle. For example, if we transmitted the pair (a, a) and (b, c) can never be mistaken for each other—even though the first symbols can be confused, the second cannot.

Use Espresso to compute a set of pairs of symbols that is maximum and has the property that no two pairs of symbols can be confused for each other. What is the communication rate (in bits/cycle) for the resulting scheme?

(Incidentally, this example was first studied by Shannon, and was critical in his development of Information Theory. It took roughly 30 years before the exact information capacity for the the set of symbols in this example was derived.)

20 marks

8. ($\star$-credit) Let $h(x, y) = f(x) + g(y)$, where x and y are disjoint sets of variables. Show that a minimum cover of h can be computed by computing minimum covers F for f and G for g, and taking the union of the cubes in F and G taking the union of the cubes in F and G.

   Now consider $l(x, y) = f(x) \cdot g(y)$. Suppose we took minimum covers F and G as before, and formed the cover L consisting of pair-wise products of cubes from F and G. Is L a minimum cover for l?

20 marks