

Course notes for EE394V

Restructured Electricity Markets: Market Power

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7

Homework solutions

- We will discuss market outcomes from group homeworks.
- Aggregated offer curves will be shown to facilitate update of offer:
 - in practice, most markets disclose aggregate or individual offers so that data is available publicly to investigate performance of market,
 - data is typically disclosed months after transaction day so as to avoid facilitating collusion.

7.1 Homework exercise due February 2: Discussion

Interval 1 supply

- Clearing price is \$52.81/MWh compared to \$50/MWh competitive price.

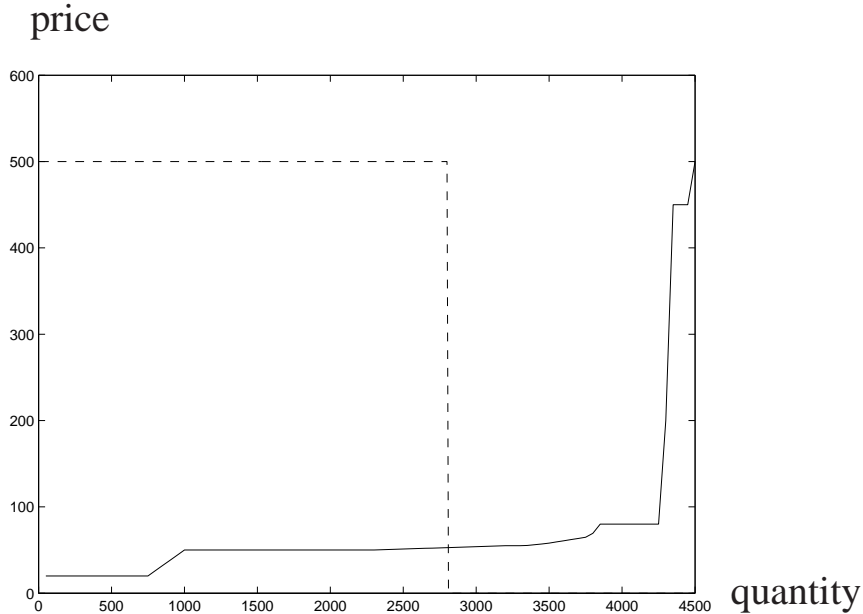


Fig. 7.1. Industry supply in interval 1.

Interval 1 supply detail

- Clearing price is \$52.81/MWh compared to \$50/MWh competitive price.

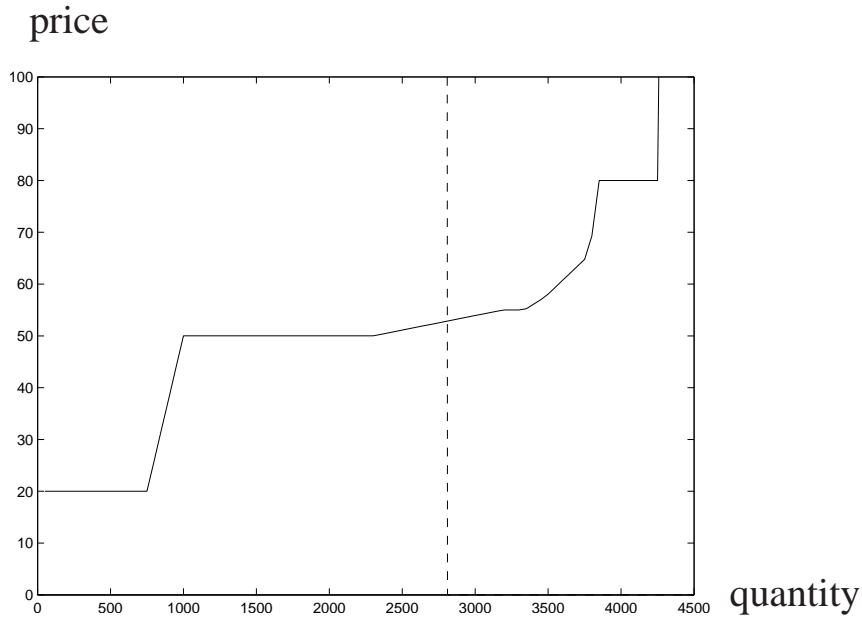


Fig. 7.2. Detail of industry supply in interval 1.

Interval 1 profits per MWh sold

Group	Capacity (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	32.81	30
1	250	32.81	30
1	250	32.81	30
2	250	32.81	30
2	250	32.81	30
2	250	32.81	30
3	150	0	0
3	150	2.81	0
3	150	0	0
4	150	2.81	0
4	150	2.81	0
4	150	2.81	0
5	50	0	0
5	50	0	0
5	50	0	0

Group	Capacity (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	0	0
6	50	0	0
6	50	0	0
7	250	32.81	30
7	150	2.81	0
7	50	0	0
8	250	32.81	30
8	150	2.81	0
8	50	0	0
9	250	32.81	30
9	150	2.81	0
9	50	0	0
10	250	32.81	30
10	150	2.81	0
10	50	0	0
Demand	2800	447.19	450

Discussion of interval 1

- Profits per MWh produced are \$2.81/MWh above competitive profit, reflecting mark-up.
- Transfer of wealth from demand to generators is \$7,868/h more than the demand payment in the competitive case of \$140,000/h.
- Dispatch used all baseload and no peaking.
 - Operating costs same as in competitive solution of \$65,000/h.
- Demand is same as in competitive case.

Discussion of interval 1, continued

- All offers were at or above marginal costs.
- Some groups with intermediate capacity identified that to maximize profit they should economically withhold, since intermediate capacity was “marginal” in the competitive case:
 - Typical offer for intermediate was to set intercept of supply offer equal to marginal cost and then choose slope of offer either zero or so that, at full output, offer was below marginal cost of peaking.
 - Rationale of slope was presumably to increase price but avoid ceding sales to peaking generation.
 - Group 3 offered in one of its intermediate units at \$450/MWh, exceeding offer price of most peakers; however, no peakers were dispatched so the high offer price did not result in deviation from economic dispatch.
- Typical offer for peaking was to set intercept of supply offer equal to marginal cost and choose slope of offer equal to zero.
 - No peaking capacity dispatched to meet demand.
 - Group 3 offered its intermediate capacity at a very high price.

Discussion of interval 1, continued

- Baseload offers less consistent:
 - Groups 6 to 10 offered baseload at or close to marginal cost.
 - Groups 1 and 2 offered its baseload above marginal.
- Withholding by intermediate was profitable, since operating profit per MWh was zero in competitive solution and non-zero with offers.
 - Owners of intermediate generation were exercising market power, according to our definition.

Interval 2 supply

- Clearing price is \$60.96/MWh compared to \$50/MWh competitive price.

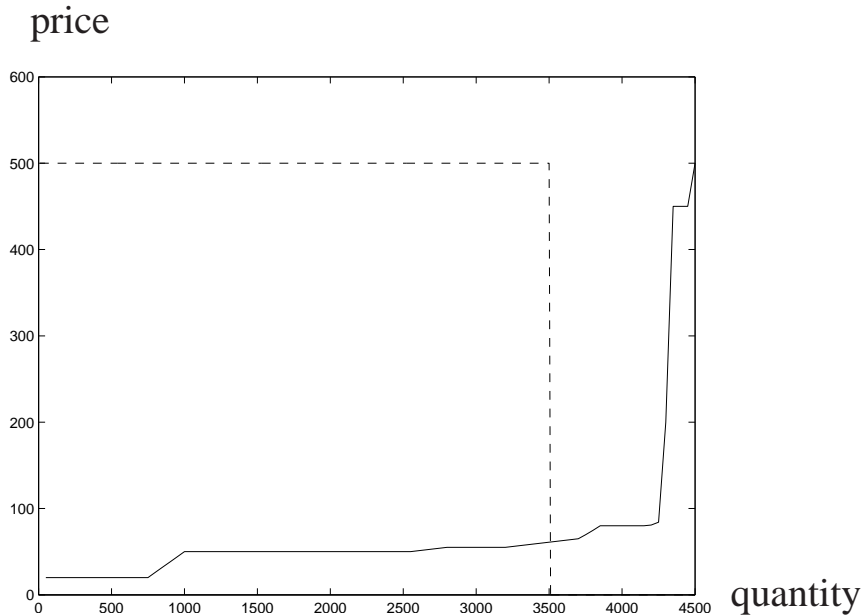


Fig. 7.3. Industry supply in interval 2.

Interval 2 supply detail

- Clearing price is \$60.96/MWh compared to \$50/MWh competitive price.

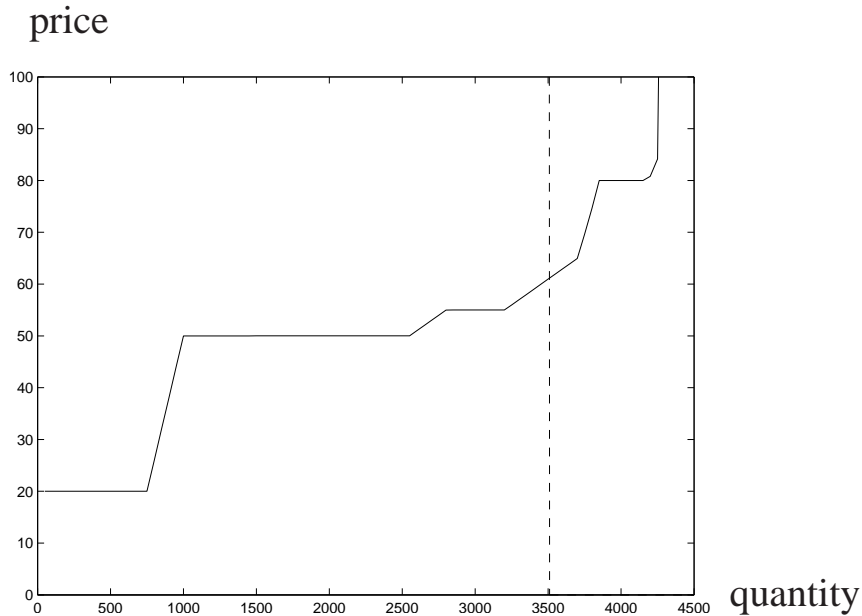


Fig. 7.4. Detail of industry supply in interval 2.

Interval 2 profits per MWh sold

Group	Capacity (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	40.96	30
1	250	40.96	30
1	250	40.96	30
2	250	40.96	30
2	250	40.96	30
2	250	40.96	30
3	150	10.96	0
3	150	10.96	0
3	150	0	0
4	150	10.96	0
4	150	10.96	0
4	150	10.96	0
5	50	0	0
5	50	0	0
5	50	0	0

Group	Capacity (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	0	0
6	50	0	0
6	50	0	0
7	250	40.96	30
7	150	10.96	0
7	50	0	0
8	250	40.96	30
8	150	10.96	0
8	50	0	0
9	250	40.96	30
9	150	10.96	0
9	50	0	0
10	250	40.96	30
10	150	10.96	0
10	50	0	0
Demand	3500	439.04	450

Discussion of interval 2

- Economic withholding by groups 1 and 2 by setting intercept at (approximately) \$50/MWh, recognizing that clearing price would be at least \$50/MWh.
- Close to competitive offers for most assets for most other groups except group 3.
- Group 3 offered one of its intermediate units in at \$450/MWh, but there was enough other intermediate capacity so that dispatch was optimal.

Discussion of interval 2, continued

- Profits per MWh produced are \$10.96/MWh above competitive profit, reflecting mark-up.
- Exercise of market power, as measured by mark-up, is greater than in interval 1.
- Transfer of wealth from demand to generators is \$38,360/h more than the demand payment in the competitive case of \$175,000/h.
- Demand is same as in competitive case.

Discussion of interval 2, continued

- Profit of group 3:
 - Given offers, group 3 generates 206.78 MW and operating profit is $206.78 \text{ MW} \times (60.96 - 50) \$/\text{MWh} = \$2,266.39/\text{h}$.
 - Exceeds operating profit in competitive case of zero.
- Profits of other groups (except groups 5 and 6) also higher than in competitive because of higher prices.

Interval 3 supply

- Clearing price is \$500/MWh compared to \$80/MWh competitive price.

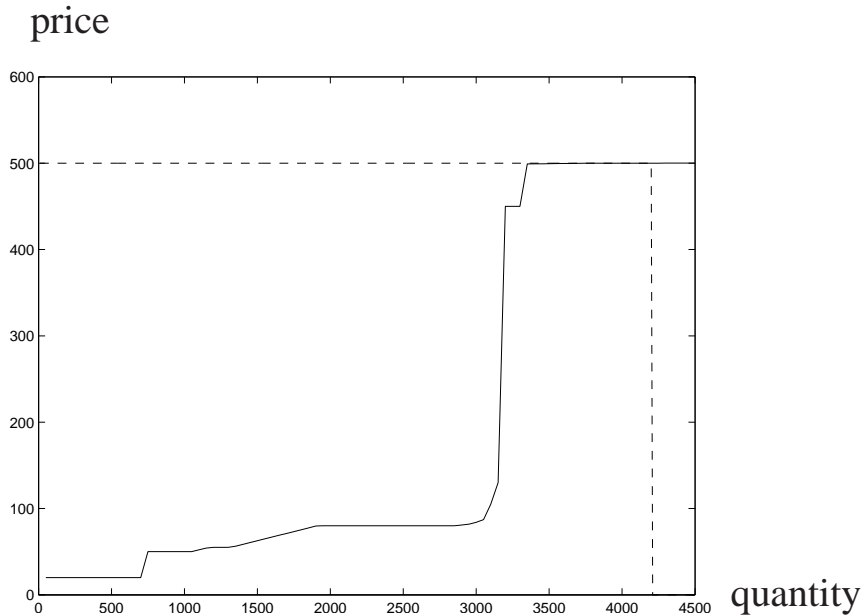


Fig. 7.5. Industry supply in interval 3.

Interval 3 supply detail

- Clearing price is \$500/MWh compared to \$80/MWh competitive price.

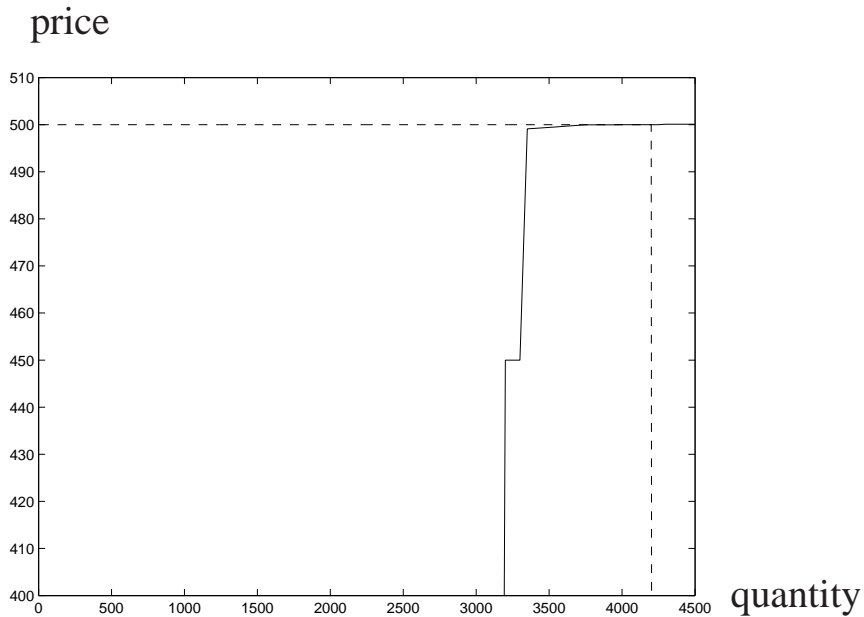


Fig. 7.6. Detail of industry supply in interval 3.

Interval 3 profits per MWh sold

Group	Capacity (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	480	60
1	250	480	60
1	250	0	60
2	250	480	60
2	250	480	60
2	250	480	60
3	150	450	30
3	150	450	30
3	150	450	30
4	150	450	30
4	150	450	30
4	150	450	30
5	50	420	0
5	50	420	0
5	50	420	0

Group	Capacity (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	420	0
6	50	420	0
6	50	420	0
7	250	480	60
7	150	450	30
7	50	420	0
8	250	480	60
8	150	450	30
8	50	420	0
9	250	480	60
9	150	450	30
9	50	420	0
10	250	480	60
10	150	450	30
10	50	420	0
Demand	4200	-0	420

Discussion of interval 3

- Group 1 offered in two of its units at close to or equal to \$500/MWh:
 - dispatched to 450 MW,
 - compared to 750 MW in competitive solution,
 - so dispatch cost is \$18,000/h more than in competitive solution.
- Profit of group 1 is $450 \text{ MW} \times (500 - 20) \$/\text{MWh} = \$216,000/\text{h}$
- Group has economically withheld and its profits are well above competitive levels.
- However, if group 1 had offered more of its capacity at a slightly lower price, it might have generated more at roughly the same price and received even higher profits!
- Large transfer of wealth from demand to generators that greatly exceeds the inefficiency of dispatch.
- Demand is same as in competitive case.

Homework exercise: Due Tuesday, February 16, by 10pm

- Suppose that the cost and demand conditions for the last homework exercise stayed exactly the same.
- Update your offers to improve your profits compared to your previous offers.

Homework exercise due Thursday, February 11: discussion

- Assume that the costs c_i and benefits b_k are differentiable and that offers and bids are competitive.
- Assume that the market clearing conditions involve a price P^* , generator quantities Q_i^* , and demand quantities D_k^* .
- Show that the market clearing quantities maximize the profits (“producer surplus”) of the generators and maximize the “consumer surplus” of the loads.
- That is, show that:
 - (i) for each generator i , for the given price P^* , the choice Q_i^* maximizes the producer surplus $P^*Q_i - c_i(Q_i)$ over choices of Q_i , and
 - (ii) for each demand k , for the given price P^* , the choice D_k^* maximizes the consumer surplus $b_k(D_k) - P^*D_k$ over choices of D_k .
- So, if profits are maximized given the prices P^* , why would any market participant choose to offer non-competitively?

Homework exercise due Thursday, February 11: discussion

- (i) Consider the producer surplus maximization problem for generator i , given the price P^* :

$$\max_{Q_i} P^* Q_i - c_i(Q_i).$$

- Since c_i is convex in its argument, the objective of this problem, $P^* Q_i - c_i(Q_i)$, is concave in Q_i .
- The objective is also differentiable.
- Therefore, the first-order necessary conditions for maximizing the objective are sufficient.
- The first-order necessary conditions are:

$$P^* = \frac{\partial c_i}{\partial Q_i}(Q_i).$$

- The market clearing conditions are that:

$$p_i(Q_i^*) = P^*.$$

- Since the offers were competitive, $p_i = \frac{\partial c_i}{\partial Q_i}$, so that

$$\frac{\partial c_i}{\partial Q_i}(Q_i^*) = P^*.$$

- That is, the market clearing quantity Q_i^* maximizes the producer surplus, given the price P^* .
- (ii) A similar argument applies to the consumer surplus.
- (iii) The argument rests on the price P^* being given. However, if a generator or load can affect the price (that is, if it has market power) then it will find that it can improve its profits by offering non-competitively and therefore change the price compared to the competitive price P^* .

Homework exercise due February 16: Discussion

Interval 1 supply

- Clearing price is \$50.0002/MWh compared to \$50/MWh competitive price and \$52.81/MWh in last week's submission.

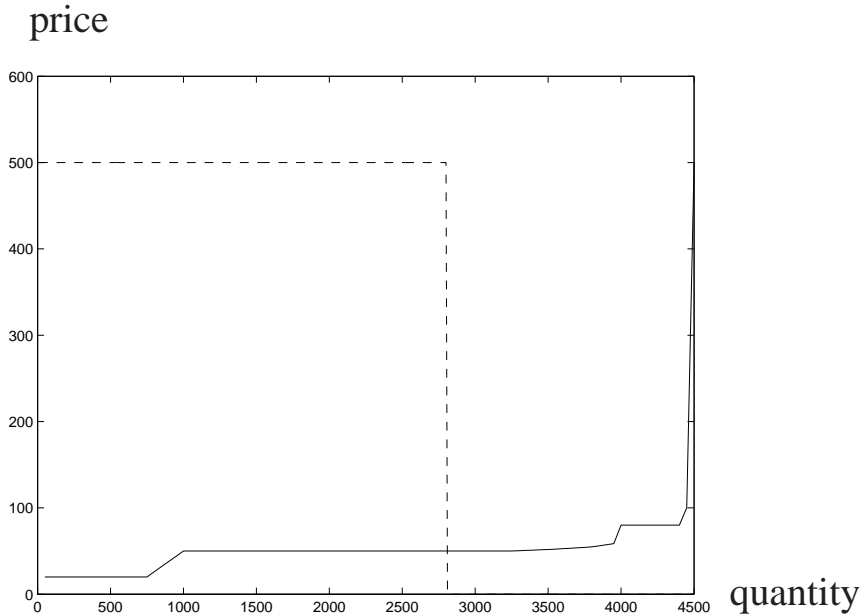


Fig. 7.7. Industry supply in interval 1.

Interval 1 supply detail

- Clearing price is \$50.0002/MWh compared to \$50/MWh competitive price and \$52.81/MWh in last week's submission.

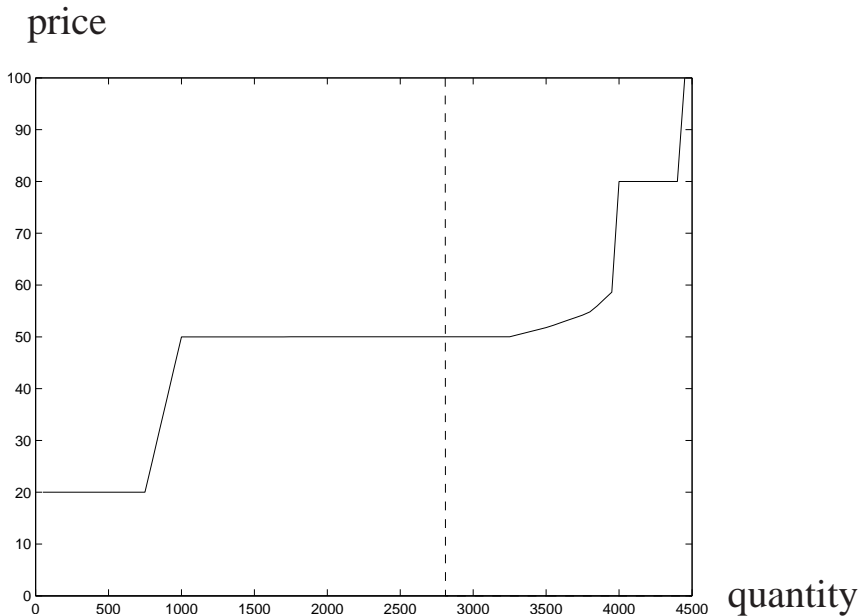


Fig. 7.8. Detail of industry supply in interval 1.

