You may ask why the gate capacitance of a MOS transistor in saturation is $\frac{2}{3}C_{ox}W_{eff}L_{eff}$. The following is the derivation. We denote W_{eff} as W and L_{eff} as L for simplicity, and ignore channel length modulation effect. Thus the channel extends from y=0 to y=L instead of L-ΔL, where ΔL is introduced by channel length modulation effect.

Note that $C_{gate} = \frac{\partial Q_{channel}(v_{GS})}{\partial v_{GS}}$, firstly we need to find the expression for $Q_{channel}(v_{GS})$.

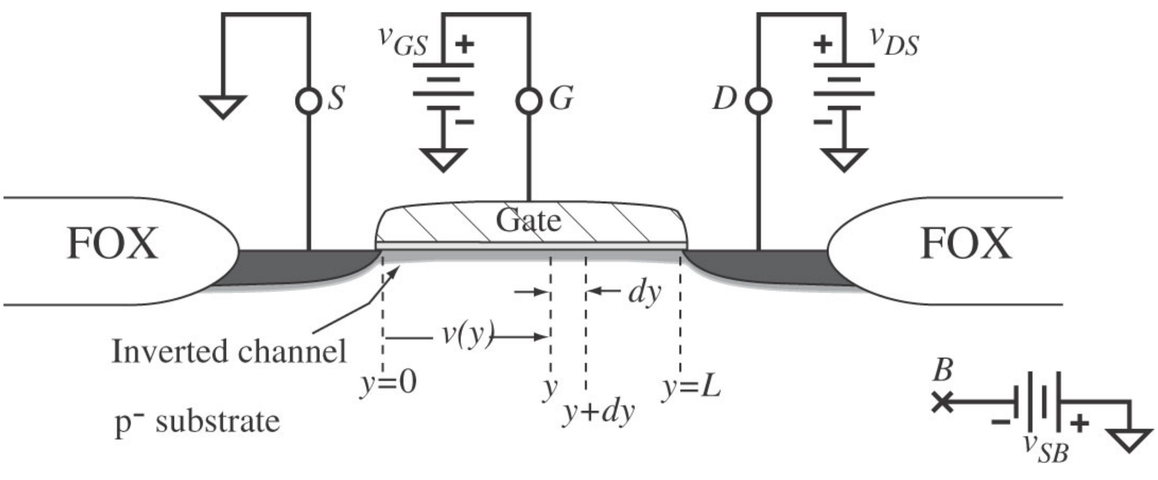

Fig. 1, An NMOS transistor with small v_{DS} and $v_{GS}>V_T$.

For $V_{GS}>V_T$, any charge placed on the gate must be mirrored by the charge with reversed polarity in the channel. The charge per unit area in the channel, $Q_I(y)$ can be expressed as,

$$Q_I(y) = C_{ox}(v_{GS} - V(y) - V_T) \tag{1}$$

Note that $E = -\frac{dV(y)}{dy}$, the carrier speed v_e is $v_e = -\mu_n E$, thus

$$i_D = WQ_I v_e = WC_{ox}(v_{GS} - V(y) - V_T)\mu_n \frac{dV(y)}{dy} \tag{2}$$

Thus

$$i_D dy = WQ_I v_e dy = WC_{ox}(v_{GS} - V(y) - V_T)\mu_n dV(y) \tag{3}$$

Integrating along the channel from y = 0 to y = L, we have

$$\int_0^L i_D dy = \int_0^{v_{GS}-V_T} WC_{ox}(v_{GS} - V(y) - V_T)\mu_n dV(y) \tag{4}$$

Note that i_D is constant along the channel, we have

$$\int_0^L i_D dy = i_D L = \int_0^{v_{GS}-V_T} WC_{ox}(v_{GS} - V(y) - V_T)\mu_n dV(y) = \frac{1}{2}\mu_n WC_{ox}(v_{GS} - V_T)^2 \tag{5}$$

Thus

$$i_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_{GS} - V_T)^2 \tag{6a}$$

Eq. (6a) applies when $v_{GS} > V_T$ and $v_{DS} > v_{GS} - V_T$, or when the transistor is in saturation.

The factor $\mu_n C_{ox}$ is called device transconductance parameter,

$$K_p = \mu_n C_{ox} = \frac{\mu_n \varepsilon_{ox}}{t_{ox}}$$

Thus

$$i_D = \frac{1}{2}K_P \frac{W}{L}(v_{GS} - V_T)^2 \tag{6b}$$

If we integrate the left side of Eq. (3) from 0 to y (y < L), we have,

$$\int_0^y i_D dy = \int_0^{V(y)} WC_{ox}(v_{GS} - V(y) - V_T)\mu_n dV(y)$$

Thus

$$i_D y = WC_{ox}\left[(v_{GS} - V_T)V(y) - \frac{V^2(y)}{2}\right] \tag{7}$$

Substitute Eq. (6b) into Eq. (7), we have,

$$\frac{1}{2}C_{ox}\frac{W}{L}(v_{GS} - V_T)^2 y = WC_{ox}\left[(v_{GS} - V_T)V(y) - \frac{V^2(y)}{2}\right] \tag{8a}$$

Eq. 8(a) can be re-written as

$$\frac{1}{2}(v_{GS}-V_T)^2\frac{y}{L}=(v_{GS}-V_T)\left(V(y)-\frac{V^2(y)}{2}\right) \tag{8b}$$

Solving Eq. 8(b), we obtain that,

$$V(y)=(v_{GS}-V_T)\left(1\pm\sqrt{1-\frac{y}{L}}\right)$$

Note that V(y) = 0 when y = 0, thus

$$V(y)=(v_{GS}-V_T)\left(1-\sqrt{1-\frac{y}{L}}\right) \tag{9}$$

From Eq. (1), we can write,

$$dQ_{channel}=Q_I(y)Wdy=C_{ox}(V_{GS}-V(y)-V_T)Wdy \tag{10a}$$

Substitute Eq. (9) into Eq. (10a),

$$dQ_{channel}=C_{ox}\left[(v_{GS}-V_T)-(v_{GS}-V_T)\left(1-\sqrt{1-\frac{y}{L}}\right)\right]Wdy=C_{ox}(v_{GS}-V_T)\left(\sqrt{1-\frac{y}{L}}\right)Wdy \tag{10b}$$

We have,

$$Q_{channel}=\int dQ_{channel}=C_{ox}(v_{GS}-V_T)W\int_0^L\left(\sqrt{1-\frac{y}{L}}\right)dy \tag{11a}$$

Note that

$$\int_0^L\left(\sqrt{1-\frac{y}{L}}\right)dy=\frac{2}{3}L$$

Thus

$$Q_{channel}=\int dQ_{channel}=\frac{2}{3}C_{ox}(v_{GS}-V_T)WL \tag{11b}$$

Finally, we arrive at

$$C_{gate}=\frac{\partial Q_{channel}(v_{GS})}{\partial v_{GS}}=\frac{2}{3}C_{ox}WL \tag{12}$$