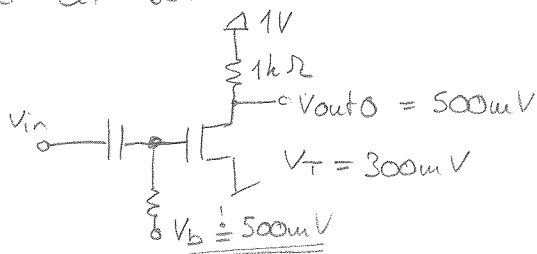
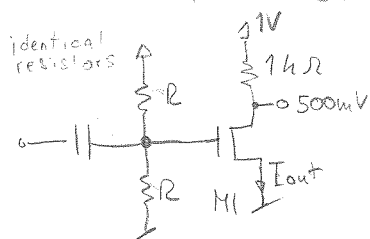


Motivation

- transistor in
- The following common source configuration needs to have its gate biased at 500mV



Solution #1: Voltage bias



Q: what happens if V_T is 50mV smaller? (Process, Temp. variation)

A:

Equations: $I_{out} = \frac{9.5V}{1000\Omega} = 500\mu A$

$$\textcircled{1} \begin{cases} 500\mu A = \frac{R}{2} (500mV - 300mV)^2 \end{cases}$$

$$\textcircled{1} \begin{cases} I_{out2} = \frac{R}{2} (500mV - 250mV)^2 \end{cases}$$

$$\textcircled{2} \frac{I_{out2}}{500\mu A} = \frac{(250mV)^2}{(200mV)^2}$$

$$I_{out2} = 500\mu A \cdot \frac{0.0625}{0.04} = 500\mu A \cdot 1.5625 \approx 781\mu A$$

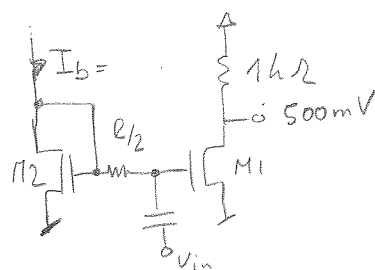
$$\Rightarrow V_{out0} = 1V - 781\mu A \cdot 1k\Omega = 219mV$$

because $V_{out0} = 219mV < V_{GS} - V_T = 250mV$

M1 is no more working in saturation!

Bad Bias Technique

Solution #2: Current bias



Q: what happens if V_T is 50mV smaller?

A: assuming $r_{o1}, r_{o2} = \infty$, M1, M2 are identical and $I_b = 500\mu A$, the output voltage will remain unchanged and the circuit will keep working.

Good bias technique

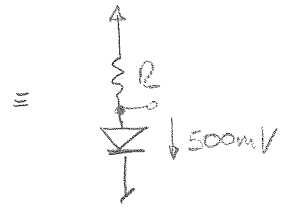
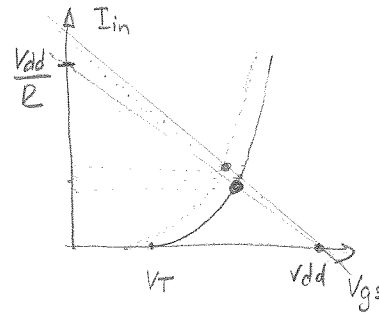
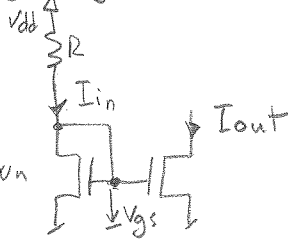
! cmos transistors are always biased using currents! Do never use absolute voltages, even when you are "playing" with simulator

Example of an easy way to generate a reference current:

Finding R :

- I_{in} is known
- $\beta = \frac{\mu_n C_{ox} W}{L}$ is also known
- $V_{gs} = \sqrt{\frac{2I_{in}}{\beta}} + V_T$

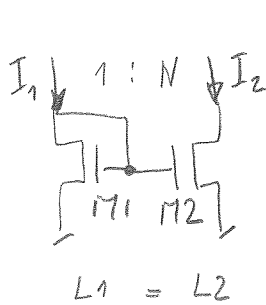
$$\Rightarrow R = \frac{V_{dd} - V_{gs}}{I_{in}} = \frac{V_{dd} - V_T - \sqrt{\frac{2I_{in}}{\beta}}}{I_{in}}$$



The resistor is usually off-chip so that one can tweak it to change the bias current.

- In real products better stability and supply voltage independency are required. A bandgap based reference generator is used.