Interval 1 profits per MWh sold

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	249.99	30	30
1	250	249.99	30	30
1	250	249.99	30	30
2	250	250	30	30
2	250	250	30	30
2	250	250	30	30
3	150	0	0	0
3	150	0	0	0
3	150	0	0	0
4	150	150	0	0
4	150	0	0	0
4	150	0	0	0
5	50	0	0	0
5	50	0	0	0
5	50	0	0	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	0	0	0
6	50	0	0	0
6	50	0	0	0
7	250	250	30	30
7	150	0.01	0	0
7	50	0	0	0
8	250	250	30	30
8	150	0.02	0	0
8	50	0	0	0
9	250	250	30	30
9	150	150	0	0
9	50	0	0	0
10	250	250	30	30
10	150	0.01	0	0
10	50	0	0	0
Demand	2800	2800	450	450

Discussion of interval 1

- Profits per MWh produced are essentially the same as competitive profit, reflecting very small mark-up.
- Transfer of wealth from demand to generators is \$0.56/h more than the demand payment in the competitive case of \$140,000/h.
- Dispatch used almost the same baseload (≈ 2499.97 MW) and almost the same intermediate (≈ 300.03 MW) as in competitive solution.
- No peaking capacity dispatched to meet demand.
 - Operating costs therefore essentially the same as in competitive solution of \$65,000/h.
- Demand is same as in competitive case.

Discussion of interval 1, continued

- Again all offers were at or above marginal costs:
 - some of the baseload offered at approximately the marginal cost of the intermediate capacity,
 - but enough of the intermediate capacity was offered close enough to marginal so that price was essentially competitive.
- Offers were more competitive than in last week's submission reflecting competition in supply between intermediate generation.
- Goal of closer-to-competitive offer was to sell more, but this also resulted in lower clearing prices.
- Overall result is less transfer from demand to generators.

Interval 2 supply

- Clearing price is \$54.29/MWh compared to \$50/MWh competitive price and \$60.96/MWh in last week's submission.

price

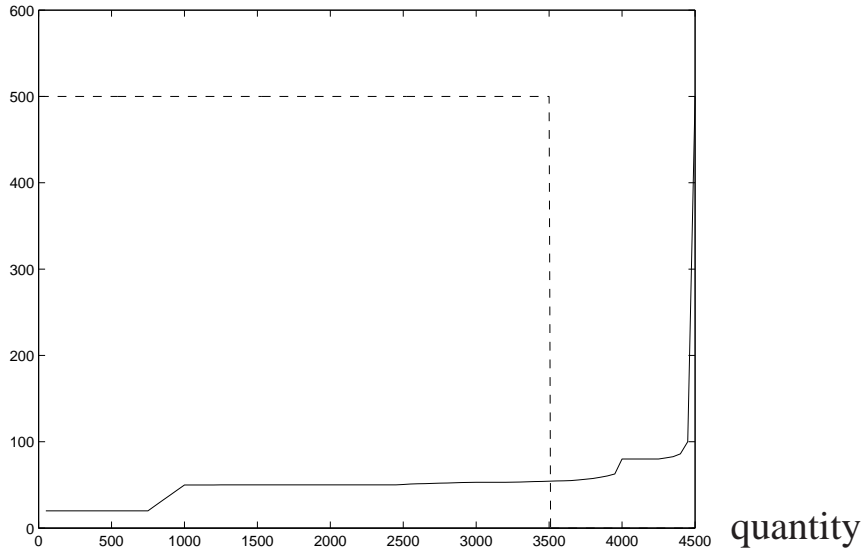


Fig. 7.9. Industry supply in interval 2.

Interval 2 supply detail

- Clearing price is \$54.29/MWh compared to \$50/MWh competitive price and \$60.96/MWh in last week's submission.

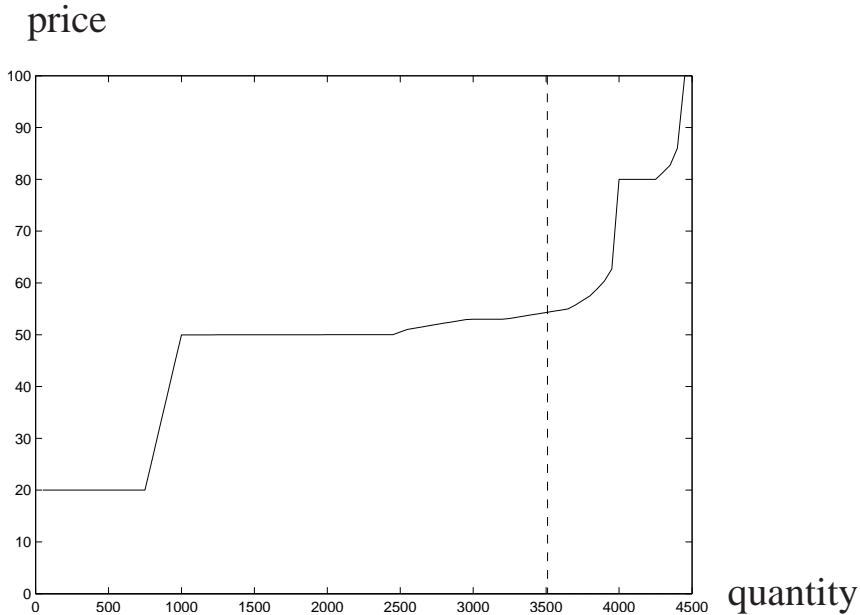


Fig. 7.10. Detail of industry supply in interval 2.

Interval 2 profits per MWh sold

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	250	34.29	30
1	250	250	34.29	30
1	250	214.70	34.29	30
2	250	250	34.29	30
2	250	250	34.29	30
2	250	250	34.29	30
3	150	122	4.29	0
3	150	122	4.29	0
3	150	122	4.29	0
4	150	150	4.29	0
4	150	150	4.29	0
4	150	26.37	4.29	0
5	50	0	0	0
5	50	0	0	0
5	50	0	0	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	0	0	0
6	50	0	0	0
6	50	0	0	0
7	250	250	34.29	30
7	150	42.94	4.29	0
7	50	0	0	0
8	250	250	34.29	30
8	150	64.09	4.29	0
8	50	0	0	0
9	250	250	34.29	30
9	150	150	4.29	0
9	50	0	0	0
10	250	250	34.29	30
10	150	85.88	4.29	0
10	50	0	0	0
Demand	3500	3500	445.71	450

Discussion of interval 2

- Demand is same as in competitive case.
- Clearing price is much lower than last week.
- Profits per MWh are \$4.29 above competitive profit.
- Transfer of wealth from demand to generators is much smaller, only \$15,015/h more than the demand payment in the competitive case of \$175,000/h:
 - smaller than in last week's solution where transfer was \$38,360/h more than in competitive solution.
- However, dispatch is not efficient since:
 - only 2464.7 MW of baseload out of 2500 MW baseload capacity dispatched,
 - but 1035.3 MW of intermediate dispatched, compared to 1000 MW in competitive solution with efficient dispatch.

Discussion of interval 2, continued

- Deviation from efficient dispatch primarily due to economic withholding by group 1.
- Profit of group 1:
 - Given offers, group 1 generates 714.7 MW and operating profit is $714.7 \text{ MW} \times (54.29 - 20) \$/\text{MWh} = \$24,507/\text{h}$,
 - Much lower than operating profit for group 1 from last week's solution of $750 \text{ MW} \times (60.96 - 20) \$/\text{MWh} = \$30,720/\text{h}$.
 - But if group 1 had offered competitively, clearing price would be closer to competitive, production would be 750 MW, and operating profit would be at least $750 \text{ MW} \times (50 - 20) \$/\text{MWh} = \$22,500/\text{h}$.
- Note that group 1 may have been able to increase profits by supplying somewhat more because economic withholding by intermediate capacity would still have set price above competitive.

Interval 3 supply

- Clearing price is \$447/MWh compared to \$80/MWh competitive price and \$500/MWh in last week's submission.

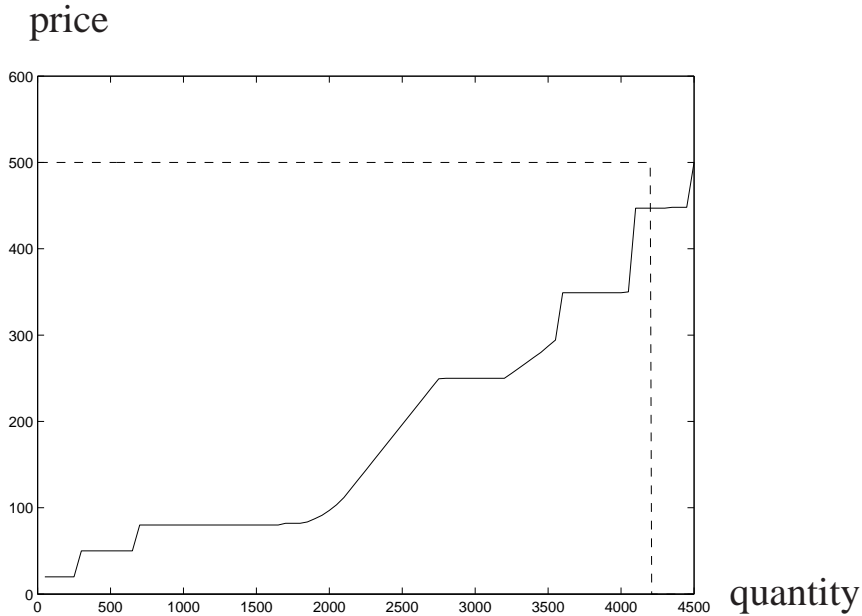


Fig. 7.11. Industry supply in interval 3.

Interval 3 supply detail

- Clearing price is \$447/MWh compared to \$80/MWh competitive price and \$500/MWh in last week's submission.

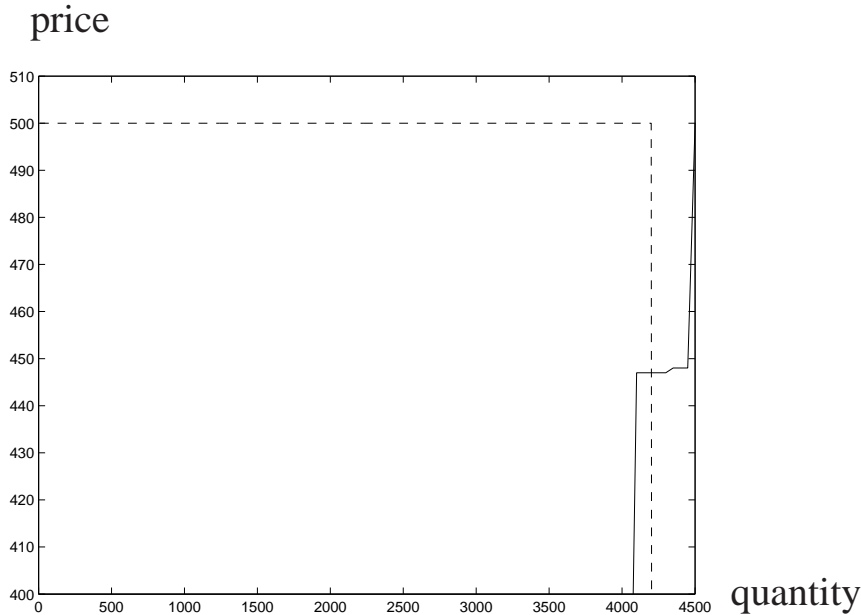


Fig. 7.12. Detail of industry supply in interval 3.

Interval 3 profits per MWh sold

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	250	427	60
1	250	250	427	60
1	250	250	427	60
2	250	250	427	60
2	250	250	427	60
2	250	250	427	60
3	150	150	397	30
3	150	150	397	30
3	150	150	397	30
4	150	150	397	30
4	150	150	397	30
4	150	150	397	30
5	50	50	367	0
5	50	50	367	0
5	50	50	367	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	50	367	0
6	50	50	367	0
6	50	50	367	0
7	250	250	427	60
7	150	150	397	30
7	50	50	367	0
8	250	250	427	60
8	150	150	397	30
8	50	50	367	0
9	250	250	427	60
9	150	150	397	30
9	50	50	367	0
10	250	150	427	60
10	150	0	397	30
10	50	0	367	0
Demand	4200	4200	53	420

Discussion of interval 3

- Demand is same as in competitive case.
- Prices have fallen compared to last week's solution.
- As in last week's solution, large transfer of wealth from demand to generators is \$1,541,400/h more than the demand payment in the competitive case of \$336,000/h.
- Inefficient dispatch due to economic withholding by group 10:
 - only 150 MW out of 250 MW baseload capacity dispatched,
 - zero MW out of 150 MW intermediate capacity dispatched,
 - resulting in additional 250 MW of peaking capacity from other groups dispatched.
 - Additional dispatch cost of \$10,500 compared to competitive dispatch cost of \$141,000.

Discussion of interval 3

- Group 10 profits $150 \text{ MW} \times (447 - 20) \text{ \$/MWh} = \$64,050$.
- If group 10 had offered competitively, and other offers stayed the same, then price would have been around $\$300/\text{MWh}$, and group 10 profits would have been over $\$100,000/\text{h}$.
- Group 10 withheld, but not profitably!
- Not exercising market power according to our definition, but not behaving competitively either!

Homework exercise due February 23: Discussion

Interval 1 supply

- Clearing price is \$50.0001/MWh compared to \$50/MWh competitive price and \$50.0002/MWh in last week's submission.

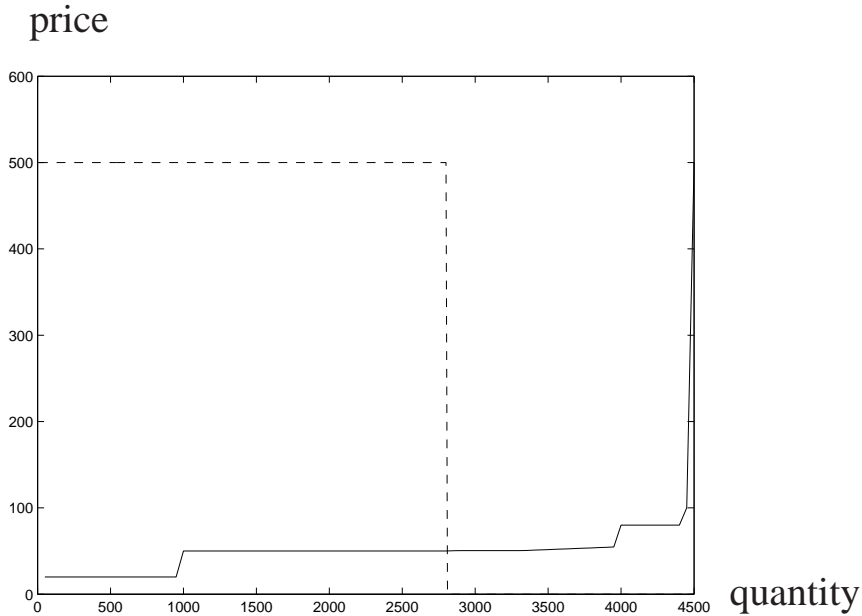


Fig. 7.13. Industry supply in interval 1.

Interval 1 supply detail

- Clearing price is \$50.0001/MWh compared to \$50/MWh competitive price and \$50.0002/MWh in last week's submission.

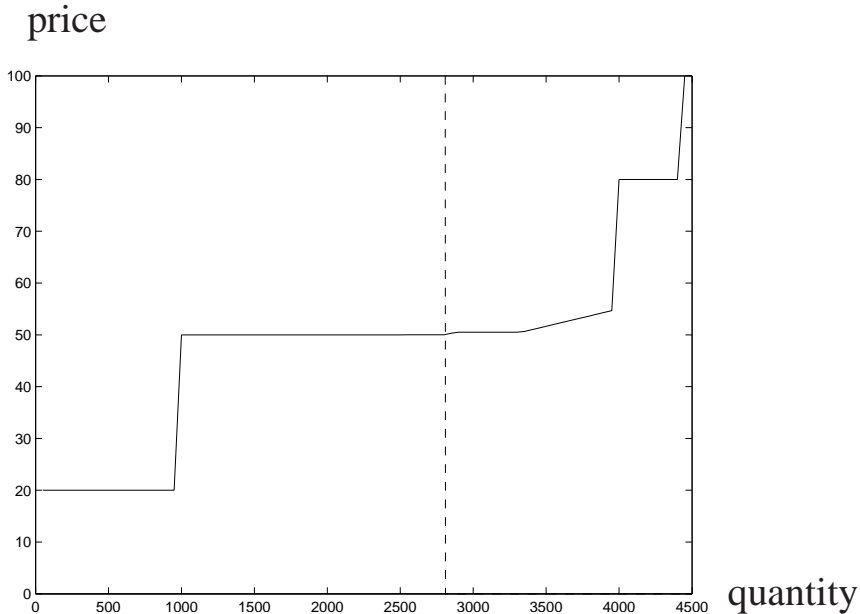


Fig. 7.14. Detail of industry supply in interval 1.

Interval 1 profits per MWh sold

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	250	30	30
1	250	250	30	30
1	250	250	30	30
2	250	250	30	30
2	250	250	30	30
2	250	250	30	30
3	150	0	0	0
3	150	0	0	0
3	150	0	0	0
4	150	0	0	0
4	150	0	0	0
4	150	0	0	0
5	50	0	0	0
5	50	0	0	0
5	50	0	0	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	0	0	0
6	50	0	0	0
6	50	0	0	0
7	250	250	30	30
7	150	0.01	0	0
7	50	0	0	0
8	250	250	30	30
8	150	0.01	0	0
8	50	0	0	0
9	250	250	30	30
9	150	149.99	0	0
9	50	0	0	0
10	250	250	30	30
10	150	149.99	0	0
10	50	0	0	0
Demand	2800	2800	450	450

Discussion of interval 1

- Profits per MWh produced are again essentially the same as competitive profit, reflecting very small mark-up.
- Transfer of wealth from demand to generators is only \$0.28/h more than the demand payment in the competitive case of \$140,000/h.
- Dispatch used the same baseload and the same intermediate as in competitive solution.
- No peaking capacity dispatched to meet demand.
 - Operating costs the same as in competitive solution of \$65,000/h.
- Demand is same as in competitive case.
- Essentially competitive outcome.

Interval 2 supply

- Clearing price is \$53.95/MWh compared to \$50/MWh competitive price and \$54.29/MWh in last week's submission.

price

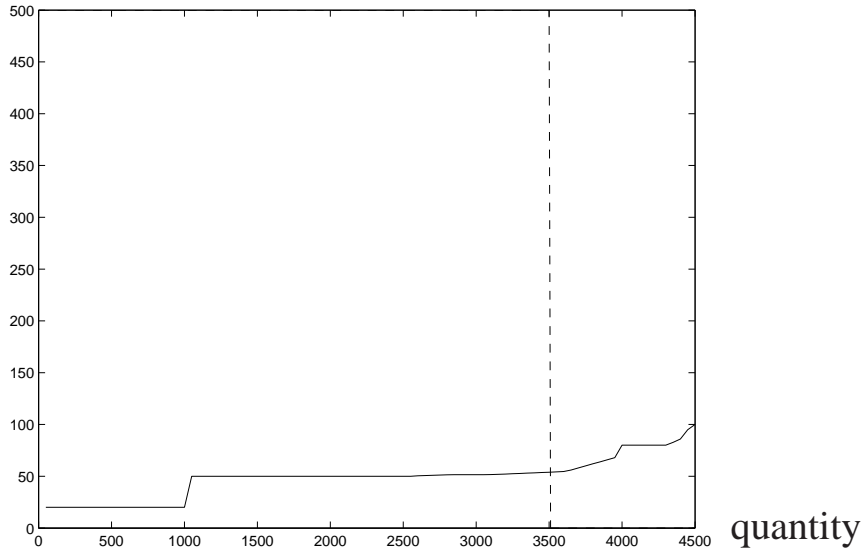


Fig. 7.15. Industry supply in interval 2.

Interval 2 supply detail

- Clearing price is \$53.95/MWh compared to \$50/MWh competitive price and \$54.29/MWh in last week's submission.

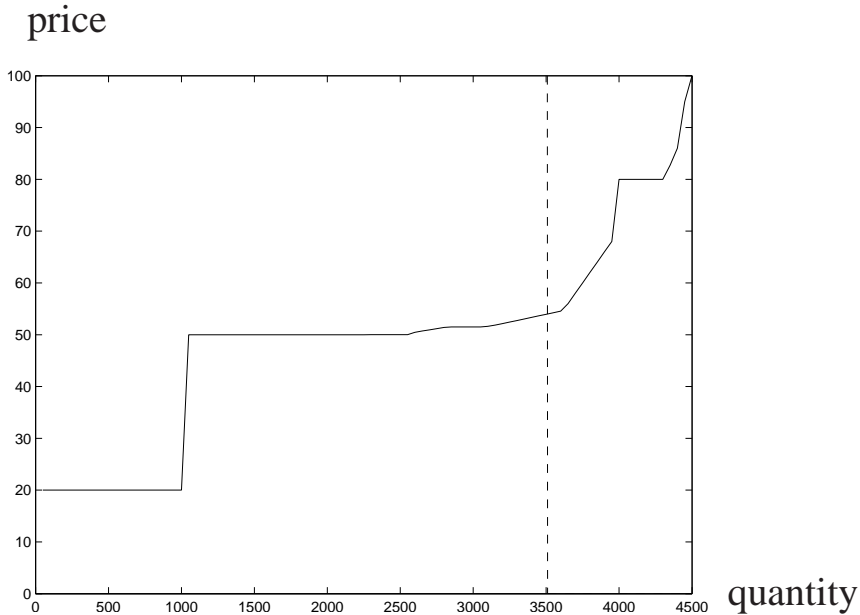


Fig. 7.16. Detail of industry supply in interval 2.

Interval 2 profits per MWh sold

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	250	33.95	30
1	250	250	33.95	30
1	250	250	33.95	30
2	250	250	33.95	30
2	250	250	33.95	30
2	250	250	33.95	30
3	150	127.64	3.95	0
3	150	127.64	3.95	0
3	150	127.64	3.95	0
4	150	16.22	3.95	0
4	150	16.22	3.95	0
4	150	16.22	3.95	0
5	50	0	0	0
5	50	0	0	0
5	50	0	0	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	0	0	0
6	50	0	0	0
6	50	0	0	0
7	250	250	33.95	30
7	150	118.40	3.95	0
7	50	0	0	0
8	250	250	33.95	30
8	150	150	3.95	0
8	50	0	0	0
9	250	250	33.95	30
9	150	150	3.95	0
9	50	0	0	0
10	250	250	33.95	30
10	150	150	3.95	0
10	50	0	0	0
Demand	3500	3500	446.05	450

Discussion of interval 2

- Demand is same as in competitive case.
- Clearing price is a little lower than last week.
- Profits per MWh are \$3.95 above competitive profit.
- Transfer of wealth from demand to generators is again small, only \$13,825/h more than the demand payment in the competitive case of \$175,000/h.
 - smaller than in last week's solution where transfer was \$15,015/h more than in competitive solution.
- Dispatch is efficient.

