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The Cascode ~~Transistor~~ Amplifier

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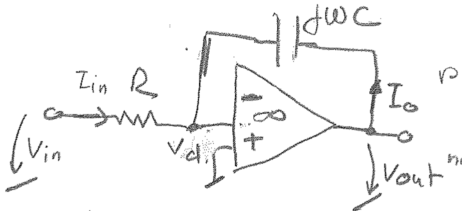
Motivation

- ~~Transistors are mainly used as amplifiers~~
  One of the main function of a transistor is the amplifier
- * ~~But~~ • transistors are not linear ($I_C \propto e^{V_{be}}$, $I_D \propto (V_{GS}-V_T)^2$)
  • transistor gain ~~cannot be controlled well~~ not well controlled
  ex. : ~~gm~~ gain depends on the bias conditions
  ~~gm~~ gain can vary by > 2x over process corners and Temp.

Q: How can we build reliable amplifiers despite these drawbacks?

A: ~~We build circuits that do not rely on the~~
the Operational Amplifier

ex.



Two stable conditions

pos. feedback → 1 - $V_d \neq 0$, $V_{out} = \pm\infty$ (Supply rails)

neg. feedback → 2 - $V_d = 0$, V_{out} = any voltage $(0, \infty)$

for 2nd case :

$$\begin{cases} I_{in} = V_{in}/R \\ I_o = -I_{in} \\ V_{out} = \dfrac{I_o}{j\omega C} \end{cases}$$

$$V_{out} = \frac{-V_{in}/R}{j\omega C}$$

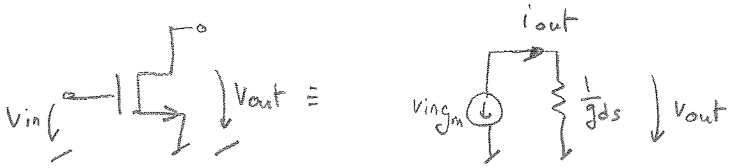
$$\boxed{\frac{V_{out}}{V_{in}} = \frac{1}{j\omega RC}}$$

The integrator

→ the function of the stage (Integrator) does not ~~• the operation of the~~ depend on the amplifier gain but on more stable, more linear passive components.

Q: How can we build amplifiers with high gain?

The simple transistor in common source ~~gen~~ configuration does not have enough gain.

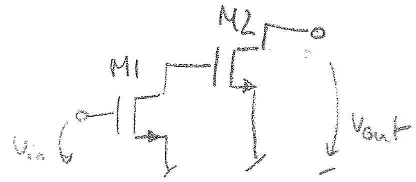


gain:

$$\begin{cases} V_{out} = i_{out} \cdot \frac{1}{g_{ds}} \\ i_{out} = -v_{in} \cdot g_m \end{cases} \Rightarrow \boxed{\frac{V_{out}}{V_{in}} = -\frac{g_m}{g_{ds}}}$$

$$\frac{V_{out}}{V_{in}} \approx 10 \ldots 40$$

A1: ~~We boost the gm~~. Ex. we cascade stages



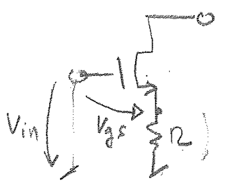
$$\frac{V_{out}}{V_{in}} = \left(-\frac{g_{m1}}{g_{ds1}}\right) \cdot \left(-\frac{g_{m2}}{g_{ds2}}\right) = \frac{g_{m1}g_{m2}}{g_{ds1}g_{ds2}}$$

A2: We lower the g_{ds} (~~increase~~ boost output impedance)

The Source degeneration technique

Output Impedance $\left(\frac{V_{out}}{i_{out}}\right)$

$r_o = \frac{1}{g_{ds}}$ [redacted]



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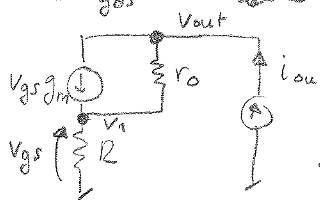


$$\begin{cases} \frac{V_1}{R} - V_{gs}g_m + \frac{V_1 - V_{out}}{r_o} = 0 \\ -i_{out} + \frac{V_{out} - V_1}{r_o} + V_{gs}g_m = 0 \\ V_{gs} = -V_1 \end{cases}$$