Principle



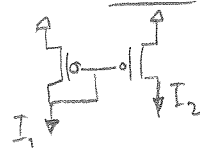
$$\begin{cases} I_1 = \frac{\mu_n C_{ox} W_1}{2 L_1} (V_{GS1} - V_T)^2 (1 + \lambda_1 V_{GS1}) \\ I_2 = \frac{\mu_n C_{ox} W_2}{2 L_2} (V_{GS1} - V_T)^2 (1 + \lambda_2 V_{DS2}) \end{cases}$$

$$\frac{I_1}{I_2} = \frac{\frac{\mu_n C_{ox} W_1}{2 L_1} (V_{GS1} - V_T)^2 (1 + \lambda_1 V_{GS1})}{\frac{\mu_n C_{ox} W_2}{2 L_2} (V_{GS1} - V_T)^2 (1 + \lambda_2 V_{DS2})}$$

← Note that M1 is always in saturation because $V_{DS1} = V_{GS1} > V_{GS1} - V_T$

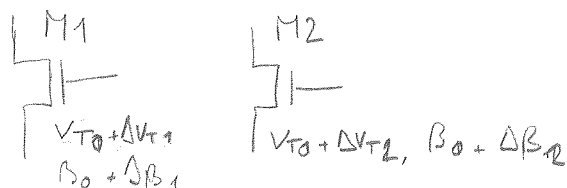
$$\boxed{\frac{I_1}{I_2} = \frac{W_1}{W_2} \frac{1 + \lambda_1 V_{GS1}}{1 + \lambda_2 V_{DS2}}}$$

- If the output impedance is ∞ , i.e., $\lambda_1 = \lambda_2 = 0$ then the current ratio is $N = \frac{W_2}{W_1}$
- Big N 's ($N=10 \dots 20$) are used to reduce the bias current I_1 assuming the operating current I_2 is fixed
- the NMOS mirror is called current sink. The PMOS mirror is called current source

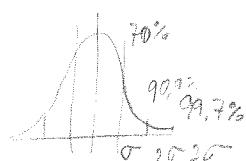


2A Transistor ~~Current~~ Mismatch in a Current Mirror

- Two real transistors are never identical



$$\begin{cases} \sigma_{\Delta V_T} = \frac{A_{VT}}{\sqrt{WL}} \\ \frac{\sigma_{\Delta \beta}}{\beta_0} = \frac{A_{\beta}}{\sqrt{WL}} \end{cases}$$



- A_{VT} and A_{β} are two empirically measured constants

- $\sigma_{VT}, \sigma_{\beta}$ are the standard deviations of corresponding ^{random} variables that are assumed to have a Gaussian distribution

- ΔV_T and $\Delta \beta$ are random variables like noise therefore they add in rms way

- The correlation between V_T and β is very low therefore their effect adds in rms way, as well

- Note that usually $\sigma_{\Delta V_T}$ and $\frac{\sigma_{\Delta \beta}}{\beta}$ are given as the σ between two devices. ^{Therefore} The σ of a single device from its nominal value is $\times \sqrt{2}$ smaller

$$\begin{aligned} \sigma_{\Delta V_T}^2 &= \sigma_{V_{T1,abs}}^2 + \sigma_{V_{T2,abs}}^2 \\ &= 2 \sigma_{V_{T,abs}}^2 \quad [\text{if devices have the same size}] \end{aligned}$$

$$\sigma_{V_{T,abs}} = \frac{\sigma_{\Delta V_T}}{\sqrt{2}}$$

- The way two transistors are laid out can affect their matching factors. The more symmetric they are the better they match

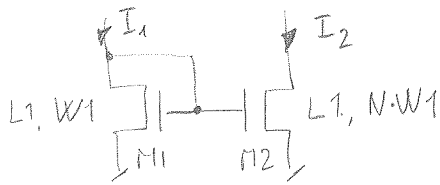
- $\sigma_{\Delta V_T} \approx 2-5 \text{ mV}$ for a deep sub-micron technology

Current Mismatch in a Mirror

(2B)

- Due to transistor mismatch, the current will never be the same in a mirror, even if $V_{DS1} = V_{DS2} = V_{GS1}$.

- current mismatch calculation;



$$\begin{cases} \sigma_{V_{GS1}}^2 = \sigma_{V_{T1}}^2 + \left(\frac{\sqrt{\beta_1}}{\beta_1}\right)^2 (V_{GS1} - V_T)^2 \\ \sigma_{V_{GS2}}^2 = \sigma_{V_{T2}}^2 + \left(\frac{\sqrt{\beta_2}}{\beta_2}\right)^2 (V_{GS2} - V_T)^2 \end{cases}$$

$$\sigma_{\Delta I}^2 = (V_{GS1}^2 + V_{GS2}^2) g_{m2}^2 =$$

$$V_{GS2} = V_{GS1}$$

$$L_2 = L_1$$

$$W_2 = N W_1$$

$$= g_{m2}^2 \cdot \left[\sigma_{V_{T1}}^2 + \sigma_{V_{T2}}^2 + \left(\frac{\sqrt{\beta_1}}{\beta_1}\right)^2 (V_{GS1} - V_T)^2 + \left(\frac{\sqrt{\beta_2}}{\beta_2}\right)^2 (V_{GS1} - V_T)^2 \right]$$