Interval 3 supply

- Clearing price is \$500/MWh compared to \$80/MWh competitive price and \$447/MWh in last week's submission.

price

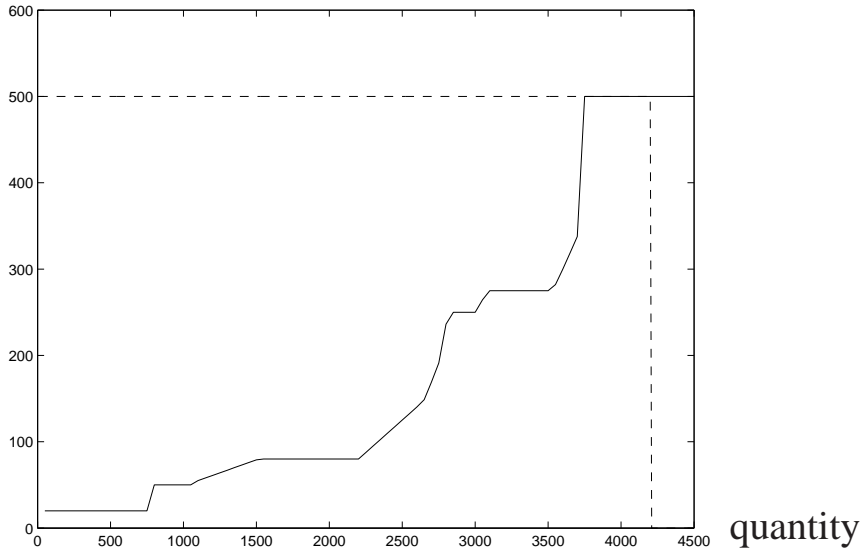


Fig. 7.17. Industry supply in interval 3.

Interval 3 supply detail

- Clearing price is \$500/MWh compared to \$80/MWh competitive price and \$447/MWh in last week's submission.

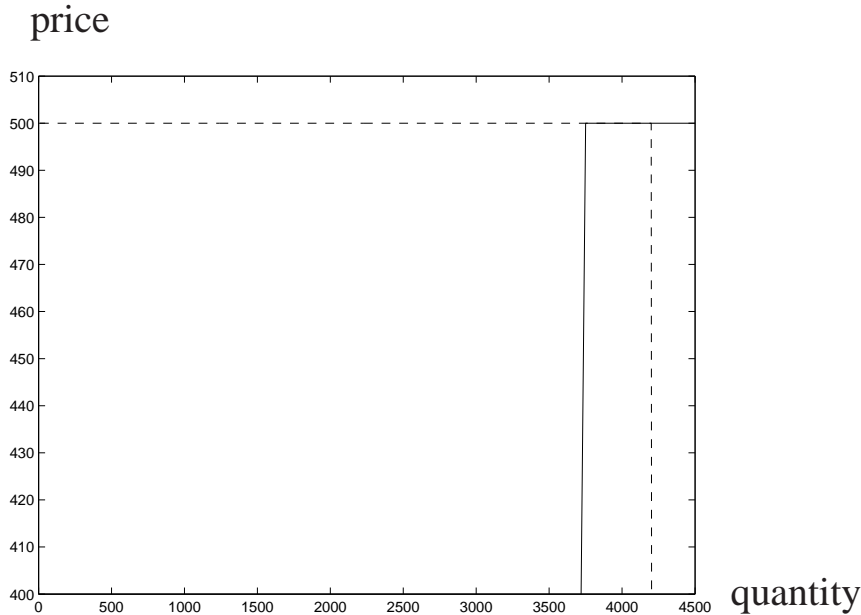


Fig. 7.18. Detail of industry supply in interval 3.

Interval 3 profits per MWh sold

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	150	480	60
1	250	150	480	60
1	250	150	480	60
2	250	250	480	60
2	250	250	480	60
2	250	250	480	60
3	150	150	450	30
3	150	150	450	30
3	150	150	450	30
4	150	150	450	30
4	150	150	450	30
4	150	150	450	30
5	50	50	420	0
5	50	50	420	0
5	50	50	420	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	50	420	0
6	50	50	420	0
6	50	50	420	0
7	250	250	480	60
7	150	150	450	30
7	50	50	420	0
8	250	250	480	60
8	150	150	450	30
8	50	50	420	0
9	250	250	480	60
9	150	150	450	30
9	50	50	420	0
10	250	250	480	60
10	150	150	450	30
10	50	50	420	0
Demand	4200	4200	0	420

Discussion of interval 3

- Demand is same as in competitive case.
- Prices have increased compared to last week's solution.
- As in last week's solution, large transfer of wealth from demand to generators is \$1,764,000/h more than the demand payment in the competitive case of \$336,000/h.
- Inefficient dispatch due to economic withholding by group 1:
 - only 450 MW out of 750 MW baseload capacity dispatched,
 - resulting in additional 300 MW of peaking capacity from other groups dispatched.
 - Additional dispatch cost of \$18,000 compared to competitive dispatch cost of \$141,000.

Discussion of interval 3

- Group 1 profits $450 \text{ MW} \times (500 - 20) \text{ \$/MWh} = \$216,000$.
- If group 1 had offered more of its capacity at a lower price, and other offers stayed the same, then market clearing price would still have been around $\$340/\text{MWh}$, and group 1 profits could have been as high as $750 \text{ MW} \times (340 - 20) \text{ \$/MWh} = \$240,000$, which is a little larger.
- Group 1 withheld, but not in a way to maximize its ex post profits!
- Group 1 is exercising market power according to our definition, but not behaving ex post optimally!
- However, given uncertainty in other offers, group 1's strategy was very good:
 - avoided risk of clearing price being lower than $\$340/\text{MWh}$.

Homework exercise due March 9: Discussion

Interval 3 supply

- Clearing price is \$500/MWh in each sub-period compared to \$80/MWh competitive price and \$500/MWh in last week's submission.

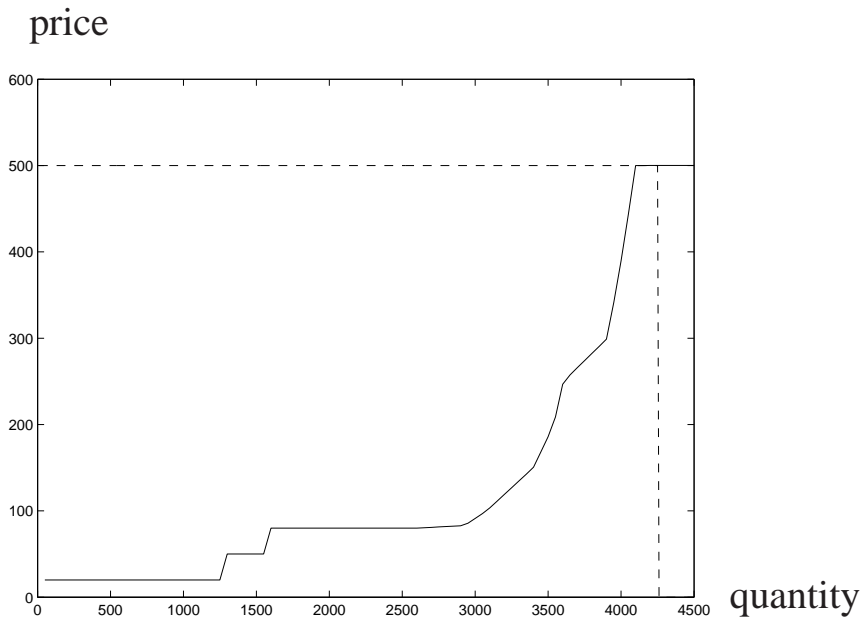


Fig. 7.19. Industry supply in interval 3, 4250 MW demand.

Interval 3 supply detail

- Clearing price is \$500/MWh in each sub-period compared to \$80/MWh competitive price and \$500/MWh in last week's submission.

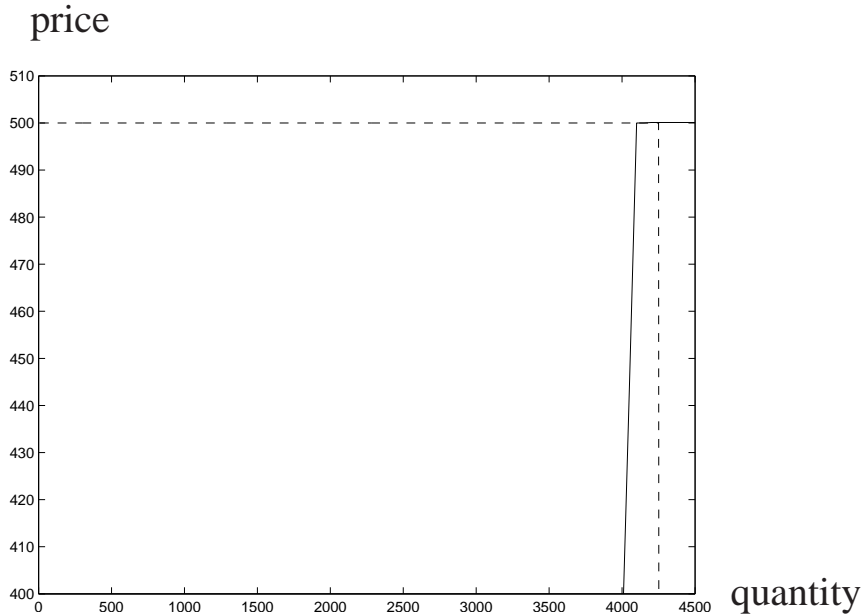


Fig. 7.20. Detail of industry supply in interval 3, 4250 MW demand.

Interval 3 profits per MWh sold

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	250	480	60
1	250	250	480	60
1	250	250	480	60
2	250	250	480	60
2	250	250	480	60
2	250	250	480	60
3	150	150	450	30
3	150	150	450	30
3	150	150	450	30
4	150	150	450	30
4	150	150	450	30
4	150	150	450	30
5	50	50	420	0
5	50	50	420	0
5	50	50	420	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	50	420	0
6	50	50	420	0
6	50	50	420	0
7	150	50	480	60
7	150	50	450	30
7	150	50	420	0
8	250	250	480	60
8	150	150	450	30
8	50	50	420	0
9	250	250	480	60
9	150	150	450	30
9	50	50	420	0
10	250	250	480	60
10	150	150	450	30
10	50	50	420	0
Demand	4200	4200	0	420

Discussion of interval 3

- Demand is same as in competitive case.
- Prices the same as in last week's solution.
- Inefficient dispatch due to economic withholding by group 7.
- Groups 1, 2, 7, 8, 9, 10 are pivotal for all three demand levels:
 - groups can achieve high prices despite variation in demand.

Homework exercise, March 9, solution

- We have a “symmetric duopoly” with each firm $i = 1, 2$ having marginal cost function:

$$\forall Q_i, c'_i(Q_i) = 20 + 60Q_i/2500.$$

- Operating range $[0, \bar{Q}_i]$, where $\bar{Q}_i = 2500$ MW.
- The inverse demand functions are as follows:
 - (i) $\forall Q, p^d(Q) = \max\{50 - (Q - 2800)/2, 0\}$,
 - (ii) $\forall Q, p^d(Q) = \max\{75 - (Q - 3500)/2, 0\}$,
 - (iii) $\forall Q, p^d(Q) = \max\{500 - (Q - 4200)/2, 0\}$,
- where Q is in MW and $p^d(Q)$ is in \$/MWh.
- For each inverse demand function, we apply the Cournot model to find the predicted prices and quantities.

Homework exercise, March 9, solution, interval (i)

- Inverse demand is:

$$\begin{aligned}\forall Q, p^d(Q) &= \max\{50 - (Q - 2800)/2, 0\}, \\ &= \max\{1450 - Q/2, 0\}, \\ &= 1450 - Q/2,\end{aligned}$$

- assuming that $1450 - Q/2 \geq 0$.
- For firm $i = 1$, we have that the profit is:

$$\begin{aligned}\pi_1(Q_1, Q_2) &= (1450 - (Q_1 + Q_2)/2)Q_1 - c_1(Q_1), \\ &= -\frac{1}{2}Q_1^2 + \left(1450 - \frac{1}{2}Q_2\right)Q_1 - c_1(Q_1).\end{aligned}$$

- Firm $i = 1$ can choose Q_1 , but accepts as fixed the value Q_2 (whatever it might actually be).

Homework exercise, March 9, solution, interval (i)

- Differentiating π_1 with respect to Q_1 and setting equal to zero to maximize profit, we obtain:

$$\begin{aligned} 0 &= \frac{\partial \pi_1}{\partial Q_1}(Q_1, Q_2), \\ &= -Q_1 + 1450 - \frac{1}{2}Q_2 - c'_i(Q_1), \\ &= -Q_1 - \frac{1}{2}Q_2 + 1450 - \left(20 + \frac{60}{2500}Q_1\right). \end{aligned}$$

- That is:

$$1.024Q_1 + 0.5Q_2 = 1430. \quad (7.1)$$

Homework exercise, March 9, solution, interval (i)

- Similarly, for firm $i = 2$, we have that:

$$\begin{aligned}\pi_2(Q_2, Q_1) &= (1450 - (Q_1 + Q_2)/2)Q_2 - c_2(Q_2), \\ &= -\frac{1}{2}Q_2^2 + \left(1450 - \frac{1}{2}Q_1\right)Q_2 - c_2(Q_2).\end{aligned}$$

- Firm $i = 2$ can choose Q_2 , but accepts as fixed the value Q_1 (whatever it might actually be).

Homework exercise, March 9, solution, interval (i)

- Differentiating π_2 with respect to Q_2 and setting equal to zero to maximize profit, we obtain:

$$\begin{aligned} 0 &= \frac{\partial \pi_2}{\partial Q_2}(Q_2, Q_1), \\ &= -Q_2 + 1450 - \frac{1}{2}Q_1 - c'_i(Q_2), \\ &= -Q_2 - \frac{1}{2}Q_1 + 1450 - \left(20 + \frac{60}{2500}Q_2\right). \end{aligned}$$

- That is:

$$0.5Q_1 + 1.024Q_2 = 1430. \quad (7.2)$$

Homework exercise, March 9, solution, interval (i)

- Solving the simultaneous equations (7.1) and (7.2), we obtain:

$$Q_1^* = 938.3\text{MW},$$

$$Q_2^* = 938.3\text{MW},$$

$$Q_1^* + Q_2^* = 1876.6\text{MW},$$

$$p^d(Q_1^* + Q_2^*) = 511.7\$/\text{MWh},$$

$$c'_i(Q_1^*) = c'_i(Q_2^*) = 42.5\$/\text{MWh}.$$

- Note that $1450 - (Q_1^* + Q_2^*)/2 \geq 0$ so that price is greater than zero, as assumed.
- In fact, price is much higher than marginal cost.
- Note that solution is “symmetric.”

Homework exercise, March 9, solution, interval (ii)

- Inverse demand is:

$$\begin{aligned}\forall Q, p^d(Q) &= \max\{75 - (Q - 3500)/2, 0\}, \\ &= \max\{1825 - Q/2, 0\}, \\ &= 1825 - Q/2,\end{aligned}$$

- assuming that $1825 - Q/2 \geq 0$.
- For firm $i = 1$, we have that the profit is:

$$\begin{aligned}\pi_1(Q_1, Q_2) &= (1825 - (Q_1 + Q_2)/2)Q_1 - c_1(Q_1), \\ &= -\frac{1}{2}Q_1^2 + \left(1825 - \frac{1}{2}Q_2\right)Q_1 - c_1(Q_1). \\ 0 &= \frac{\partial \pi_1}{\partial Q_1}(Q_1, Q_2), \\ &= -Q_1 + 1825 - \frac{1}{2}Q_2 - c'_1(Q_1), \\ &= -Q_1 - \frac{1}{2}Q_2 + 1825 - \left(20 + \frac{60}{2500}Q_1\right).\end{aligned}$$

Homework exercise, March 9, solution, interval (ii)

- That is:

$$1.024Q_1 + 0.5Q_2 = 1805. \quad (7.3)$$

- Similarly, for firm $i = 2$, we have:

$$0.5Q_1 + 1.024Q_2 = 1805. \quad (7.4)$$

- Solving the simultaneous equations (7.3) and (7.4), we obtain:

$$\begin{aligned} Q_1^* &= 1184.4\text{MW}, \\ Q_2^* &= 1184.4\text{MW}, \\ Q_1^* + Q_2^* &= 2368.8\text{MW}, \\ p^d(Q_1^* + Q_2^*) &= 640.6\$/\text{MWh}, \\ c_i'(Q_1^*) = c_i'(Q_2^*) &= 48.4\$/\text{MWh}. \end{aligned}$$

- Note that $1825 - (Q_1^* + Q_2^*)/2 \geq 0$ so that price is greater than zero.
- Price is even higher.

Homework exercise, March 9, solution, interval (iii)

- Inverse demand is:

$$\begin{aligned}\forall Q, p^d(Q) &= \max\{500 - (Q - 4200)/2, 0\}, \\ &= \max\{2600 - Q/2, 0\}, \\ &= 2600 - Q/2,\end{aligned}$$

- assuming that $2600 - Q/2 \geq 0$.
- For firm $i = 1$, we have that the profit is:

$$\begin{aligned}\pi_1(Q_1, Q_2) &= (2600 - (Q_1 + Q_2)/2)Q_1 - c_1(Q_1), \\ &= -\frac{1}{2}Q_1^2 + \left(2600 - \frac{1}{2}Q_2\right)Q_1 - c_1(Q_1). \\ 0 &= \frac{\partial \pi_1}{\partial Q_1}(Q_1, Q_2), \\ &= -Q_1 + 2600 - \frac{1}{2}Q_2 - c'_1(Q_1), \\ &= -Q_1 - \frac{1}{2}Q_2 + 2600 - \left(20 + \frac{60}{2500}Q_1\right).\end{aligned}$$

Homework exercise, March 9, solution, interval (iii)

- That is:

$$1.024Q_1 + 0.5Q_2 = 2580. \quad (7.5)$$

- Similarly, for firm $i = 2$, we have:

$$0.5Q_1 + 1.024Q_2 = 2580. \quad (7.6)$$

- Solving the simultaneous equations (7.5) and (7.6), we obtain:

$$\begin{aligned} Q_1^* &= 1692.9\text{MW}, \\ Q_2^* &= 1692.9\text{MW}, \\ Q_1^* + Q_2^* &= 3385.8\text{MW}, \\ p^d(Q_1^* + Q_2^*) &= 907.1\$/\text{MWh}, \\ c_i'(Q_1^*) = c_i'(Q_2^*) &= 60.63\$/\text{MWh}. \end{aligned}$$

- Note that $2600 - (Q_1^* + Q_2^*)/2 \geq 0$ so that price is greater than zero.
- Price is even higher.

Homework exercise: Discussion

Interval 3, sub-interval 1 supply from March 21

- Clearing price \$277/MWh compared to \$500/MWh last week and \$80/MWh competitive price.

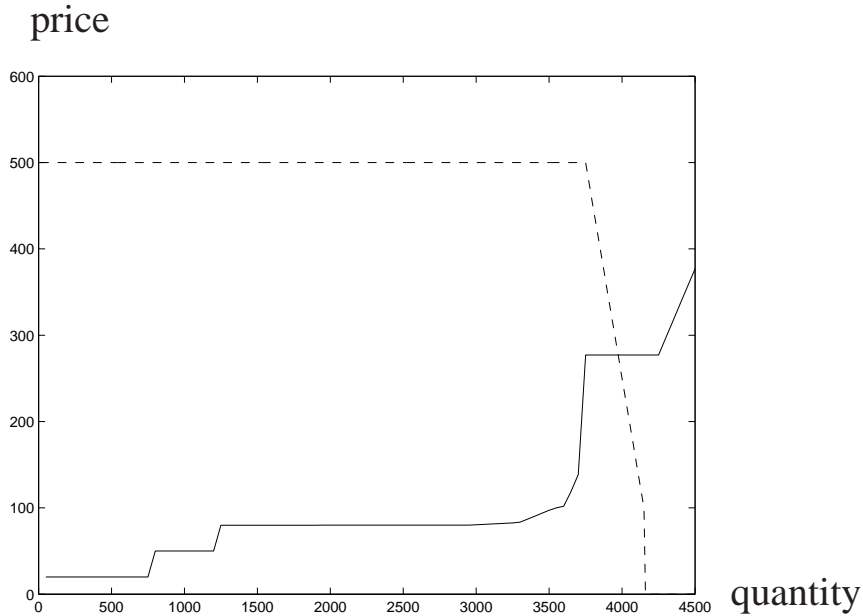


Fig. 7.21. Industry supply in interval 3, sub-interval 1.

Interval 3, sub-interval 1 supply detail from March 21

- Clearing price \$277/MWh compared to \$500/MWh last week and \$80/MWh competitive price.

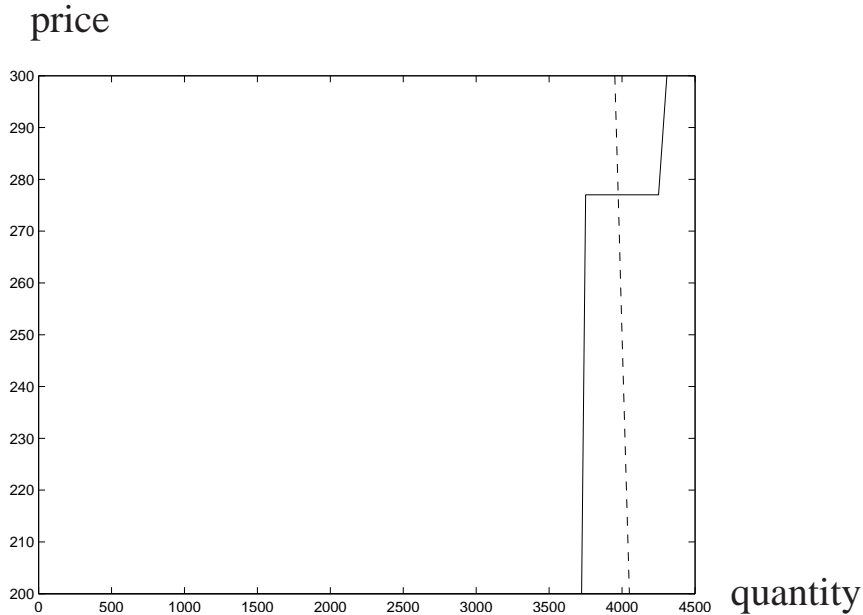


Fig. 7.22. Detail of industry supply in interval 3, sub-interval 1.

Interval 3, sub-interval 1 profits per MWh sold from March 21

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	250	257	60
1	250	250	257	60
1	250	250	257	60
2	250	111.55	257	60
2	250	111.55	257	60
2	250	0	257	60
3	150	150	227	30
3	150	150	227	30
3	150	150	227	30
4	150	150	227	30
4	150	150	227	30
4	150	150	227	30
5	50	50	197	0
5	50	50	197	0
5	50	50	197	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	50	197	0
6	50	50	197	0
6	50	50	197	0
7	250	250	257	60
7	150	150	227	30
7	50	50	197	0
8	250	250	257	60
8	150	150	227	30
8	50	50	197	0
9	250	250	257	60
9	150	150	227	30
9	50	50	197	0
10	250	250	257	60
10	150	150	227	30
10	50	50	197	0
Demand	4150	3973	0	420

Discussion of interval 3, sub-interval 1 from March 21

- Results closer to competitive than last week!
- Group 2 offered all of its capacity at \$277/MWh or above, which set the price in all intervals:
 - dispatch is inefficient, since not all baseload is dispatched and peaking is at full capacity.
- Not all demand served:
 - price responsive demand partially served.
- Note that stated benefit for demand of zero is *marginal* benefit:
 - average benefit of served demand is closer to $(500 - 277) = \$223/\text{MWh}$.