Subs. v_{gs} (1)

$$\begin{cases} -\frac{V_{gs}}{R} - V_{gs}g_m - \frac{V_{gs} + V_{out}}{r_o} = 0 \\ -i_{out} + \frac{V_{out} + V_{gs}}{r_o} + V_{gs} \cdot g_m = 0 \end{cases}$$

isol v_{gs} (2)

$$\begin{cases} V_{gs}\left(1/R + g_m + 1/r_o\right) = -\frac{V_{out}}{r_o} \\ V_{gs}\left(1/r_o + g_m\right) = i_{out} - \frac{V_{out}}{r_o} \end{cases}$$

$$\frac{\cancel{v_{gs}}\left(1/r_o + g_m\right)}{\cancel{v_{gs}}\left(1/R + g_m + \frac{1}{r_o}\right)} = \frac{i_{out} - \frac{V_{out}}{r_o}}{-\frac{V_{out}}{r_o}}$$

$$\frac{1/r_o + g_m}{1/R + g_m + \frac{1}{r_o}} = 1 - \frac{i_{out}}{V_{out}} r_o$$

⋮

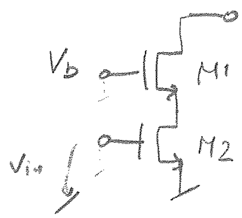
$$\boxed{r_o' \frac{V_{out}}{i_{out}} = r_o + R + R \cdot g_m r_o}$$

Unfortunately the stage equivalent transconductance is: $g_m' = \frac{g_m r_o}{r_o + R + g_m R r_o}$ $(< g_m)$

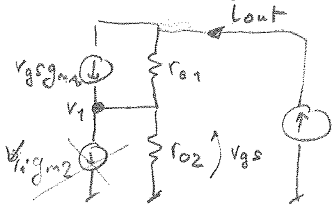
and the total gain is unchanged! : $g_m' r_o' = g_m r_o \frac{r_o + \cancel{R + R g_m r_o}}{r_o + \cancel{R + R g_m r_o}} = \underline{\underline{g_m r_o}}$

The cascode transistor

- beside the unchanged gain the source degeneration technique has the disadvantage that the max R is limited by larg signal considerations.
- Substitute the degeneration resistance with a transistor in saturation region:



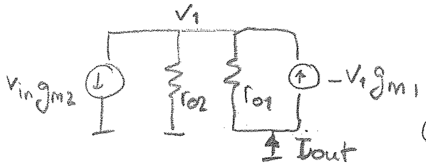
Output Impedance



the small signal circuit is the same as the prev one therefore:

$$r_o' = \frac{V_{out}}{i_{out}} = r_{o1} + r_{o2} + r_{o2}\, g_{m1} r_{o1}$$

The effective g_m' is different:



(1) $\begin{cases} I_{out} = -V_1 g_{m1} - \frac{V_1}{r_{o1}} \\ V_{in} g_{m2} + \frac{V_1}{r_{o2}} + \frac{V_1}{r_{o1}} + V_1 g_{m1} = 0 \end{cases}$ (2) $\begin{cases} V_1 \left(g_{m1} + \frac{1}{r_{o1}}\right) = -I_{out} \\ V_1 \left(\frac{1}{r_{o2}} + \frac{1}{r_{o1}} + g_{m1}\right) = -V_{in} g_{m2} \end{cases}$

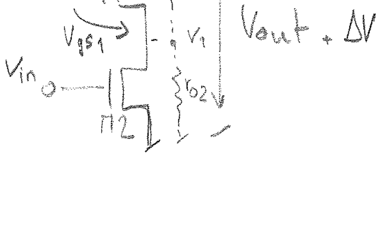
(3) $\frac{V_1 \left(g_{m1} + \frac{1}{r_{o1}}\right)}{V_1 \left(\frac{1}{r_{o2}} + \frac{1}{r_{o1}} + g_{m1}\right)} = \frac{+i_{out}}{+V_{in} g_{m2}}$; (4)

$$\frac{i_{out}}{V_{in}} = g_{m2} \frac{g_{m1} + \frac{1}{r_{o1}}}{g_{m1} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}} \simeq g_{m2}$$

Therefore the gain is:

$$A = g_{m2} \cdot (r_{o1} + r_{o2} + r_{o2}\, g_{m1} r_{o1}) \simeq g_{m2}\, r_{o2}\, g_{m1} r_{o1}$$