$$= 2 I \kappa' \frac{N W_1}{L_1} \left[\frac{A_{\Delta V_T}^2}{2 W_1 L_1} + \frac{A_{\Delta V_T}^2}{2 N W_1 L_1} + \left(\frac{A_{\beta}^2}{2 W_1 L_1} + \frac{A_{\beta}^2}{2 N W_1 L_1} \right) (V_{GS1} - V_T)^2 \right]$$

$$= \frac{I \kappa' N}{L_1^2} A_{\Delta V_T}^2 \left(1 + \frac{1}{N} \right) + \frac{I \kappa' N A_{\beta}^2}{L_1^2} (V_{GS1} - V_T)^2 \left(1 + \frac{1}{N} \right)$$

$$= \boxed{\frac{I \kappa' N}{L_1^2} \left(1 + \frac{1}{N} \right) \left[A_{\Delta V_T}^2 + A_{\beta}^2 (V_{GS} - V_T)^2 \right]}$$

$$\Rightarrow \boxed{\left(\frac{\sigma_{\Delta I}}{I_2} \right)^2 = \frac{\kappa'}{L_1^2} \left(1 + \frac{1}{N} \right) \left[A_{\Delta V_T}^2 + A_{\beta}^2 (V_{GS} - V_T)^2 \right]}$$

- matching improves $L_1 \uparrow$
- matching gets worse when $\sqrt{N} \uparrow$
- the second term in the brackets is usually negligible for normal $V_{GS} - V_T$, however it can contribute if $V_{GS} - V_T$ is ~~big~~ big.
- if more mirrors are connected to the chain, the ^{current} errors will sum in an rms way.

3A

Mismatch due to voltage unsymmetry

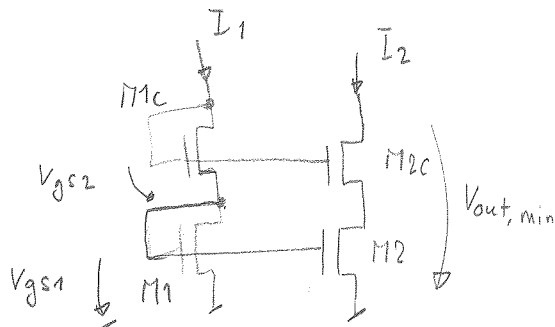
- As we have seen before, if the transistor $\lambda_1, \lambda_2 \neq 0$ and $V_{GS1} \neq V_{DS2}$ there is a deterministic mismatch of the mirror currents. $\rightarrow$ maximize r_o !

$$\frac{I_1}{I_2} = \frac{1 + \lambda_1 V_{GS1}}{1 + \lambda_2 V_{DS2}}$$

Cascode Current Mirrors

- In the last class we showed that the cascode structure can boost the transistor impedance

Cascode Mirror #1

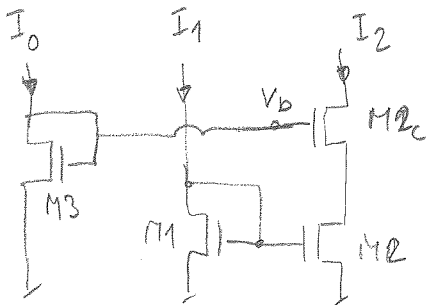


+ : λ is reduced by a factor $\sim g_{m2c} r_{o2c}$

- : large minimum V_{out} to keep $M2, M2c$ in saturation

$$\rightarrow V_{out,min} = V_{GS1} + V_{GS2} - V_{T2} = \underline{V_{DSat1} + V_{DSat2} + V_{T1}}$$

Cascode Mirror #2



+ : V_b can be selected such that

$$V_{out,min} = V_{DSat2} + V_{DSat2c} \rightarrow V_{out,min} \text{ is small}$$

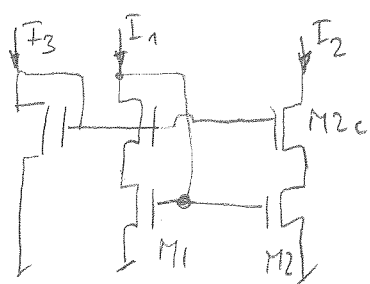
+ : High output impedance

$\rightarrow I_2$ is insensitive to variations of V_{out}

- : not precise because λ_1 is still high

- : needs an additional bias current

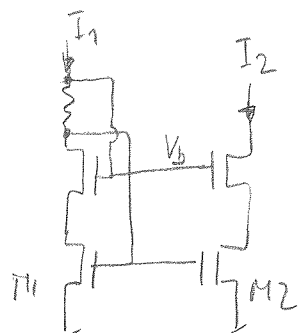
Cascode Mirror #3



→ Most widely used

- + : both sides of the mirror are high impedance
- + : $\min V_{out, \min} = V_{Dsat2} + V_{Dsat2c}$ can be obtained
- : needs an additional bias current

Cascode Mirror #4



- + : both sides are high imp.
- + : $\min V_{out, \min}$
- + : no additional bias current needed
- : V_b does not always track process / temp variations
 $V_{out, \min}$ is not always optimum