Interval 3, sub-interval 2 supply from March 21

- Clearing price \$277/MWh compared to \$500/MWh last week and \$80/MWh competitive price.

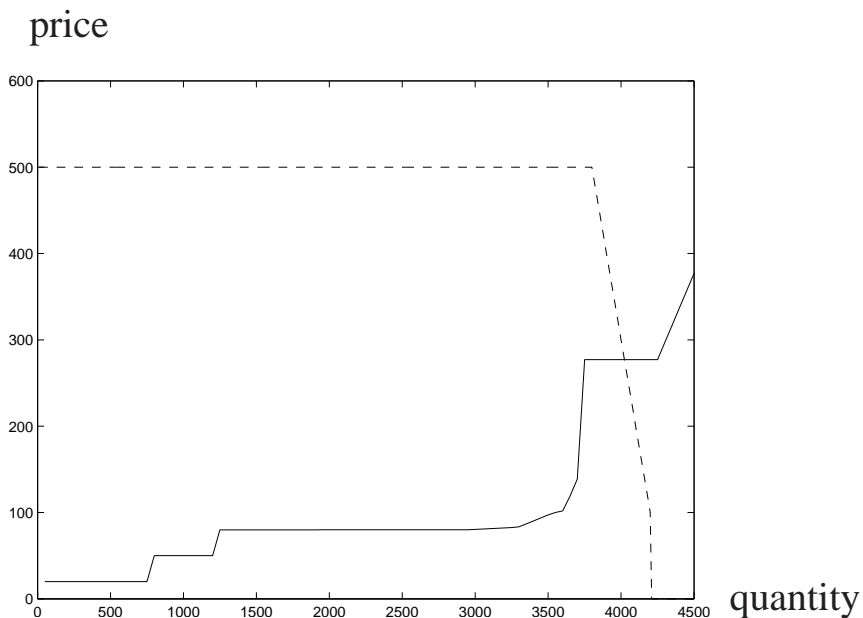


Fig. 7.23. Industry supply in interval 3, sub-interval 2.

Interval 3, sub-interval 2 supply detail from March 21

- Clearing price \$277/MWh compared to \$500/MWh last week and \$80/MWh competitive price.

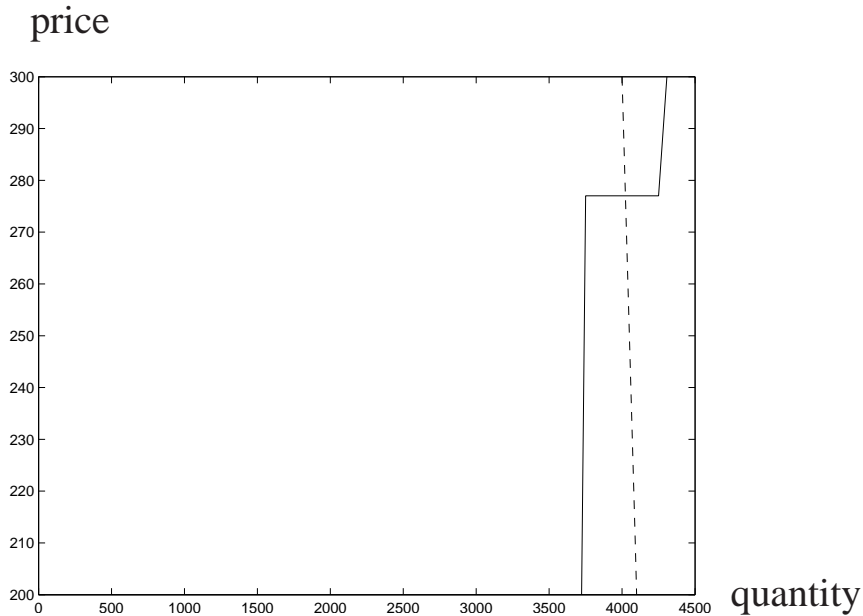


Fig. 7.24. Detail of industry supply in interval 3, sub-interval 2.

Interval 3, sub-interval 2 profits per MWh sold from March 21

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	250	257	60
1	250	250	257	60
1	250	250	257	60
2	250	136.55	257	60
2	250	136.55	257	60
2	250	0	257	60
3	150	150	227	30
3	150	150	227	30
3	150	150	227	30
4	150	150	227	30
4	150	150	227	30
4	150	150	227	30
5	50	50	197	0
5	50	50	197	0
5	50	50	197	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	50	197	0
6	50	50	197	0
6	50	50	197	0
7	250	250	257	60
7	150	150	227	30
7	50	50	197	0
8	250	250	257	60
8	150	150	227	30
8	50	50	197	0
9	250	250	257	60
9	150	150	227	30
9	50	50	197	0
10	250	250	257	60
10	150	150	227	30
10	50	50	197	0
Demand	4200	4023	0	420

Discussion of interval 3, sub-interval 2 from March 21

- Similar to sub-interval 1.
- Dispatch is inefficient since not all of the 2500 MW of baseload is dispatched.
- Not all demand served.

Interval 3, sub-interval 3 supply from March 21

- Clearing price \$277/MWh compared to \$500/MWh last week and \$80/MWh competitive price.

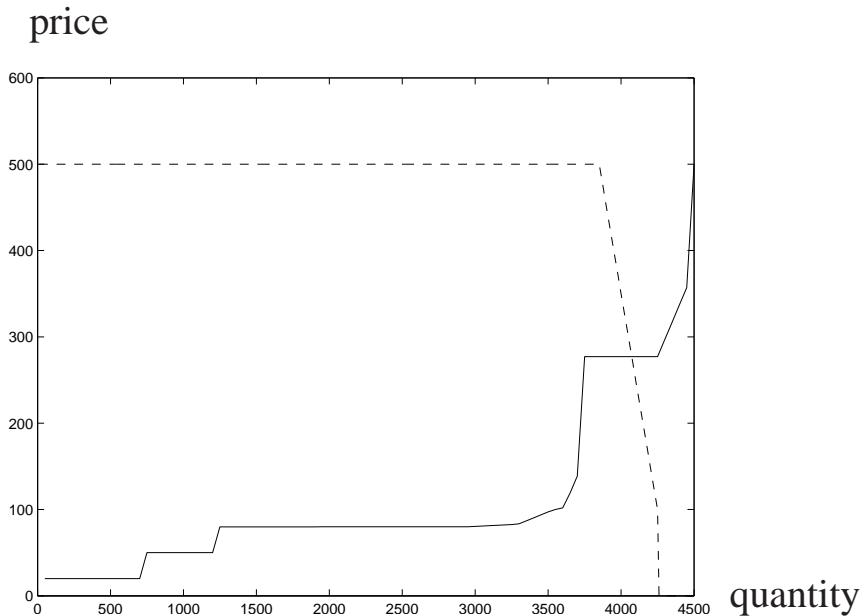


Fig. 7.25. Industry supply in interval 3, sub-interval 3.

Interval 3, sub-interval 3 supply detail from March 21

- Clearing price \$277/MWh compared to \$500/MWh last week and \$80/MWh competitive price.

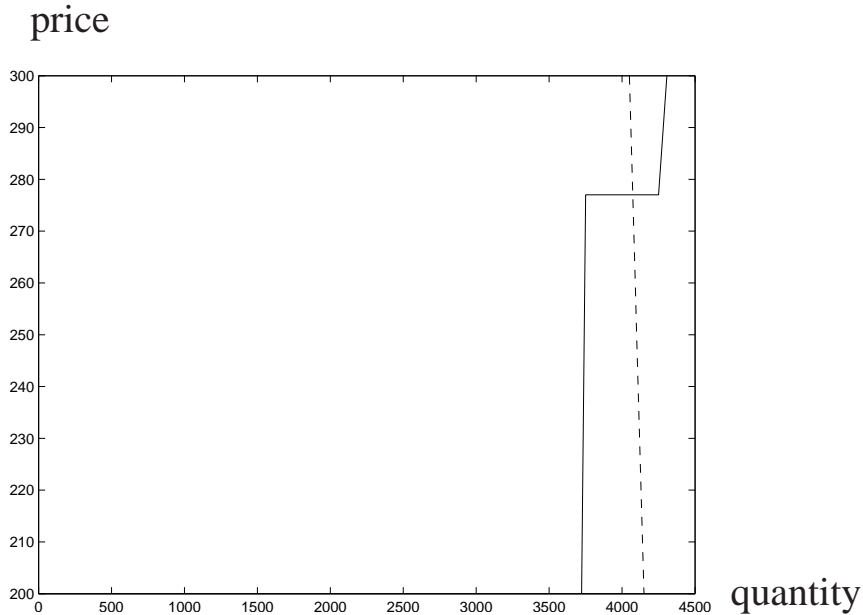


Fig. 7.26. Detail of industry supply in interval 3, sub-interval 3.

Interval 3, sub-interval 3 profits per MWh sold from March 21

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	250	257	60
1	250	250	257	60
1	250	250	257	60
2	250	161.55	257	60
2	250	161.55	257	60
2	250	0	257	60
3	150	150	227	30
3	150	150	227	30
3	150	150	227	30
4	150	150	227	30
4	150	150	227	30
4	150	150	227	30
5	50	50	197	0
5	50	50	197	0
5	50	50	197	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	50	197	0
6	50	50	197	0
6	50	50	197	0
7	250	250	257	60
7	150	150	227	30
7	50	50	197	0
8	250	250	257	60
8	150	150	227	30
8	50	50	197	0
9	250	250	257	60
9	150	150	227	30
9	50	50	197	0
10	250	250	257	60
10	150	150	227	30
10	50	50	197	0
Demand	4250	4073	0	420

Discussion of interval 3, sub-interval from March 21

- Similar to sub-intervals 1 and 2.
- Dispatch is inefficient since not all of the 2500 MW of baseload is dispatched.
- Not all demand served.

Homework exercise: Discussion

Interval 3, sub-interval 1 supply from March 30

- Clearing price \$277/MWh compared to \$277/MWh last week and \$80/MWh competitive price.

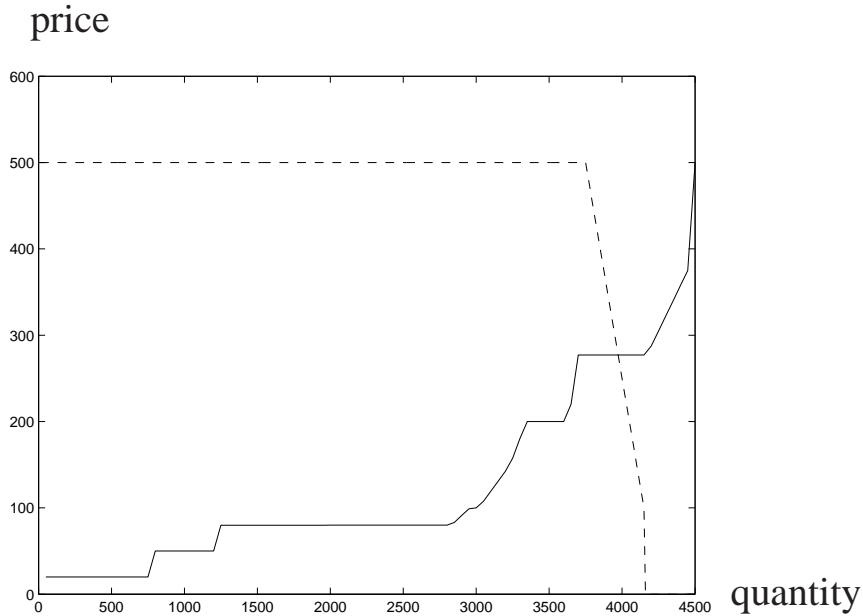


Fig. 7.27. Industry supply in interval 3, sub-interval 1.

Interval 3, sub-interval 1 supply detail from March 30

- Clearing price \$277/MWh compared to \$277/MWh last week and \$80/MWh competitive price.

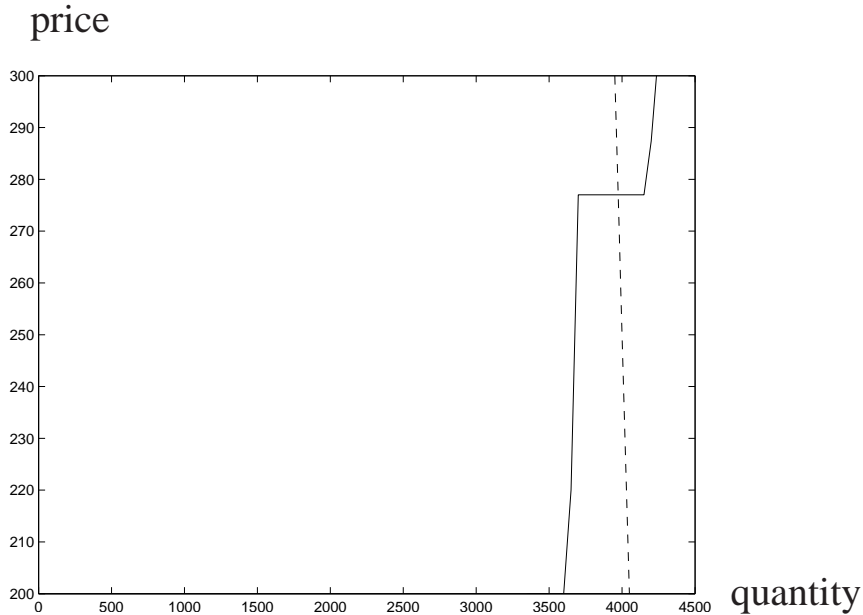


Fig. 7.28. Detail of industry supply in interval 3, sub-interval 1.

Interval 3, sub-interval 1 profits per MWh sold from March 30

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	250	257	60
1	250	250	257	60
1	250	250	257	60
2	250	151.37	257	60
2	250	151.37	257	60
2	250	0	257	60
3	150	150	227	30
3	150	150	227	30
3	150	150	227	30
4	150	150	227	30
4	150	150	227	30
4	150	150	227	30
5	50	50	197	0
5	50	50	197	0
5	50	50	197	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	23.45	197	0
6	50	23.45	197	0
6	50	23.45	197	0
7	250	250	257	60
7	150	150	227	30
7	50	50	197	0
8	250	250	257	60
8	150	150	227	30
8	50	50	197	0
9	250	250	257	60
9	150	150	227	30
9	50	50	197	0
10	250	250	257	60
10	150	150	227	30
10	50	50	197	0
Demand	4150	3973.10	223	420

Discussion of interval 3, sub-interval 1 from March 30

- Results almost same as last week!
- Group 2 again offered all of its capacity at \$277/MWh or above, which set the price in all intervals:
 - dispatch is inefficient, since not all baseload is dispatched and peaking is at full capacity.
- Not all demand served:
 - price responsive demand partially served.
- Note that stated benefit for demand of zero is *marginal* benefit:
 - average benefit of served demand is closer to $(500 - 277) = \$223/\text{MWh}$.

Interval 3, sub-interval 2 supply from March 30

- Clearing price \$277/MWh compared to \$277/MWh last week and \$80/MWh competitive price.

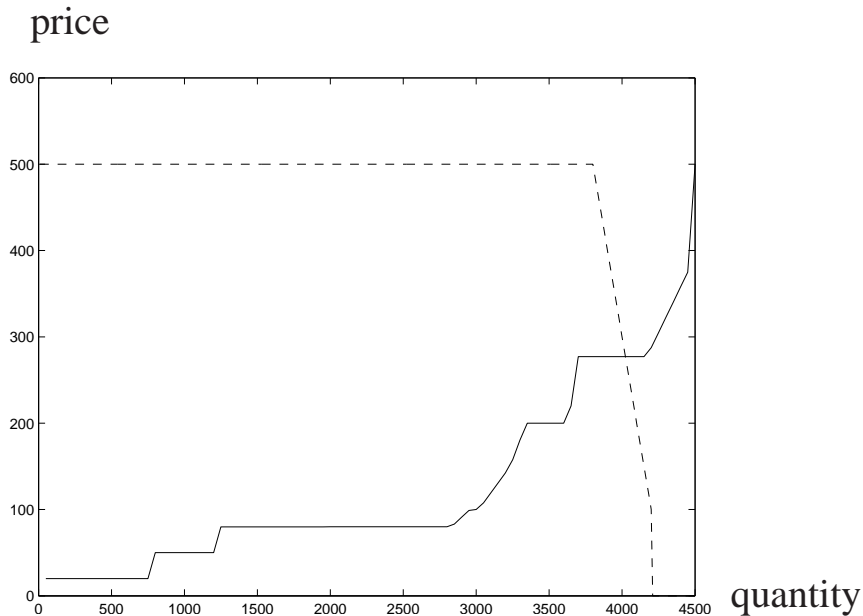


Fig. 7.29. Industry supply in interval 3, sub-interval 2.

Interval 3, sub-interval 2 supply detail from March 30

- Clearing price \$277/MWh compared to \$277/MWh last week and \$80/MWh competitive price.

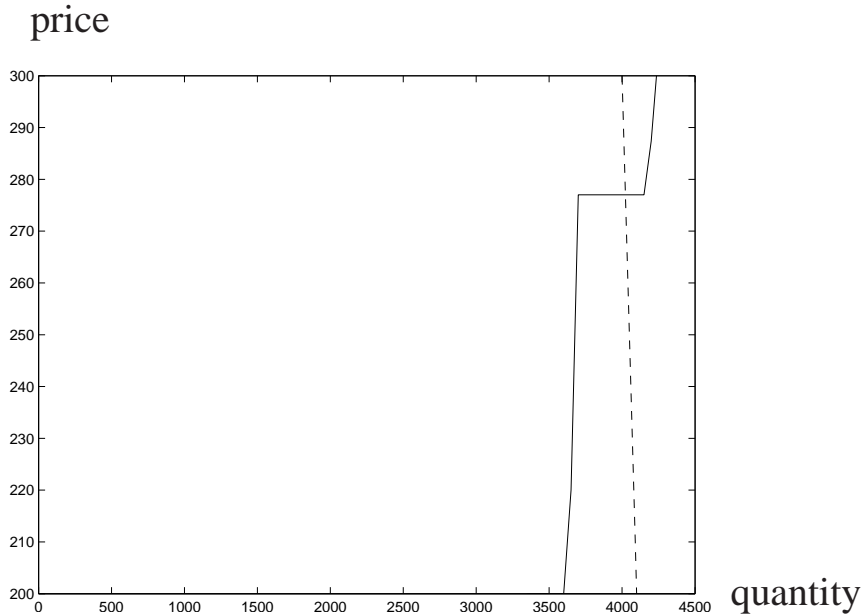


Fig. 7.30. Detail of industry supply in interval 3, sub-interval 2.

Interval 3, sub-interval 2 profits per MWh sold from March 30

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	250	257	60
1	250	250	257	60
1	250	250	257	60
2	250	176.37	257	60
2	250	176.37	257	60
2	250	0	257	60
3	150	150	227	30
3	150	150	227	30
3	150	150	227	30
4	150	150	227	30
4	150	150	227	30
4	150	150	227	30
5	50	50	197	0
5	50	50	197	0
5	50	50	197	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	23.45	197	0
6	50	23.45	197	0
6	50	23.45	197	0
7	250	250	257	60
7	150	150	227	30
7	50	50	197	0
8	250	250	257	60
8	150	150	227	30
8	50	50	197	0
9	250	250	257	60
9	150	150	227	30
9	50	50	197	0
10	250	250	257	60
10	150	150	227	30
10	50	50	197	0
Demand	4200	4023.10	223	420

Discussion of interval 3, sub-interval 2 from March 30

- Similar to sub-interval 1.
- Dispatch is inefficient since not all of the 2500 MW of baseload is dispatched.
- Not all demand served.

Interval 3, sub-interval 3 supply from March 30

- Clearing price \$277/MWh compared to \$277/MWh last week and \$80/MWh competitive price.

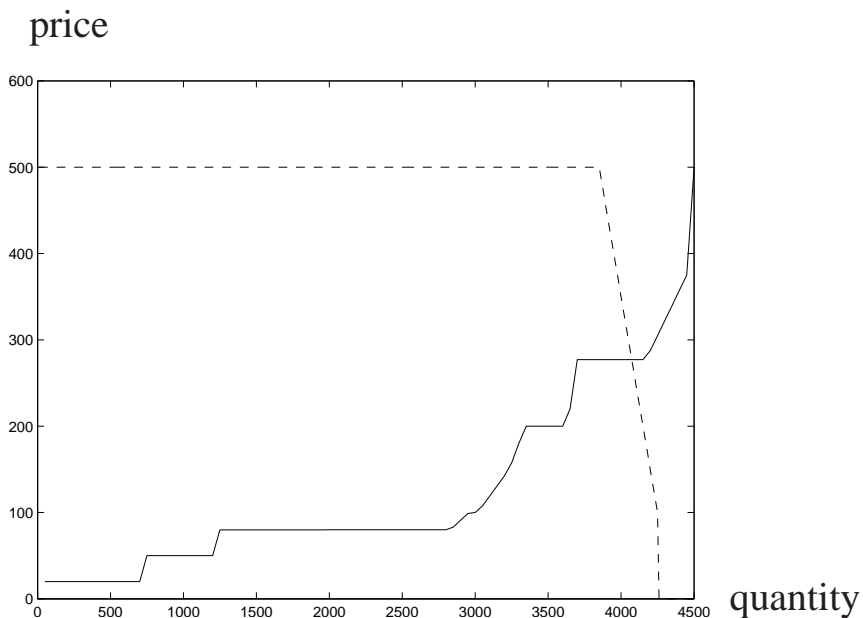


Fig. 7.31. Industry supply in interval 3, sub-interval 3.

Interval 3, sub-interval 3 supply detail from March 30

- Clearing price \$277/MWh compared to \$277/MWh last week and \$80/MWh competitive price.

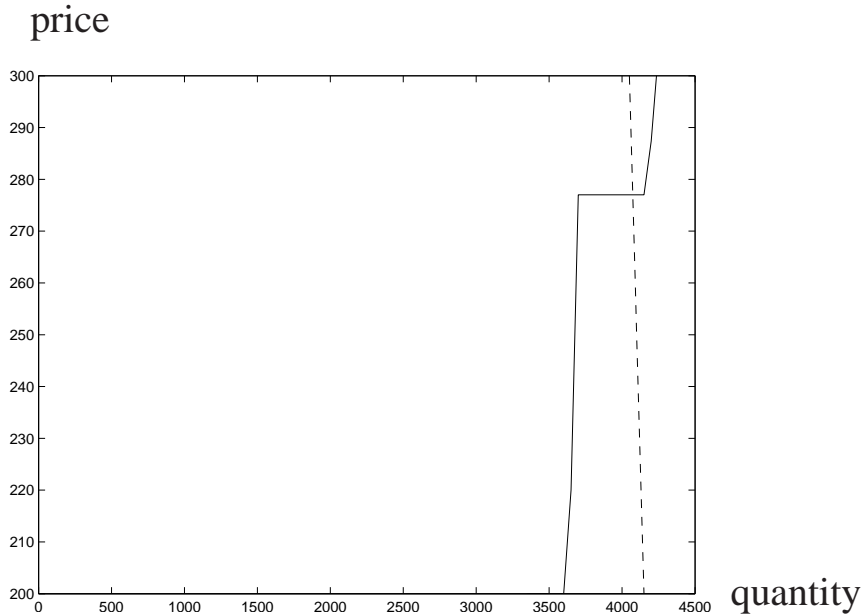


Fig. 7.32. Detail of industry supply in interval 3, sub-interval 3.