Intuitive explanation

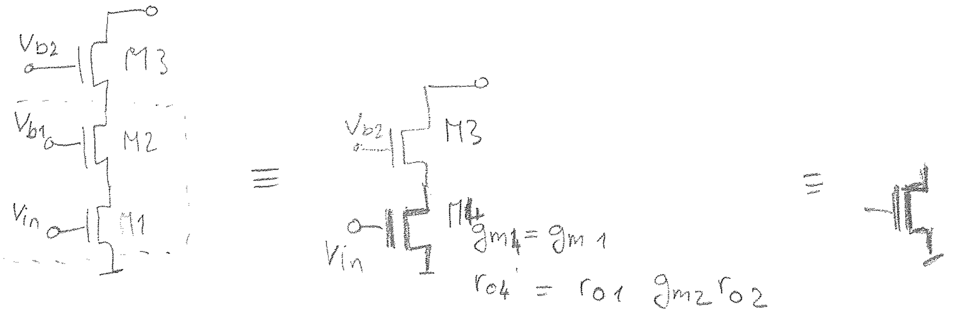


- when Vout changes, V_1 tends to follow it over the r_{o1} / r_{o2} voltage divider. However, V_1 activates Transistor M1 that provides additional current that flows through r_{o1} therefore most of the voltage drop is on r_{o1} and r_{o2} is kept stable reducing the net current and hence increasing the impedance

Derivatives of the basic cascode

1) The double cascode

- recursive use of the cascode idea

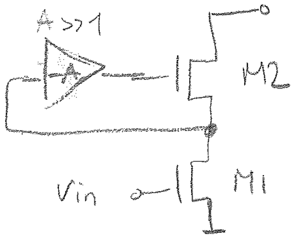


$g_m \cong g_{m1}$

$r_o \cong r_{o1} \cdot g_{m2} r_{o2}\, g_{m3} r_{o3}$

2) The regulated cascode

- g_{m2} is boosted by an operational amplifier

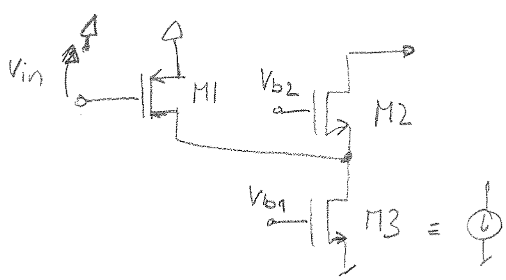


$g_m \cong g_{m1}$

$r_o \cong r_{o1} \cdot \underbrace{A g_{m2}}_{\text{boosted } g_m} \cdot r_{o2}$

3) The folded cascode

- PMOS input transistor and NMOS cascode transistor give more freedom in the choice of the common mode voltages



(assuming M3 is an ideal current sink)

$g_m = g_{m1}$

$r_o = r_{o1} \cdot g_{m2} r_{o2}$

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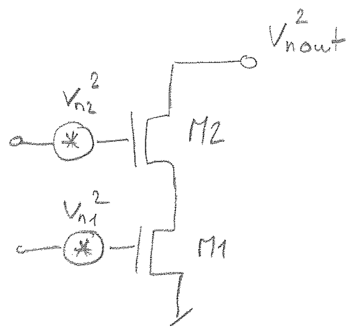
Noise contribution of the cascode transistor



$$V_{nout}^2 = V_{n2}^2 \cdot (g_m r_{o2})^2 + V_{n1}^2 (g_{m2} r_{o2} g_{m1} r_{o1})^2$$

Input referred noise:

$$V_{nin}^2 = \frac{V_{nout}^2}{(g_{m2} r_{o2} g_{m1} r_{o1})^2}$$

$$= \frac{V_{n2}^2 (g_m r_{o2})^2 + V_{n1}^2 (g_{m2} r_{o2} g_{m1} r_{o1})^2}{(g_{m2} r_{o2} g_{m1} r_{o1})^2}$$

$$= V_{n2}^2 \frac{1}{(g_{m1} r_{o1})^2} + V_{n1}^2$$

⚠ The noise contribution of M2 is scaled by the square of the gain of M1.

⟹ M2 is usually a negligible source of noise!

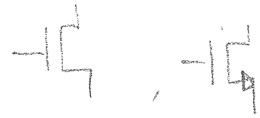
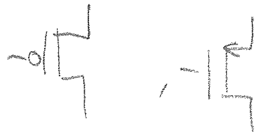
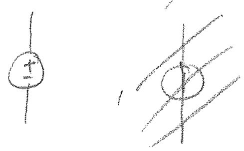
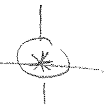
Conclusion

1) The cascode ~~circuit~~ is a very common and useful topology in CMOS analog circuits

2) The gm of the circuit is the same as the one of bottom transistor. The output impedance is the ro of the bottom transistor boosted by the gain of the upper transisror.

3) The upper transistor does usually not contribute to the total noise

Notation

NMOS Transistor :	
PMOS Transistor :	
current source :	
voltage source :	
voltage controlled current source (gm) :	
~~noise source~~ :	
gnd	
Vdd