Interval 3, sub-interval 3 profits per MWh sold from March 30

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	250	257	60
1	250	250	257	60
1	250	250	257	60
2	250	201.37	257	60
2	250	201.37	257	60
2	250	0	257	60
3	150	150	227	30
3	150	150	227	30
3	150	150	227	30
4	150	150	227	30
4	150	150	227	30
4	150	150	227	30
5	50	50	197	0
5	50	50	197	0
5	50	50	197	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	23.45	197	0
6	50	23.45	197	0
6	50	23.45	197	0
7	250	250	257	60
7	150	150	227	30
7	50	50	197	0
8	250	250	257	60
8	150	150	227	30
8	50	50	197	0
9	250	250	257	60
9	150	150	227	30
9	50	50	197	0
10	250	250	257	60
10	150	150	227	30
10	50	50	197	0
Demand	4250	4073.10	223	420

Discussion of interval 3, sub-interval from March 30

- Similar to sub-intervals 1 and 2.
- Dispatch is inefficient since not all of the 2500 MW of baseload is dispatched.
- Not all demand served.

Homework exercise, March 30, solution

- As previously, we have a “symmetric duopoly” with each firm $i = 1, 2$ having marginal cost function:

$$\forall Q_i, c'_i(Q_i) = 20 + 60Q_i/2500.$$

- Operating range $[0, \bar{Q}_i]$, where $\bar{Q}_i = 2500$ MW.
- The inverse demand functions are as follows:
 - (i) $\forall Q, p^d(Q) = \max\{50 - (Q - 2800)/20, 0\}$,
 - (ii) $\forall Q, p^d(Q) = \max\{75 - (Q - 3500)/20, 0\}$,
 - (iii) $\forall Q, p^d(Q) = \max\{500 - (Q - 4200)/20, 0\}$,
- where Q is in MW and $p^d(Q)$ is in \$/MWh.
- For each inverse demand function, we again apply the Cournot model to find the predicted prices and quantities.

Homework exercise, March 30, solution, interval (i)

- Inverse demand is:

$$\begin{aligned}\forall Q, p^d(Q) &= \max\{50 - (Q - 2800)/20, 0\}, \\ &= \max\{190 - Q/20, 0\}, \\ &= 190 - Q/20,\end{aligned}$$

- assuming that $190 - Q/20 \geq 0$.
- For firm $i = 1$, we have that the profit is:

$$\begin{aligned}\pi_1(Q_1, Q_2) &= (190 - (Q_1 + Q_2)/20)Q_1 - c_i(Q_1), \\ &= -\frac{1}{20}Q_1^2 + \left(190 - \frac{1}{20}Q_2\right)Q_1 - c_i(Q_1). \\ 0 &= \frac{\partial \pi_1}{\partial Q_1}(Q_1, Q_2), \\ &= -\frac{1}{10}Q_1 + 190 - \frac{1}{20}Q_2 - c'_i(Q_1), \\ &= -\frac{1}{10}Q_1 - \frac{1}{20}Q_2 + 190 - \left(20 + \frac{60}{2500}Q_1\right).\end{aligned}$$

Homework exercise, March 30, solution, interval (i)

- That is:

$$0.124Q_1 + 0.05Q_2 = 170. \quad (7.7)$$

- Similarly, for firm $i = 2$, we have that:

$$0.05Q_1 + 0.124Q_2 = 170. \quad (7.8)$$

- Solving the simultaneous equations (7.7) and (7.8), we obtain:

$$\begin{aligned} Q_1^* &= 977\text{MW}, \\ Q_2^* &= 977\text{MW}, \\ Q_1^* + Q_2^* &= 1954\text{MW}, \\ p^d(Q_1^* + Q_2^*) &= 92.3\$/\text{MWh}, \\ c'_i(Q_1^*) = c'_i(Q_2^*) &= 43.4\$/\text{MWh}. \end{aligned}$$

- Note that $190 - (Q_1^* + Q_2^*)/20 \geq 0$ so that price is greater than zero.
- Price is much lower than from solution to similar previous homework, reflecting mitigation of market power due to demand price elasticity.
- Generation is higher than previous solution.

Homework exercise, March 30, solution, interval (ii)

- Inverse demand is:

$$\begin{aligned}\forall Q, p^d(Q) &= \max\{75 - (Q - 3500)/20, 0\}, \\ &= \max\{250 - Q/20, 0\}, \\ &= 250 - Q/20,\end{aligned}$$

- assuming that $250 - Q/20 \geq 0$.
- For firm $i = 1$, we have that the profit is:

$$\begin{aligned}\pi_1(Q_1, Q_2) &= (250 - (Q_1 + Q_2)/20)Q_1 - c_i(Q_1), \\ &= -\frac{1}{20}Q_1^2 + \left(250 - \frac{1}{20}Q_2\right)Q_1 - c_i(Q_1). \\ 0 &= \frac{\partial \pi_1}{\partial Q_1}(Q_1, Q_2), \\ &= -\frac{1}{10}Q_1 + 250 - \frac{1}{20}Q_2 - c'_i(Q_1), \\ &= -\frac{1}{10}Q_1 - \frac{1}{20}Q_2 + 250 - \left(20 + \frac{60}{2500}Q_1\right).\end{aligned}$$

Homework exercise, March 30, solution, interval (ii)

- That is:

$$0.124Q_1 + 0.05Q_2 = 230. \quad (7.9)$$

- Similarly, for firm $i = 2$, we have:

$$0.05Q_1 + 0.124Q_2 = 230. \quad (7.10)$$

- Solving the simultaneous equations (7.9) and (7.10), we obtain:

$$\begin{aligned} Q_1^* &= 1321.8\text{MW}, \\ Q_2^* &= 1321.8\text{MW}, \\ Q_1^* + Q_2^* &= 2643.7\text{MW}, \\ p^d(Q_1^* + Q_2^*) &= 117.8\$/\text{MWh}, \\ c_i'(Q_1^*) = c_i'(Q_2^*) &= 51.7\$/\text{MWh}. \end{aligned}$$

- Note that $250 - (Q_1^* + Q_2^*)/20 \geq 0$ so that price is greater than zero.
- Price is considerably higher than marginal cost, but *much* lower than in previous solution.
- Generation is again higher than previous solution.

Homework exercise, March 30, solution, interval (iii)

- Inverse demand is:

$$\begin{aligned}\forall Q, p^d(Q) &= \max\{500 - (Q - 4200)/20, 0\}, \\ &= \max\{710 - Q/20, 0\}, \\ &= 710 - Q/20,\end{aligned}$$

- assuming that $710 - Q/20 \geq 0$.
- For firm $i = 1$, we have that the profit is:

$$\begin{aligned}\pi_1(Q_1, Q_2) &= (710 - (Q_1 + Q_2)/20)Q_1 - c_i(Q_1), \\ &= -\frac{1}{20}Q_1^2 + \left(710 - \frac{1}{20}Q_2\right)Q_1 - c_i(Q_1). \\ 0 &= \frac{\partial \pi_1}{\partial Q_1}(Q_1, Q_2), \\ &= -\frac{1}{10}Q_1 + 710 - \frac{1}{20}Q_2 - c'_i(Q_1), \\ &= -\frac{1}{10}Q_1 - \frac{1}{20}Q_2 + 710 - \left(20 + \frac{60}{2500}Q_1\right).\end{aligned}$$

Homework exercise, March 30, solution, interval (iii)

- That is:

$$0.124Q_1 + 0.05Q_2 = 690. \quad (7.11)$$

- Similarly, for firm $i = 2$, we have:

$$0.05Q_1 + 0.124Q_2 = 690. \quad (7.12)$$

- Solving the simultaneous equations (7.11) and (7.12), we obtain $Q_1 = Q_2 = 3965.5$ MW; however, this violates the capacity constraint of the generators.
- Therefore:

$$\begin{aligned} Q_1^* &= 2500\text{MW}, \\ Q_2^* &= 2500\text{MW}, \\ Q_1^* + Q_2^* &= 5000\text{MW}, \\ p^d(Q_1^* + Q_2^*) &= 460\$/\text{MWh}, \\ c'_i(Q_1^*) = c'_i(Q_2^*) &= 80\$/\text{MWh}. \end{aligned}$$

- Note that $710 - (Q_1^* + Q_2^*)/20 \geq 0$ so that price is greater than zero.

Homework exercise, March 30, solution, interval (iii)

- Price is considerably higher than marginal cost, but reflects scarcity since generators are operating at capacity and price is being determined by demand bid.
- Generation is again higher than previous solution.
- Increased demand response to price has significantly mitigated market power!

Homework exercise: Discussion

Interval 3, sub-interval 1 supply from April 6

- Clearing price \$260/MWh compared to \$277/MWh last week and \$80/MWh competitive price.

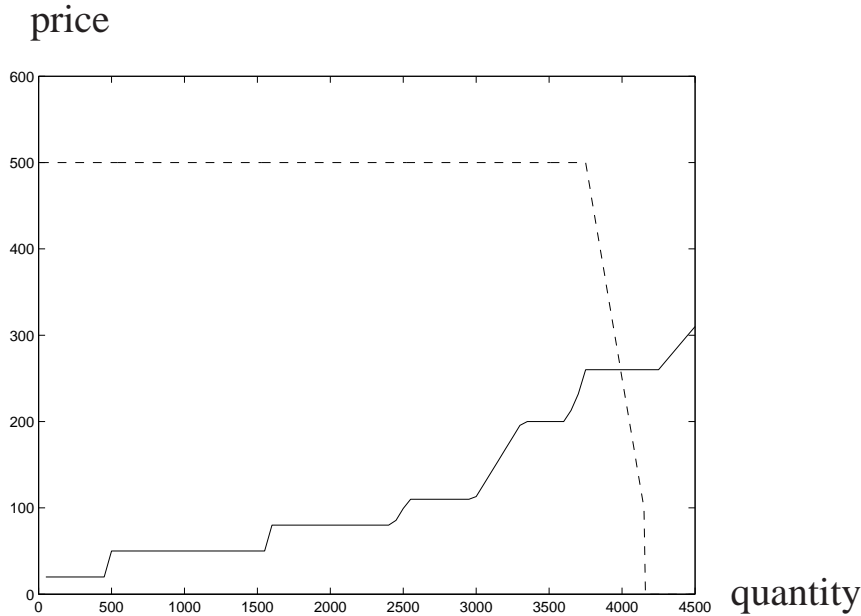


Fig. 7.33. Industry supply in interval 3, sub-interval 1.

Interval 3, sub-interval 1 supply detail from April 6

- Clearing price \$260/MWh compared to \$277/MWh last week and \$80/MWh competitive price.

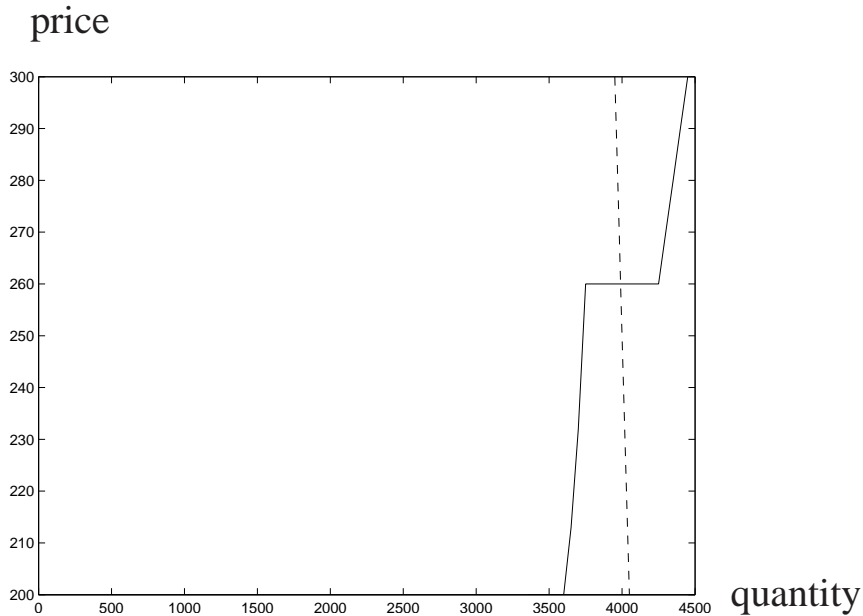


Fig. 7.34. Detail of industry supply in interval 3, sub-interval 1.

Interval 3, sub-interval 1 profits per MWh sold from April 6

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	250	240	60
1	250	250	240	60
1	250	250	240	60
2	250	120.05	240	60
2	250	120.05	240	60
2	250	0	240	60
3	150	150	210	30
3	150	150	210	30
3	150	150	210	30
4	150	150	210	30
4	150	150	210	30
4	150	150	210	30
5	50	50	180	0
5	50	50	180	0
5	50	50	180	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	50	180	0
6	50	50	180	0
6	50	50	180	0
7	250	250	240	60
7	150	150	210	30
7	50	50	180	0
8	250	250	240	60
8	150	150	210	30
8	50	50	180	0
9	250	250	240	60
9	150	150	210	30
9	50	50	180	0
10	250	250	240	60
10	150	150	210	30
10	50	50	180	0
Demand	4150	3990.10	240	420

Discussion of interval 3, sub-interval 1 from April 6

- Somewhat lower price and higher demand than last week!
- Group 2 again offered all of its capacity at \$260/MWh or above, which set the price in all intervals:
 - dispatch is inefficient, since not all baseload is dispatched and peaking is at full capacity.
- Not all demand served:
 - price responsive demand partially served.
- Note that stated benefit for demand of zero is *marginal* benefit.

Interval 3, sub-interval 2 supply from April 6

- Clearing price \$285/MWh compared to \$277/MWh last week and \$80/MWh competitive price.

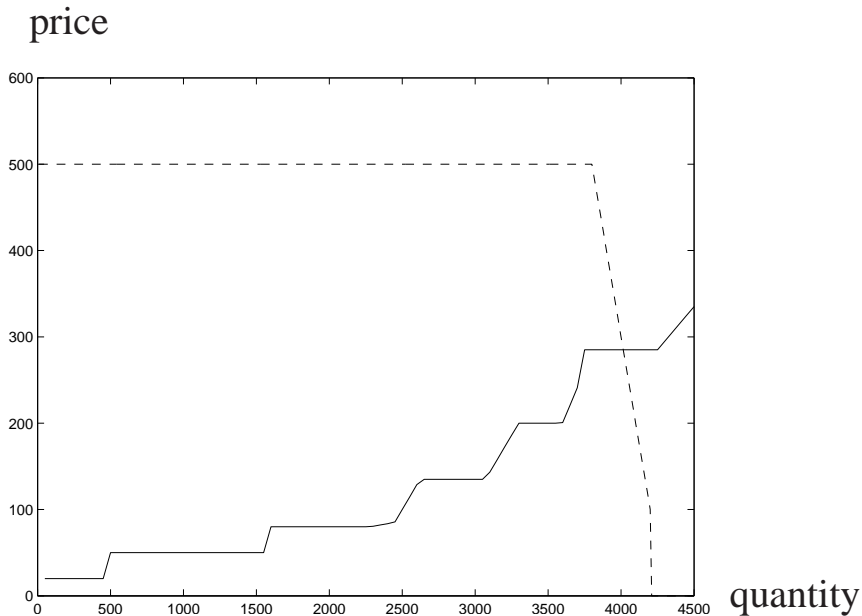


Fig. 7.35. Industry supply in interval 3, sub-interval 2.

Interval 3, sub-interval 2 supply detail from April 6

- Clearing price \$285/MWh compared to \$277/MWh last week and \$80/MWh competitive price.

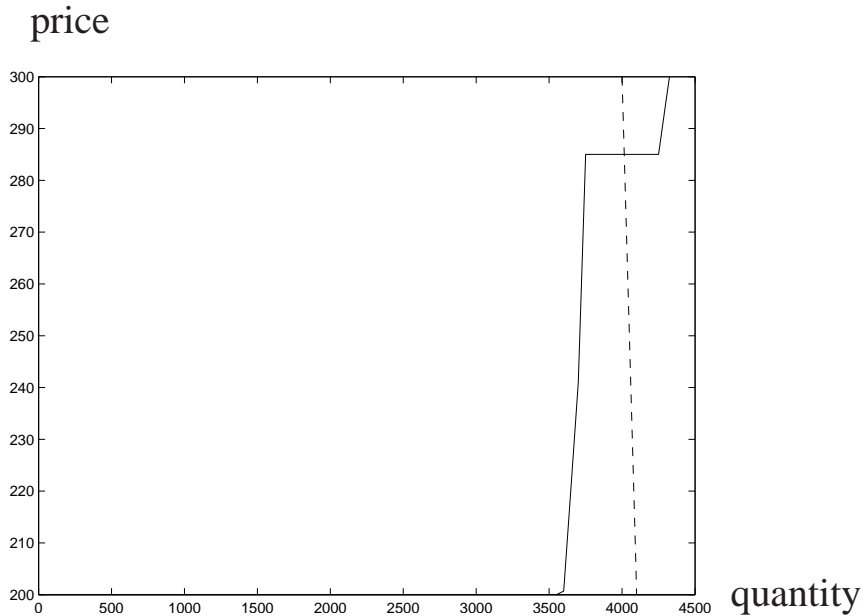


Fig. 7.36. Detail of industry supply in interval 3, sub-interval 2.

Interval 3, sub-interval 2 profits per MWh sold from April 6

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	250	265	60
1	250	250	265	60
1	250	250	265	60
2	250	132.55	265	60
2	250	132.55	265	60
2	250	0	265	60
3	150	150	235	30
3	150	150	235	30
3	150	150	235	30
4	150	150	235	30
4	150	150	235	30
4	150	150	235	30
5	50	50	205	0
5	50	50	205	0
5	50	50	205	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	50	205	0
6	50	50	205	0
6	50	50	205	0
7	250	250	265	60
7	150	150	235	30
7	50	50	205	0
8	250	250	265	60
8	150	150	235	30
8	50	50	205	0
9	250	250	265	60
9	150	150	235	30
9	50	50	205	0
10	250	250	265	60
10	150	150	235	30
10	50	50	205	0
Demand	4200	4015.10	215	420

Discussion of interval 3, sub-interval 2 from April 6

- Somewhat higher price and lower demand than last week!
- Similar to sub-interval 1, but in this case group 2 offered all of its capacity at \$285/MWh or above, which set the price in all intervals.
- Dispatch is inefficient since not all of the 2500 MW of baseload is dispatched.
- Not all demand served.

Interval 3, sub-interval 3 supply from April 6

- Clearing price \$310/MWh compared to \$277/MWh last week and \$80/MWh competitive price.

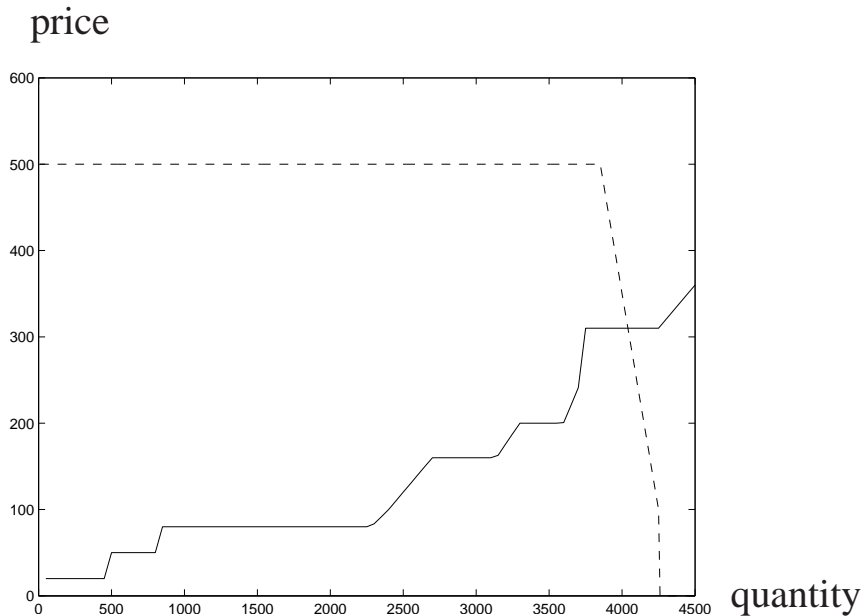


Fig. 7.37. Industry supply in interval 3, sub-interval 3.

Interval 3, sub-interval 3 supply detail from April 6

- Clearing price \$310/MWh compared to \$277/MWh last week and \$80/MWh competitive price.

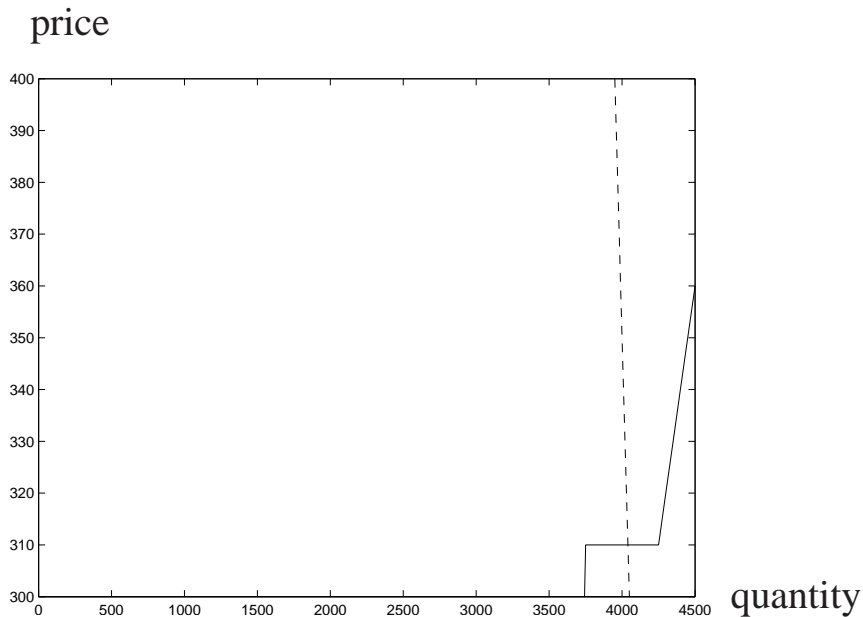


Fig. 7.38. Detail of industry supply in interval 3, sub-interval 3.

Interval 3, sub-interval 3 profits per MWh sold from April 6

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	250	290	60
1	250	250	290	60
1	250	250	290	60
2	250	145.05	290	60
2	250	145.05	290	60
2	250	0	290	60
3	150	150	260	30
3	150	150	260	30
3	150	150	260	30
4	150	150	260	30
4	150	150	260	30
4	150	150	260	30
5	50	50	230	0
5	50	50	230	0
5	50	50	230	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	50	230	0
6	50	50	230	0
6	50	50	230	0
7	250	250	290	60
7	150	150	260	30
7	50	50	230	0
8	250	250	290	60
8	150	150	260	30
8	50	50	230	0
9	250	250	290	60
9	150	150	260	30
9	50	50	230	0
10	250	250	290	60
10	150	150	260	30
10	50	50	230	0
Demand	4250	4040.10	190	420

Discussion of interval 3, sub-interval from April 6

- Even higher price and lower demand than last week!
- Similar to sub-interval 1, but in this case group 2 offered all of its capacity at \$310/MWh or above, which set the price in all intervals.
- Dispatch is inefficient since not all of the 2500 MW of baseload is dispatched.
- Not all demand served.

Homework exercise: Discussion

Interval 3, sub-interval 1 supply from April 13

- Clearing price \$260/MWh compared to \$260/MWh last week and \$80/MWh competitive price.

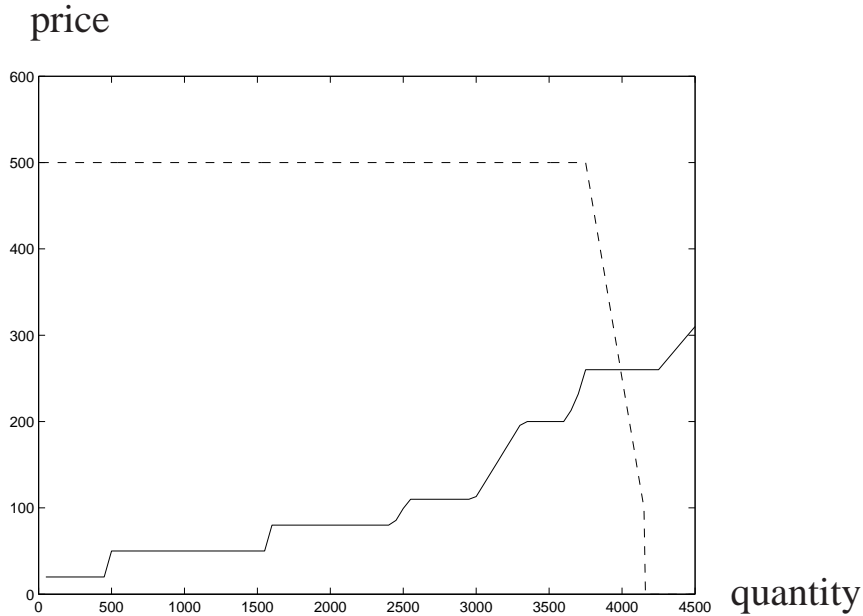


Fig. 7.39. Industry supply in interval 3, sub-interval 1.

Interval 3, sub-interval 1 supply detail from April 13

- Clearing price \$260/MWh compared to \$260/MWh last week and \$80/MWh competitive price.

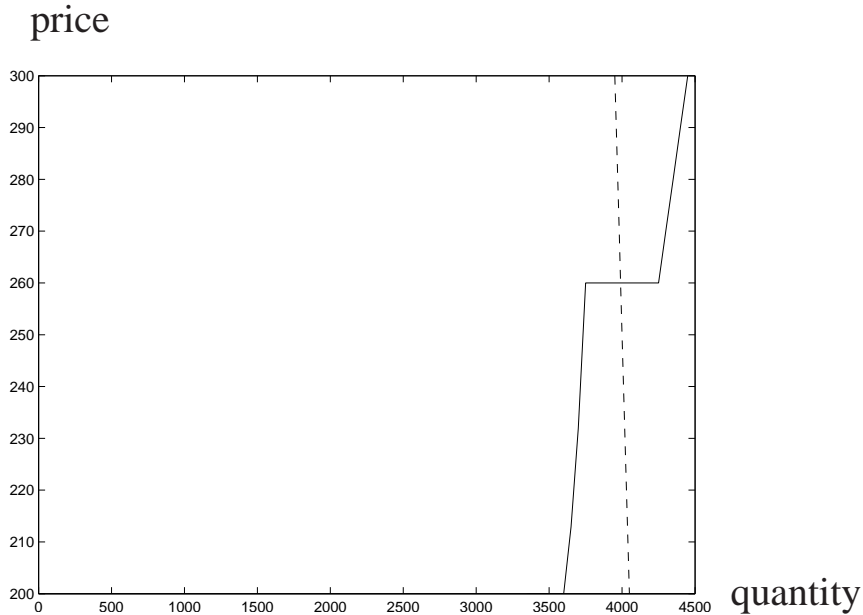


Fig. 7.40. Detail of industry supply in interval 3, sub-interval 1.

Interval 3, sub-interval 1 profits per MWh sold from April 13

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	250	240	60
1	250	250	240	60
1	250	250	240	60
2	250	120.05	240	60
2	250	120.05	240	60
2	250	0	240	60
3	150	150	210	30
3	150	150	210	30
3	150	150	210	30
4	150	150	210	30
4	150	150	210	30
4	150	150	210	30
5	50	50	180	0
5	50	50	180	0
5	50	50	180	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	50	180	0
6	50	50	180	0
6	50	50	180	0
7	250	250	240	60
7	150	150	210	30
7	50	50	180	0
8	250	250	240	60
8	150	150	210	30
8	50	50	180	0
9	250	250	240	60
9	150	150	210	30
9	50	50	180	0
10	250	250	240	60
10	150	150	210	30
10	50	50	180	0
Demand	4150	3990.10	240	420

Discussion of interval 3, sub-interval 1 from April 13

- Same as last week!
- Group 2 again offered all of its capacity at \$260/MWh or above, which set the price in all intervals:
 - dispatch is inefficient, since not all baseload is dispatched and peaking is at full capacity.
- Not all demand served:
 - price responsive demand partially served.
- Note that stated benefit for demand of zero is *marginal* benefit.

Interval 3, sub-interval 2 supply from April 13

- Clearing price \$285/MWh compared to \$285/MWh last week and \$80/MWh competitive price.

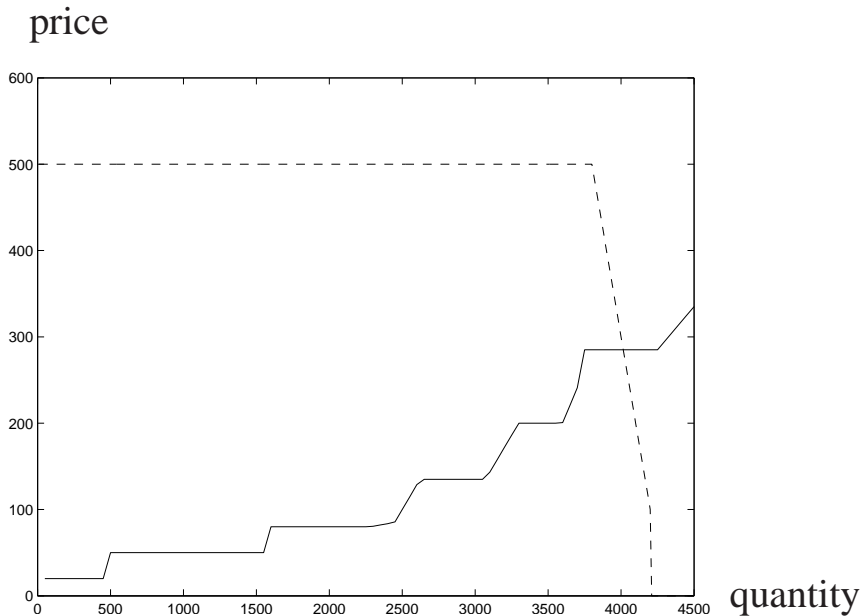


Fig. 7.41. Industry supply in interval 3, sub-interval 2.

Interval 3, sub-interval 2 supply detail from April 13

- Clearing price \$285/MWh compared to \$285/MWh last week and \$80/MWh competitive price.

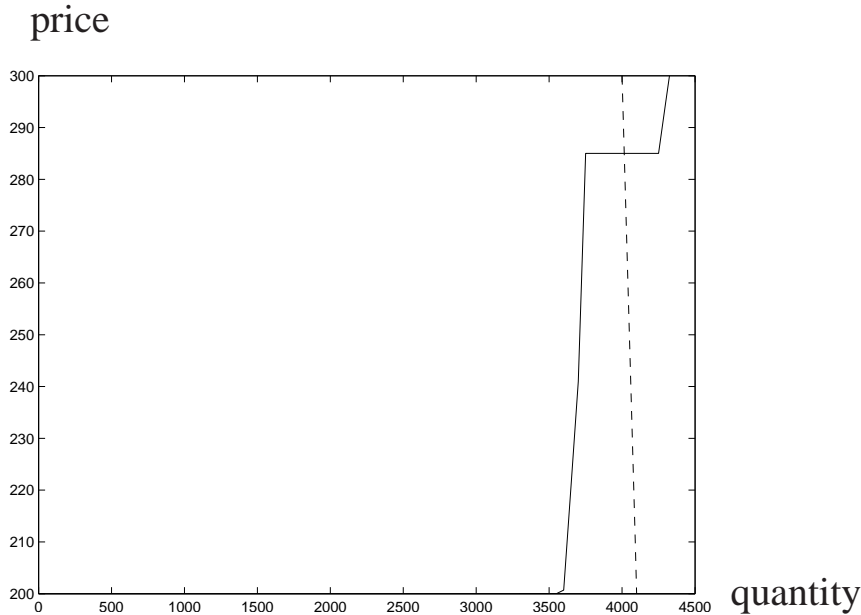


Fig. 7.42. Detail of industry supply in interval 3, sub-interval 2.

Interval 3, sub-interval 2 profits per MWh sold from April 13

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	250	265	60
1	250	250	265	60
1	250	250	265	60
2	250	132.55	265	60
2	250	132.55	265	60
2	250	0	265	60
3	150	150	235	30
3	150	150	235	30
3	150	150	235	30
4	150	150	235	30
4	150	150	235	30
4	150	150	235	30
5	50	50	205	0
5	50	50	205	0
5	50	50	205	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	50	205	0
6	50	50	205	0
6	50	50	205	0
7	250	250	265	60
7	150	150	235	30
7	50	50	205	0
8	250	250	265	60
8	150	150	235	30
8	50	50	205	0
9	250	250	265	60
9	150	150	235	30
9	50	50	205	0
10	250	250	265	60
10	150	150	235	30
10	50	50	205	0
Demand	4200	4015.10	215	420

Discussion of interval 3, sub-interval 2 from April 13

- Same last week!
- Group 2 offered all of its capacity at \$285/MWh or above, which set the price in all intervals.
- Dispatch is inefficient since not all of the 2500 MW of baseload is dispatched.
- Not all demand served.

Interval 3, sub-interval 3 supply from April 13

- Clearing price \$310/MWh compared to \$310/MWh last week and \$80/MWh competitive price.

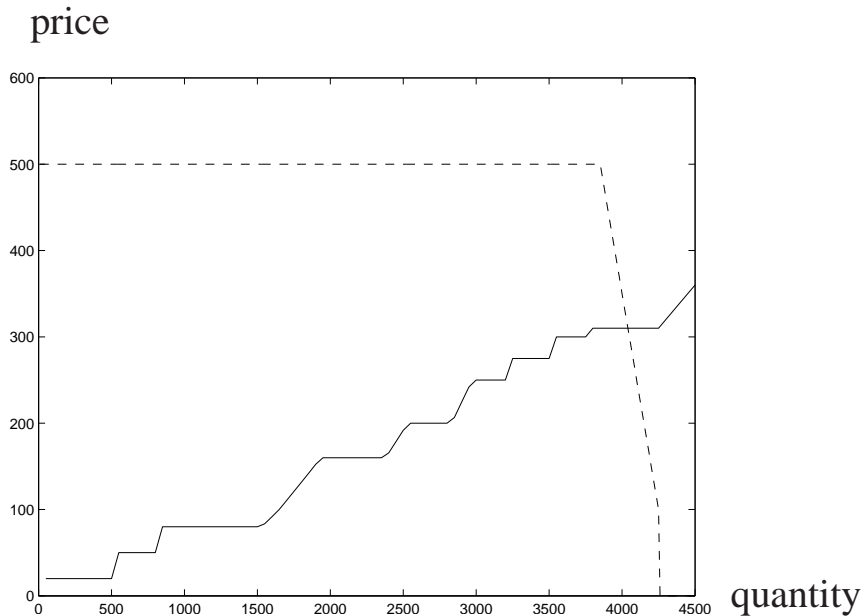


Fig. 7.43. Industry supply in interval 3, sub-interval 3.

Interval 3, sub-interval 3 supply detail from April 13

- Clearing price \$310/MWh compared to \$310/MWh last week and \$80/MWh competitive price.

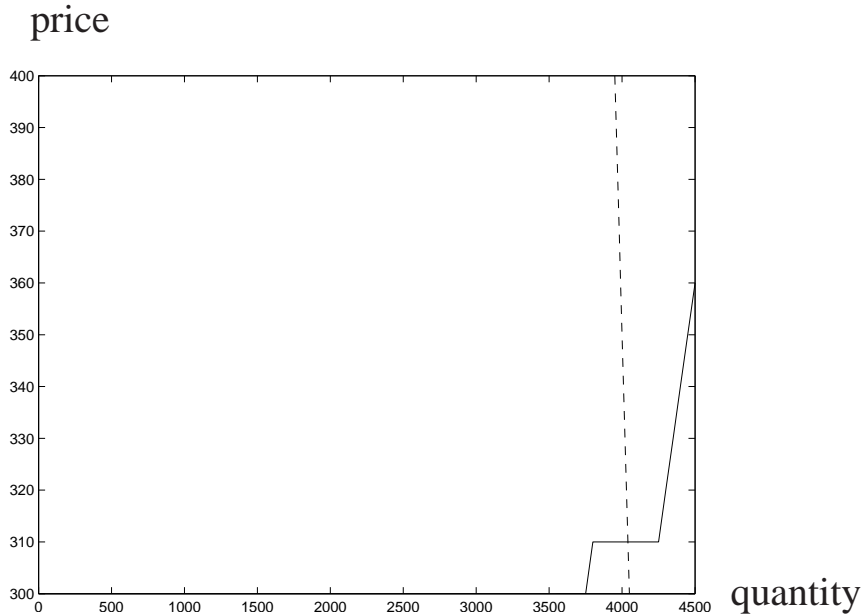


Fig. 7.44. Detail of industry supply in interval 3, sub-interval 3.

Interval 3, sub-interval 3 profits per MWh sold from April 13

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	250	290	60
1	250	250	290	60
1	250	250	290	60
2	250	145.05	290	60
2	250	145.05	290	60
2	250	0	290	60
3	150	150	260	30
3	150	150	260	30
3	150	150	260	30
4	150	150	260	30
4	150	150	260	30
4	150	150	260	30
5	50	50	230	0
5	50	50	230	0
5	50	50	230	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	50	230	0
6	50	50	230	0
6	50	50	230	0
7	250	250	290	60
7	150	150	260	30
7	50	50	230	0
8	250	250	290	60
8	150	150	260	30
8	50	50	230	0
9	250	250	290	60
9	150	150	260	30
9	50	50	230	0
10	250	250	290	60
10	150	150	260	30
10	50	50	230	0
Demand	4250	4040.10	190	420

Discussion of interval 3, sub-interval from April 13

- Same as week!
- Group 2 offered all of its capacity at \$310/MWh or above, which set the price in all intervals.
- Dispatch is inefficient since not all of the 2500 MW of baseload is dispatched.
- Not all demand served.

Homework exercise: Discussion

Interval 3, sub-interval 1 supply from April 20

- Clearing price \$196.57/MWh compared to \$260/MWh last week and \$80/MWh competitive price.

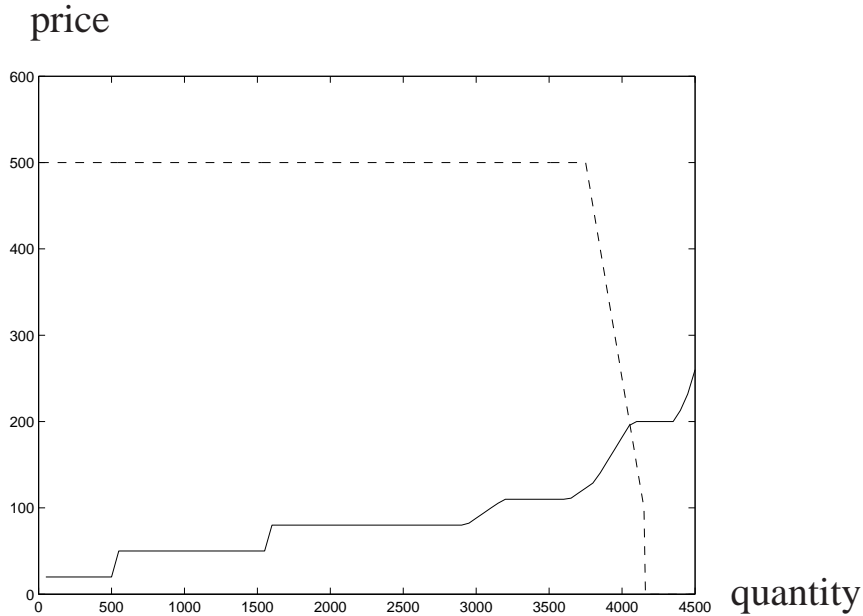


Fig. 7.45. Industry supply in interval 3, sub-interval 1.

Interval 3, sub-interval 1 supply detail from April 20

- Clearing price \$196.57/MWh compared to \$260/MWh last week and \$80/MWh competitive price.

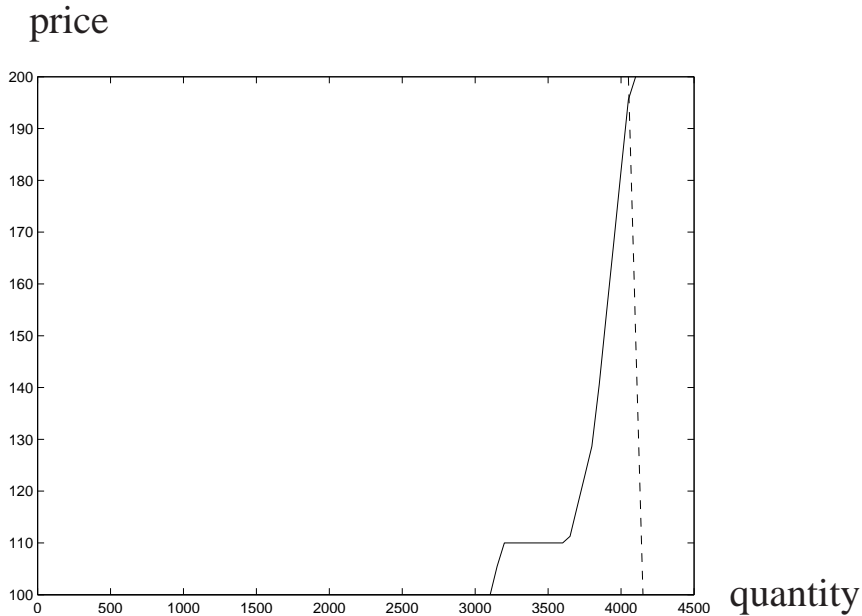


Fig. 7.46. Detail of industry supply in interval 3, sub-interval 1.

Interval 3, sub-interval 1 profits per MWh sold from April 20

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	250	176.57	60
1	250	250	176.57	60
1	250	250	176.57	60
2	250	250	176.57	60
2	250	250	176.57	60
2	250	250	176.57	60
3	150	150	146.57	30
3	150	150	146.57	30
3	150	150	146.57	30
4	150	146.57	146.57	30
4	150	0	146.57	30
4	150	0	146.57	30
5	50	50	116.57	0
5	50	50	116.57	0
5	50	50	116.57	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	50	116.57	0
6	50	50	116.57	0
6	50	50	116.57	0
7	250	250	176.57	60
7	150	150	146.57	30
7	50	50	116.57	0
8	250	171.42	176.57	60
8	150	103.16	146.57	30
8	50	32.38	116.57	0
9	250	250	176.57	60
9	150	150	146.57	30
9	50	50	116.57	0
10	250	250	176.57	60
10	150	150	146.57	30
10	50	50	116.57	0
Demand	4150	4053.53	303.43	420

Discussion of interval 3, sub-interval 1 from April 20

- Price collapsed compared to last week!
- Change is due to Group 2 offering its capacity at \$79.99 or above, instead of \$260/MWh or above, and they no longer set the price:
 - dispatch is still inefficient, since not all baseload is dispatched,
 - however, dispatch efficiency has improved since all Group 2 capacity is used!
- Not all demand served:
 - price responsive demand partially served.
- Group 2 has improved its profits:
 - last week, dispatched to 240.1 MW, with profit \$240/MWh, \$57,624/h,
 - this week, dispatched to 750 MW, with profit \$176.57/MWh, \$132,427.50/h.
- Note that stated benefit for demand of zero is *marginal* benefit.

Interval 3, sub-interval 2 supply from April 20

- Clearing price \$200/MWh compared to \$285/MWh last week and \$80/MWh competitive price.

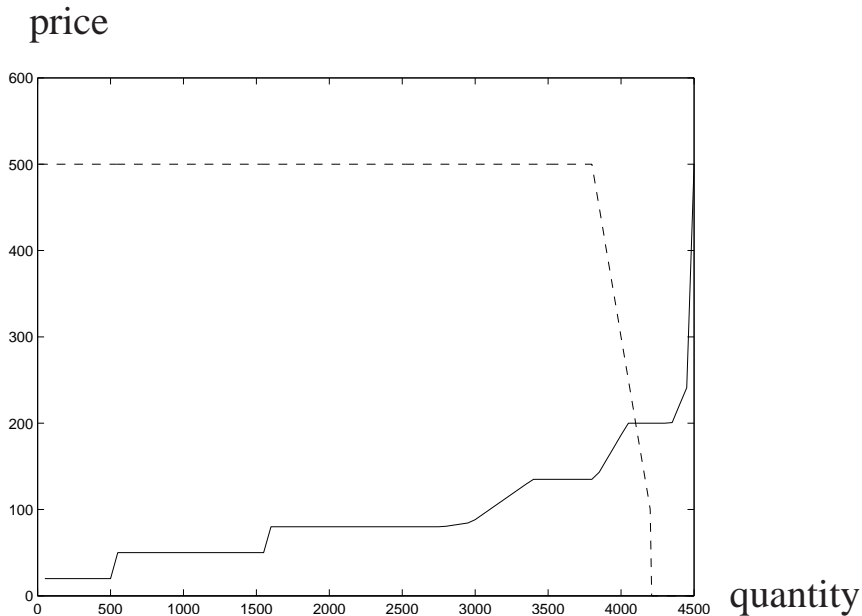


Fig. 7.47. Industry supply in interval 3, sub-interval 2.

Interval 3, sub-interval 2 supply detail from April 20

- Clearing price \$200/MWh compared to \$285/MWh last week and \$80/MWh competitive price.

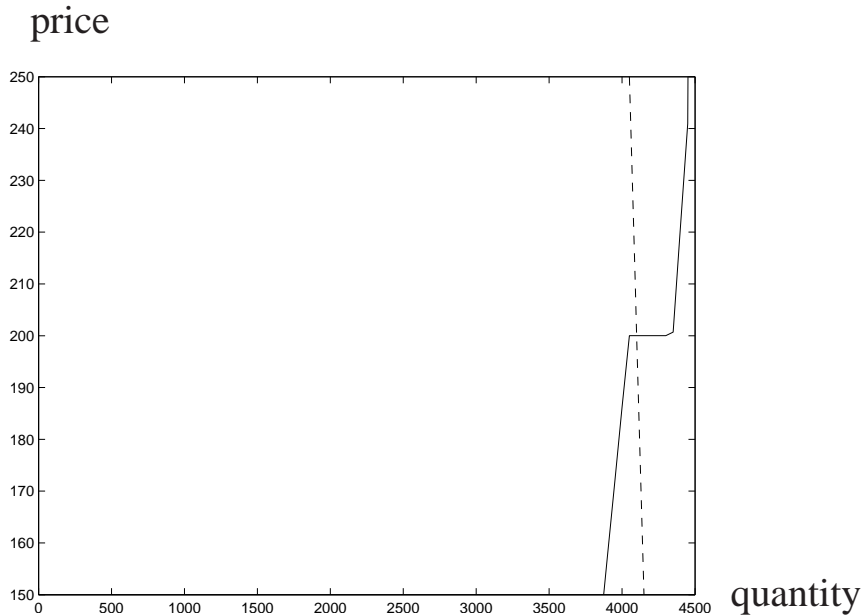


Fig. 7.48. Detail of industry supply in interval 3, sub-interval 2.

Interval 3, sub-interval 2 profits per MWh sold from April 20

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	250	180	60
1	250	250	180	60
1	250	250	180	60
2	250	250	180	60
2	250	250	180	60
2	250	250	180	60
3	150	150	150	30
3	150	150	150	30
3	150	150	150	30
4	150	150	150	30
4	150	25.93	150	30
4	150	25.93	150	30
5	50	50	120	0
5	50	50	120	0
5	50	50	120	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	50	120	0
6	50	50	120	0
6	50	50	120	0
7	250	250	180	60
7	150	150	150	30
7	50	50	120	0
8	250	166.67	180	60
8	150	100	150	30
8	50	31.58	120	0
9	250	250	180	60
9	150	150	150	30
9	50	50	120	0
10	250	250	180	60
10	150	150	150	30
10	50	50	120	0
Demand	4200	4100	300	420

Discussion of interval 3, sub-interval 2 from April 20

- Price also collapsed compared to last week!
- Change is again due to Group 2 offering its capacity at \$79.99 or above, instead of \$260/MWh or above, and they no longer set the price:
 - dispatch is still inefficient, since not all baseload is dispatched,
 - however, dispatch efficiency has improved since all Group 2 capacity is used!
- Not all demand served:
 - price responsive demand partially served.
- Group 2 has improved its profits:
 - last week, dispatched to 265.1 MW, with profit \$265/MWh, \$70,251.50/h,
 - this week, dispatched to 750 MW, with profit \$180/MWh, \$135,000/h.
- Note that stated benefit for demand of zero is *marginal* benefit.

Interval 3, sub-interval 3 supply from April 20

- Clearing price \$274.99/MWh compared to \$310/MWh last week and \$80/MWh competitive price.

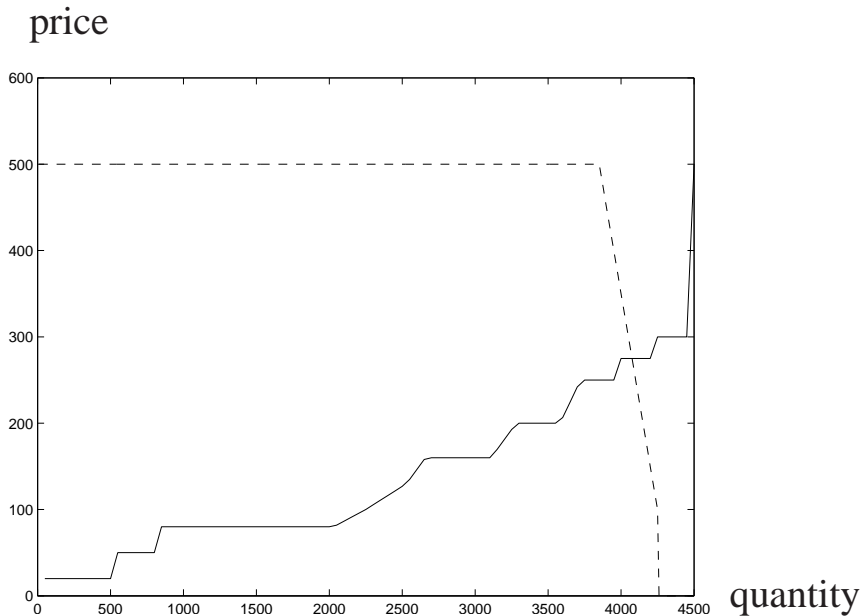


Fig. 7.49. Industry supply in interval 3, sub-interval 3.

Interval 3, sub-interval 3 supply detail from April 20

- Clearing price \$274.99/MWh compared to \$310/MWh last week and \$80/MWh competitive price.

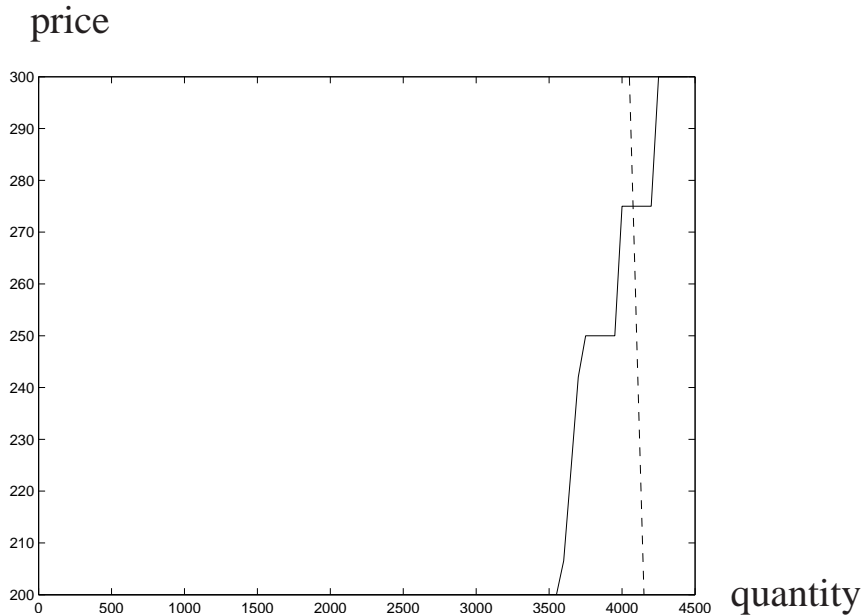


Fig. 7.50. Detail of industry supply in interval 3, sub-interval 3.

Interval 3, sub-interval 3 profits per MWh sold from April 20

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	250	254.99	60
1	250	75.11	254.99	60
1	250	0	254.99	60
2	250	250	254.99	60
2	250	250	254.99	60
2	250	250	254.99	60
3	150	150	224.99	30
3	150	150	224.99	30
3	150	150	224.99	30
4	150	150	224.99	30
4	150	150	224.99	30
4	150	150	224.99	30
5	50	50	194.99	0
5	50	50	194.99	0
5	50	50	194.99	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	50	194.99	0
6	50	50	194.99	0
6	50	50	194.99	0
7	250	250	254.99	60
7	150	150	224.99	30
7	50	50	194.99	0
8	250	250	254.99	60
8	150	150	224.99	30
8	50	50	194.99	0
9	250	250	254.99	60
9	150	150	224.99	30
9	50	50	194.99	0
10	250	250	254.99	60
10	150	150	224.99	30
10	50	50	194.99	0
Demand	4250	4075.11	225.01	420

Discussion of interval 3, sub-interval from April 20

- Price reduced compared to last week!
- Change is again due to Group 2 offering its capacity at \$79.99 or above, instead of \$260/MWh or above, and they no longer set the price:
 - dispatch is still inefficient, since not all baseload is dispatched,
 - however, dispatch efficiency has improved since all Group 2 capacity is used,
 - but not all Group 2 capacity is used.
- Not all demand served:
 - price responsive demand partially served.
- Group 2 has improved its profits:
 - last week, dispatched to 290.1 MW, with profit \$290/MWh, \$84,129/h,
 - this week, dispatched to 750 MW, with profit \$254.99/MWh, \$191,242.50/h.

Discussion of interval 3, sub-interval from April 20

- Group 1 has reduced profits:
 - last week, dispatched to 750 MW, with profit \$290/MWh, \$217,500/h,
 - this week, dispatched to 325.1 MW, with profit \$254.99/MWh, \$82,899.80/h.
- Note that stated benefit for demand of zero is *marginal* benefit.

Homework exercise: Discussion

Interval 3, sub-interval 1 supply from April 27

- Clearing price \$128.58/MWh compared to \$196.57/MWh last week and \$80/MWh competitive price.

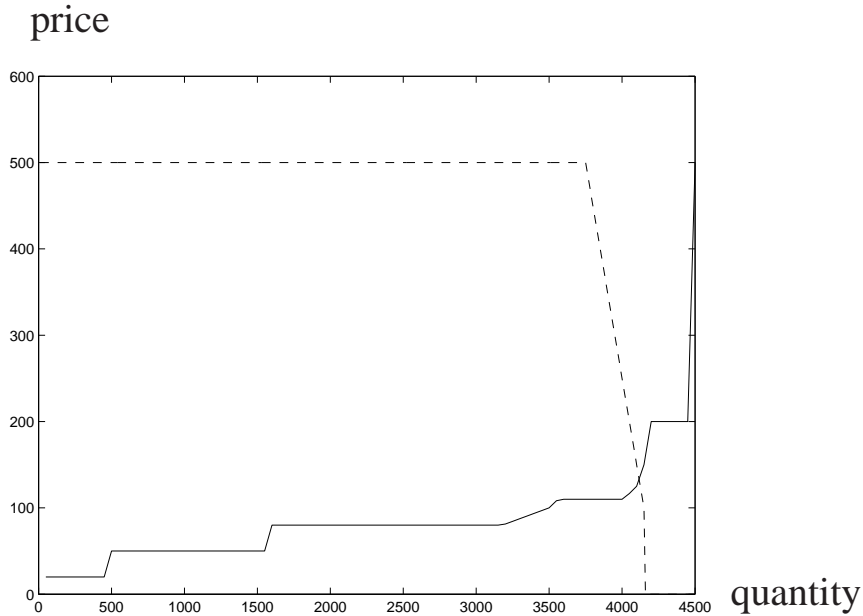


Fig. 7.51. Industry supply in interval 3, sub-interval 1.

Interval 3, sub-interval 1 supply detail from April 27

- Clearing price \$128.58/MWh compared to \$196.57/MWh last week and \$80/MWh competitive price.

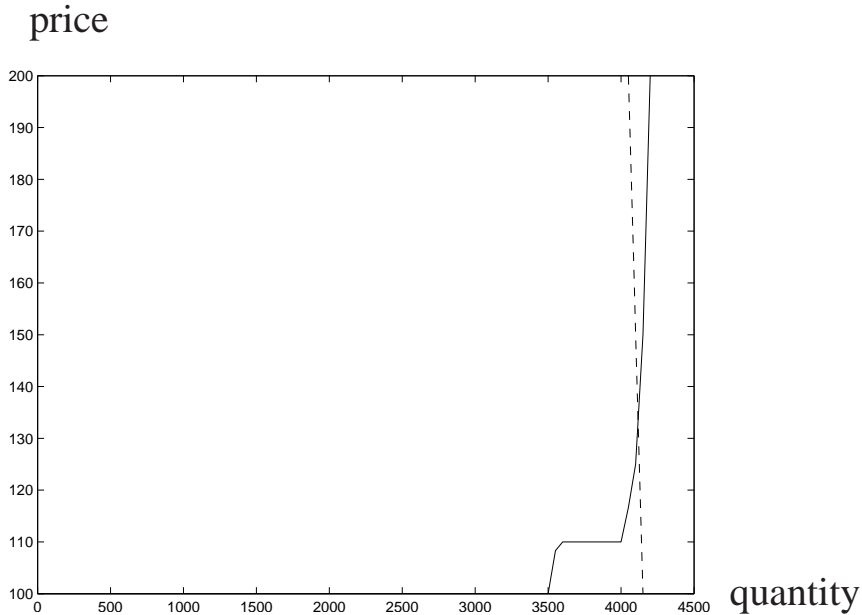


Fig. 7.52. Detail of industry supply in interval 3, sub-interval 1.

Interval 3, sub-interval 1 profits per MWh sold from April 27

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	250	108.58	60
1	250	250	108.58	60
1	250	250	108.58	60
2	250	250	108.58	60
2	250	250	108.58	60
2	250	242.94	108.58	60
3	150	150	78.58	30
3	150	150	78.58	30
3	150	150	78.58	30
4	150	78.58	78.58	30
4	150	0	78.58	30
4	150	0	78.58	30
5	50	50	48.58	0
5	50	50	48.58	0
5	50	50	48.58	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	50	48.58	0
6	50	50	48.58	0
6	50	50	48.58	0
7	250	250	108.58	60
7	150	150	78.58	30
7	50	50	48.58	0
8	250	250	108.58	60
8	150	150	78.58	30
8	50	50	48.58	0
9	250	250	108.58	60
9	150	150	78.58	30
9	50	50	48.58	0
10	250	250	108.58	60
10	150	150	78.58	30
10	50	50	48.58	0
Demand	4150	4121.52	371.42	420

Discussion of interval 3, sub-interval 1 from April 27

- Price collapsed again compared to last week!
- Change is due to Group 8 offering its capacity with lower slope of offer:
 - dispatch is still inefficient, since not all baseload is dispatched,
 - however, dispatch efficiency has improved since all of Group 8 baseload and intermediate capacity is used!
- Not quite all demand served:
 - price responsive demand almost all served.
- Group 8 has not improved its profits:
 - last week, dispatched to 306.96 MW, with profit \$49,162.32/h,
 - this week, dispatched to 450 MW, with profit \$41,361/h.
- Note that stated benefit for demand of zero is *marginal* benefit.

Interval 3, sub-interval 2 supply from April 27

- Clearing price \$150.05/MWh compared to \$200/MWh last week and \$80/MWh competitive price.

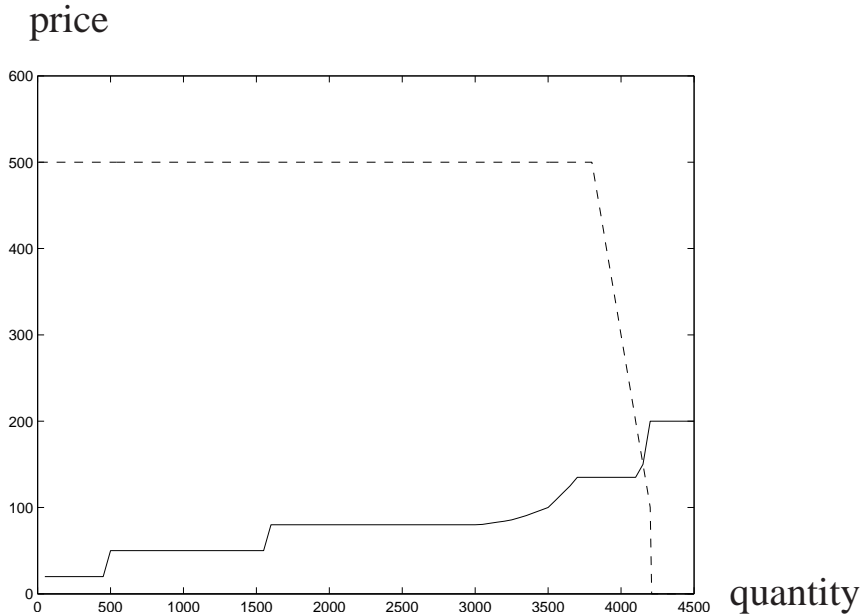


Fig. 7.53. Industry supply in interval 3, sub-interval 2.

Interval 3, sub-interval 2 supply detail from April 27

- Clearing price \$150.05/MWh compared to \$200/MWh last week and \$80/MWh competitive price.

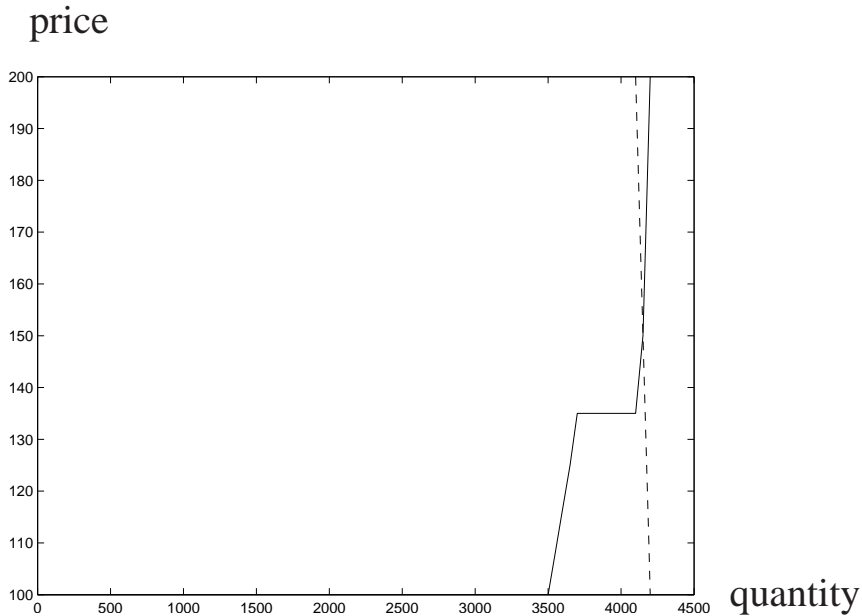


Fig. 7.54. Detail of industry supply in interval 3, sub-interval 2.

Interval 3, sub-interval 2 profits per MWh sold from April 27

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	250	130.05	60
1	250	250	130.05	60
1	250	250	130.05	60
2	250	250	130.05	60
2	250	250	130.05	60
2	250	250	130.05	60
3	150	150	100.05	30
3	150	150	100.05	30
3	150	150	100.05	30
4	150	100.05	100.05	30
4	150	0	100.05	30
4	150	0	100.05	30
5	50	50	70.05	0
5	50	50	70.05	0
5	50	50	70.05	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	50	70.05	0
6	50	50	70.05	0
6	50	50	70.05	0
7	250	250	130.05	60
7	150	150	100.05	30
7	50	50	70.05	0
8	250	250	130.05	60
8	150	150	100.05	30
8	50	50	70.05	0
9	250	250	130.05	60
9	150	150	100.05	30
9	50	50	70.05	0
10	250	250	130.05	60
10	150	150	100.05	30
10	50	50	70.05	0
Demand	4200	4150.05	349.95	420

Discussion of interval 3, sub-interval 2 from April 27

- Price also collapsed again compared to last week!
- Change is again due to Group 8 offering its capacity with lower slope of offer:
 - all baseload is dispatched,
 - dispatch efficiency has improved since all of Group 8 baseload and intermediate capacity is used!
- Not all demand served:
 - price responsive demand partially served.
- Group 8 has improved its profits:
 - last week, dispatched to 298.25 MW, with profit \$48,790.20/h,
 - this week, dispatched to 450 MW, with profit \$51,022.50/h.
- Note that stated benefit for demand of zero is *marginal* benefit.

Interval 3, sub-interval 3 supply from April 27

- Clearing price \$274.99/MWh compared to \$274.99/MWh last week and \$80/MWh competitive price.

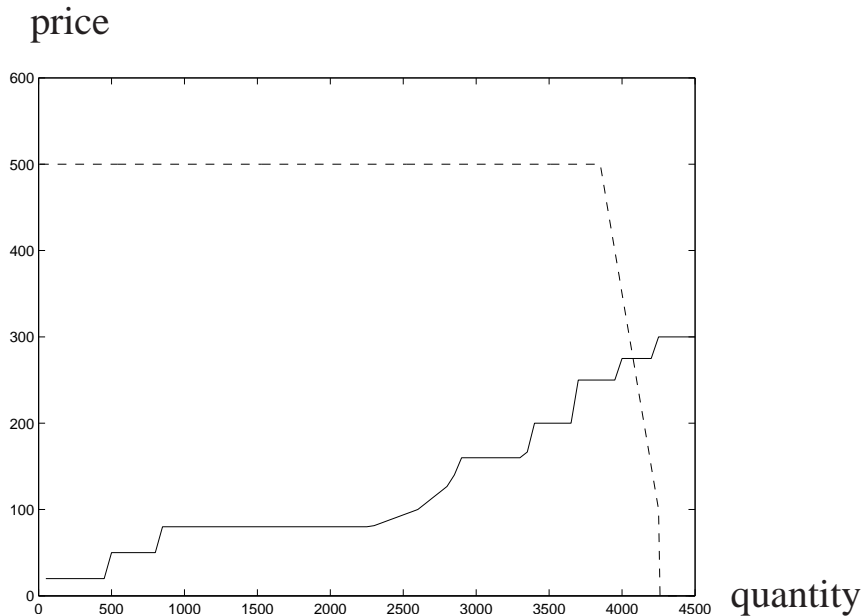


Fig. 7.55. Industry supply in interval 3, sub-interval 3.

Interval 3, sub-interval 3 supply detail from April 27

- Clearing price \$274.99/MWh compared to \$274.99/MWh last week and \$80/MWh competitive price.

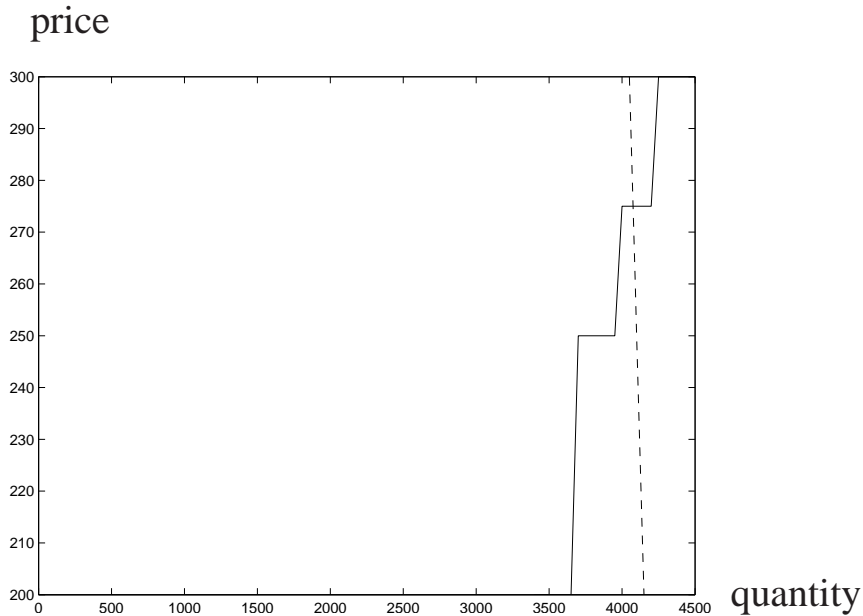


Fig. 7.56. Detail of industry supply in interval 3, sub-interval 3.

Interval 3, sub-interval 3 profits per MWh sold from April 27

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	250	254.99	60
1	250	75.11	254.99	60
1	250	0	254.99	60
2	250	250	254.99	60
2	250	250	254.99	60
2	250	250	254.99	60
3	150	150	224.99	30
3	150	150	224.99	30
3	150	150	224.99	30
4	150	150	224.99	30
4	150	150	224.99	30
4	150	150	224.99	30
5	50	50	194.99	0
5	50	50	194.99	0
5	50	50	194.99	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	50	194.99	0
6	50	50	194.99	0
6	50	50	194.99	0
7	250	250	254.99	60
7	150	150	224.99	30
7	50	50	194.99	0
8	250	250	254.99	60
8	150	150	224.99	30
8	50	50	194.99	0
9	250	250	254.99	60
9	150	150	224.99	30
9	50	50	194.99	0
10	250	250	254.99	60
10	150	150	224.99	30
10	50	50	194.99	0
Demand	4250	4075.11	225.01	420

Discussion of interval 3, sub-interval from April 27

- Price the same as last week!
- Some changes in offers, but Group 1 again set price:
 - dispatch is still inefficient, since not all baseload is dispatched.
- Not all demand served:
 - price responsive demand partially served.
- No change in profits.

Homework exercise, due April 29: Solution

- “Symmetric duopoly” with each firm $i = 1, 2$ having marginal cost function:

$$\forall Q_i, c'_i(Q_i) = 20 + 60Q_i/2500.$$

- Operating range $[0, \bar{Q}_i]$, where $\bar{Q}_i = 2500$ MW.
- Note that, in the context of the affine supply function equilibrium formulation, $e_i = 60/2500$ and $a_i = 20$ for each firm.
- The inverse demand in each of three intervals is:

Interval 1 $\forall Q, p^d(Q) = \max\{50 - (Q - 2800)/2, 0\}$,

Interval 2 $\forall Q, p^d(Q) = \max\{75 - (Q - 3500)/2, 0\}$,

Interval 3 $\forall Q, p^d(Q) = \max\{500 - (Q - 4200)/2, 0\}$,

- where Q is in MW and $p^d(Q)$ is in \$/MWh.
- That is, the demand slope is $\gamma = 2$ MW per (\$/MWh).
- Find the affine supply function equilibrium solution and the clearing prices and quantities.

Homework exercise, due April 29: Solution

- The governing equation for the affine supply function equilibrium is (4.9):

$$\forall i, \beta_i = (1 - e_i \beta_i) \left(\gamma + \sum_{j \neq i} \beta_j \right).$$

- We can solve this using the MATLAB function `fsolve` or we can solve it more directly by noting that:
 - the affine supply function equilibrium is unique, and
 - the equations are symmetric in i because the cost functions are symmetric.
- Therefore, if there is a symmetric solution with non-negative values, it is the unique solution having non-negative values.

Homework exercise, due April 29: Solution, continued

- So, suppose $\beta_1 = \beta_2 \geq 0$.
- On substituting from the data into (4.9) and setting $\beta_1 = \beta_2$, we have that:

$$\beta_1 = \left(1 - \frac{60}{2500}\beta_1\right)(2 + \beta_1).$$

- Expanding and re-arranging, we have:

$$\begin{aligned}\frac{60}{2500}(\beta_1)^2 + \frac{60 \times 2}{2500}\beta_1 - 2 &= 0, \\ \frac{1}{2}(\beta_1)^2 + \beta_1 - \frac{2500}{60} &= 0.\end{aligned}$$

- Using the “quadratic equation,” we obtain:

$$\begin{aligned}\beta_1^* &= \beta_2^*, \\ &= -1 \pm \sqrt{(1)^2 + \frac{2 \times 2500}{60}}, \\ &\approx -1 \pm 9.1833.\end{aligned}$$

Homework exercise, due April 29: Solution, continued

- Since we require $\beta_1^* = \beta_2^* \geq 0$, we have that $\beta_1^* = \beta_2^* = 8.1833$ is the unique non-negative solution.
- The supply functions are:

$$\forall P \in [20, \infty), s^{\text{affine}}(P) = \begin{bmatrix} 8.1833(P - 20) \\ 8.1833(P - 20) \end{bmatrix}.$$

- This is the affine supply function equilibrium.

Homework exercise, due April 29: Solution, continued

- The clearing price and quantities in each interval are shown in the following table together with the Cournot and competitive results.
- The affine SFE quantities are much larger and the affine SFE prices are much lower than the corresponding Cournot results:
 - Choice of strategic model is extremely significant!
 - Affine SFE is generally between competitive and Cournot outcomes.

Interval	Affine SFE		Cournot		Competitive	
	Price (\$/MWh)	Quantity (MW)	Price (\$/MWh)	Quantity (MW)	Price (\$/MWh)	Quantity (MW)
1	175.7	2548.6	511.7	1876.6	53.5	2793.0
2	217.6	3234.7	640.6	2368.8	62.3	3525.4
3	300.9	4598.1	907.1	3385.8	80.5	5039.1

Table 7.1. *Clearing price and quantity and comparison to Cournot and competitive.*

Homework exercise: Discussion

Interval 3, sub-interval 1 supply from May 4

- Clearing price \$260/MWh compared to \$128.58/MWh last week and \$80/MWh competitive price.

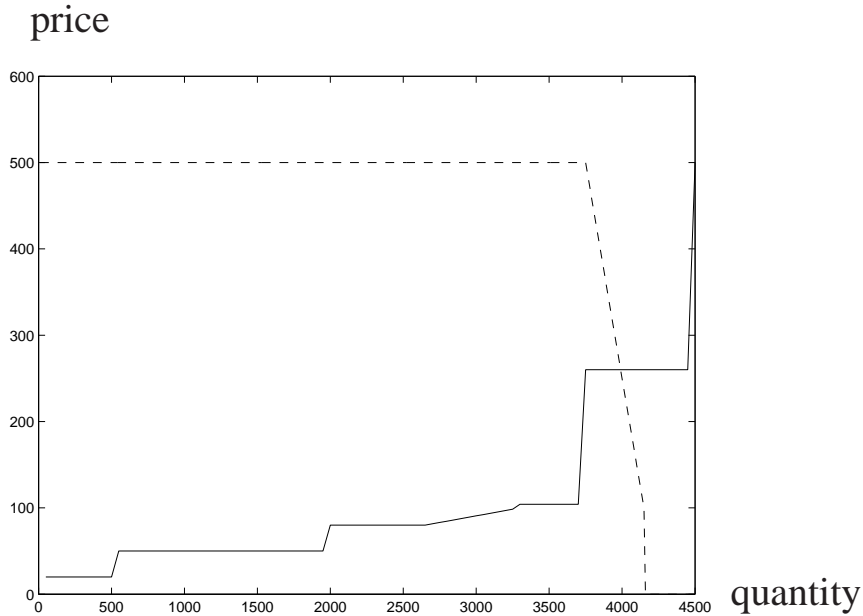


Fig. 7.57. Industry supply in interval 3, sub-interval 1.

Interval 3, sub-interval 1 supply detail from May 4

- Clearing price \$260/MWh compared to \$128.58/MWh last week and \$80/MWh competitive price.

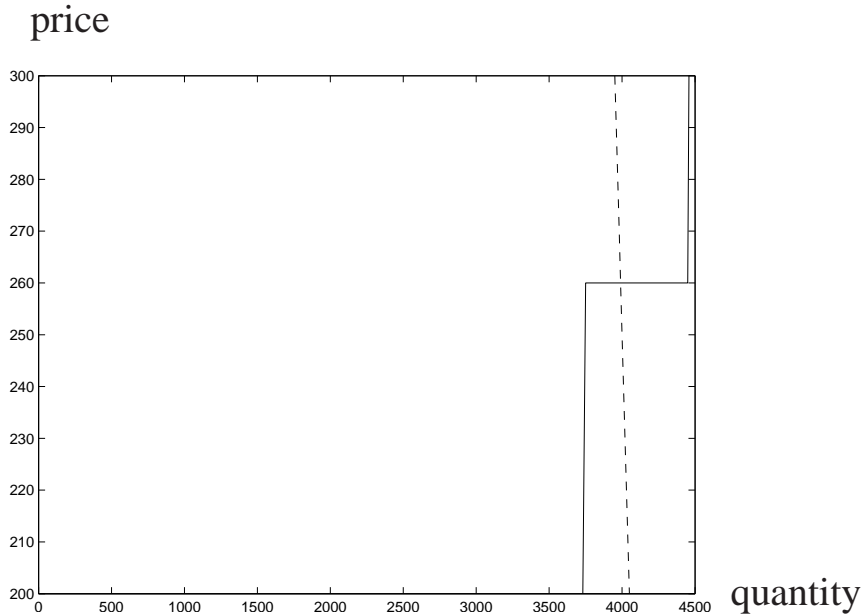


Fig. 7.58. Detail of industry supply in interval 3, sub-interval 1.

Interval 3, sub-interval 1 profits per MWh sold from May 4

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	250	240	60
1	250	250	240	60
1	250	250	240	60
2	250	80	240	60
2	250	80	240	60
2	250	80	240	60
3	150	150	210	30
3	150	150	210	30
3	150	150	210	30
4	150	150	210	30
4	150	150	210	30
4	150	150	210	30
5	50	50	180	0
5	50	50	180	0
5	50	50	180	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	50	180	0
6	50	50	180	0
6	50	50	180	0
7	250	250	240	60
7	150	150	210	30
7	50	50	180	0
8	250	250	240	60
8	150	150	210	30
8	50	50	180	0
9	250	250	240	60
9	150	150	210	30
9	50	50	180	0
10	250	250	240	60
10	150	150	210	30
10	50	50	180	0
Demand	4150	3990	240	420

Discussion of interval 3, sub-interval 1 from May 4

- Price increased compared to last week!
- Change is due to Group 2 offering all of its capacity at \$260/MWh:
 - dispatch is inefficient, since not all baseload is dispatched.
- Much less demand served than last week.
- Group 2 has not improved its profits:
 - last week, dispatched to 742.94 MW, with profit \$80,668.43/h,
 - this week, dispatched to 240 MW, with profit \$57,620/h.
- Note that stated benefit for demand of zero is *marginal* benefit.

Interval 3, sub-interval 2 supply from May 4

- Clearing price \$123.80/MWh compared to \$150.05/MWh last week and \$80/MWh competitive price.

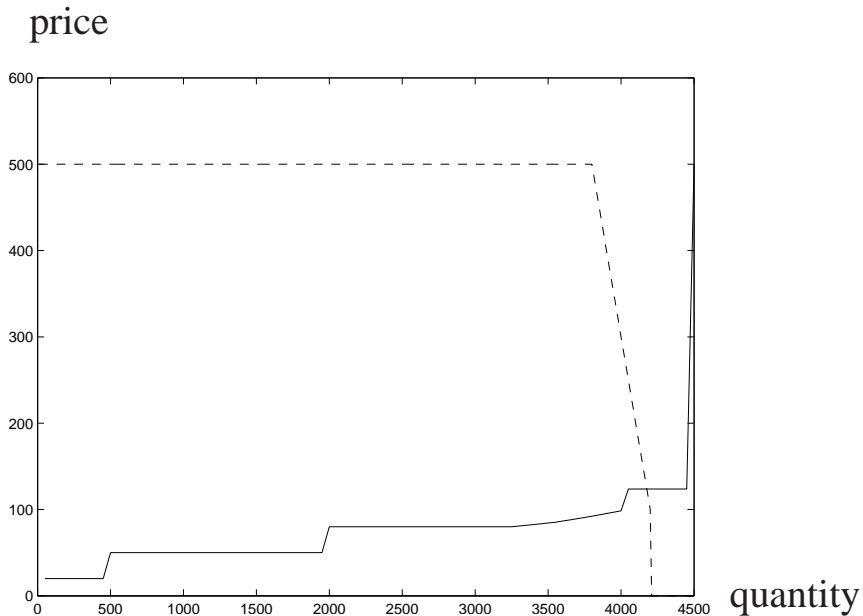


Fig. 7.59. Industry supply in interval 3, sub-interval 2.

Interval 3, sub-interval 2 supply detail from May 4

- Clearing price \$123.80/MWh compared to \$150.05/MWh last week and \$80/MWh competitive price.

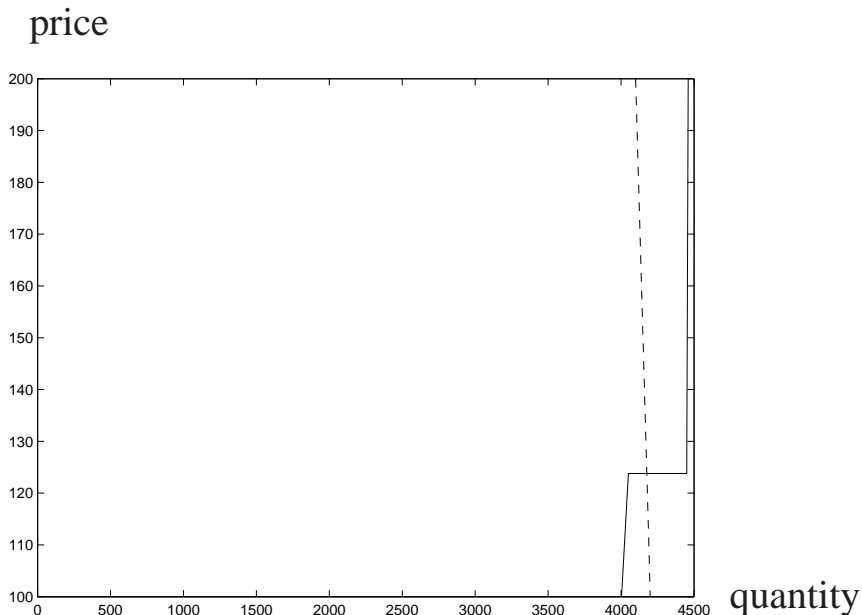


Fig. 7.60. Detail of industry supply in interval 3, sub-interval 2.

Interval 3, sub-interval 2 profits per MWh sold from May 4

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	250	103.8	60
1	250	250	103.8	60
1	250	250	103.8	60
2	250	250	103.8	60
2	250	250	103.8	60
2	250	250	103.8	60
3	150	150	73.8	30
3	150	150	73.8	30
3	150	150	73.8	30
4	150	150	73.8	30
4	150	150	73.8	30
4	150	150	73.8	30
5	50	50	43.8	0
5	50	50	43.8	0
5	50	50	43.8	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	50	43.8	0
6	50	50	43.8	0
6	50	50	43.8	0
7	250	42.1	103.8	60
7	150	42.1	73.8	30
7	50	42.1	43.8	0
8	250	250	103.8	60
8	150	150	73.8	30
8	50	50	43.8	0
9	250	250	103.8	60
9	150	150	73.8	30
9	50	50	43.8	0
10	250	250	103.8	60
10	150	150	73.8	30
10	50	50	43.8	0
Demand	4200	4176.30	376.20	420

Discussion of interval 3, sub-interval 2 from May 4

- Price has dropped yet again compared to last week and is lower than sub-interval 2!
- Change is due to Group 7 offering at lower prices and Groups 2 and 4 offering closer to competitive during this interval:
 - dispatch not efficient since not all baseload used,
 - less of Group 7 is dispatched than last week because reductions in offer prices of Groups 2 and 4 were even greater.
- Not all demand served:
 - price responsive demand partially served.
- Group 7 profits have decreased:
 - last week, dispatched to 450 MW, with profit \$51,022.50/h,
 - this week, dispatched to 126.3 MW, with profit \$9,320.94/h.
- Despite decrease in Group 7 offer prices, production of Group 7 decreased compared to last week.
- Note that stated benefit for demand of zero is *marginal* benefit.

Interval 3, sub-interval 3 supply from May 4

- Clearing price \$145/MWh compared to \$274.99/MWh last week and \$80/MWh competitive price.

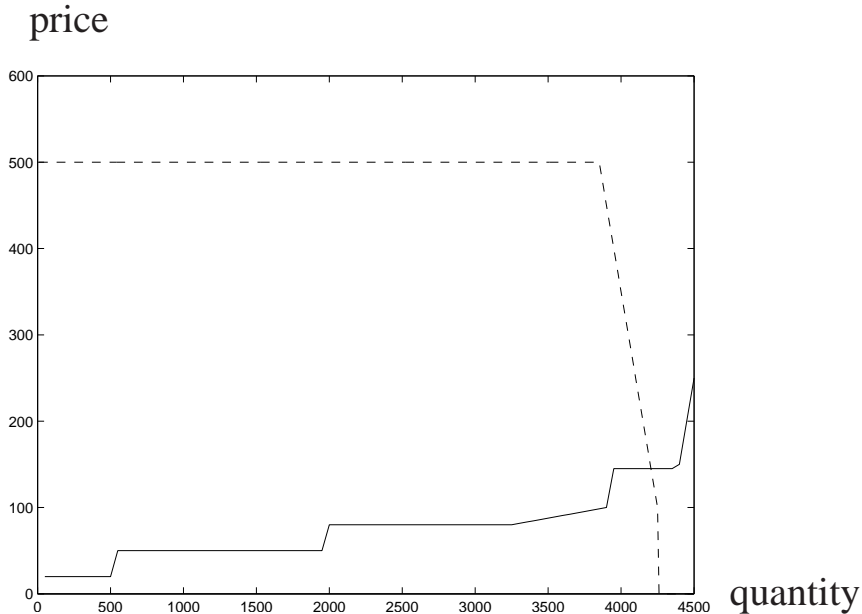


Fig. 7.61. Industry supply in interval 3, sub-interval 3.

Interval 3, sub-interval 3 supply detail from May 4

- Clearing price \$145/MWh compared to \$274.99/MWh last week and \$80/MWh competitive price.

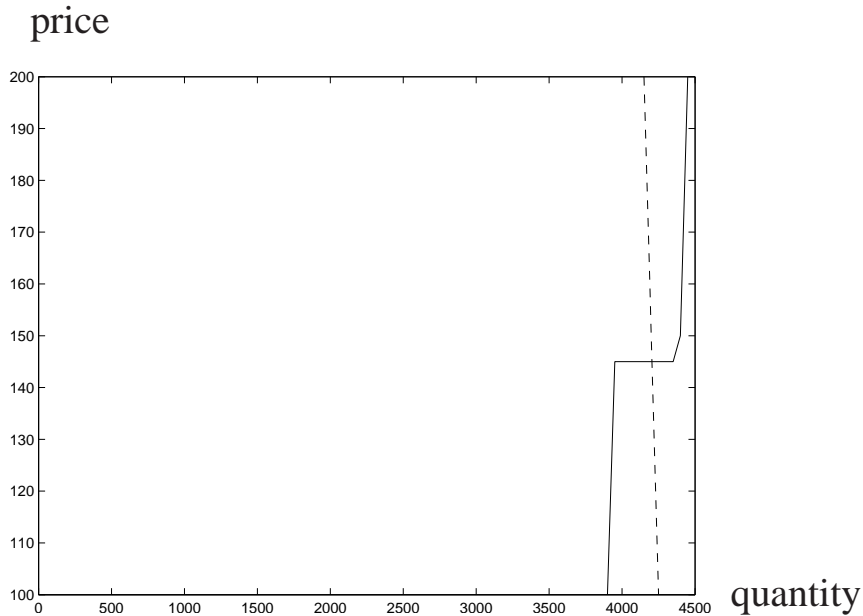


Fig. 7.62. Detail of industry supply in interval 3, sub-interval 3.

Interval 3, sub-interval 3 profits per MWh sold from May 4

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
1	250	250	125	60
1	250	250	125	60
1	250	250	125	60
2	250	250	125	60
2	250	250	125	60
2	250	250	125	60
3	150	150	95	30
3	150	150	95	30
3	150	150	95	30
4	150	150	95	30
4	150	150	95	30
4	150	150	95	30
5	50	50	65	0
5	50	0	65	0
5	50	0	65	0

Group	Capacity (MW)	Production (MW)	Profit or Benefit (\$/MWh)	Competitive Profit or Benefit (\$/MWh)
6	50	50	65	0
6	50	50	65	0
6	50	50	65	0
7	250	105.05	125	60
7	150	105.05	95	30
7	50	50	65	0
8	250	250	125	60
8	150	150	95	30
8	50	45	65	0
9	250	250	125	60
9	150	150	95	30
9	50	50	65	0
10	250	250	125	60
10	150	150	95	30
10	50	50	65	0
Demand	4250	4205.10	355	420

Discussion of interval 3, sub-interval from May 4

- Price collapsed compared to last week!
- Group 1 offer price significantly reduced:
 - dispatch is still inefficient, since not all baseload is dispatched,
 - dispatch closer to efficient than last week.
- Group 1 profits have decreased:
 - last week, dispatched to 325.11 MW, with profit \$82,899.80/h,
 - this week, dispatched to 750 MW, with profit \$93,750/h.
- Not all demand served:
 - price responsive demand partially served.
- Still have not achieved equilibrium!

Homework exercise, due May 4: Solution

- Consider the five firm example system with costs shown in the table.
- Solve (4.9) for this data using the MATLAB function `fsolve` (or any other technique of your choice) with initial guess given by the inverses of the e_i .
- That is, solve $g(\beta) = \mathbf{0}$, where:

$$\forall \beta, g_i(\beta) = \beta_i - (1 - e_i \beta_i) \left(\gamma + \sum_{j \neq i} \beta_j \right).$$

- Assume that $\gamma = 0.1$ GW per (\$/MWh).

Firm i	1	2	3	4	5
e_i ((\$/MWh)/GW)	2.687	4.615	1.789	1.93	4.615
a_i (\$/MWh)	12	12	8	8	12

Table 7.2.
Five firm cost data from Baldick, Grant, and Kahn.

Homework exercise, due May 4: Solution, continued

- The following MATLAB code fragment shows how to call the MATLAB function `g`.

```
e = [2.687; 4.615; 1.789; 1.93; 4.615];  
gamma = 0.1;  
beta0 = 1./e;  
beta = fsolve('g', beta0, [], e, gamma);
```

```
function g = g(beta, e, gamma)  
% Calculates g as defined for homework exercise  
% given quadratic cost coefficients (in vector e)  
% and negative of demand slope (in gamma).  
betajnoti = (ones(length(beta), length(beta)) ...  
    - eye(length(beta)))*beta;  
g = beta - (1 - e.*beta).*(gamma + betajnoti);  
return
```

Fig. 7.63.
Listing of
MATLAB M-
file to evaluate
`g`.

Homework exercise, due May 4: Solution, continued

- Using the MATLAB code to solve (4.9) for the cost parameters in table 7.2 and a demand slope of $\gamma = 0.1$ GW per (\$/MWh), we find that the slopes of the affine solutions are:

$$\beta = \begin{bmatrix} 0.2840 \\ 0.1857 \\ 0.3718 \\ 0.3550 \\ 0.1857 \end{bmatrix}.$$

- The affine SFE is given by:

$$\forall P \in [12, \infty), s^{\text{affine}}(P) = \begin{bmatrix} 0.2840(P - 12) \\ 0.1857(P - 12) \\ 0.3718(P - 8) \\ 0.3550(P - 8) \\ 0.1857(P - 12) \end{bmatrix}. \quad (7.13)$$

Homework exercise, due May 4: Solution, continued

- The slopes of the affine SFE (7.13) are somewhat lower than the inverses of the corresponding e_i , which are:

$$\begin{bmatrix} 0.3722 \\ 0.2167 \\ 0.5590 \\ 0.5181 \\ 0.2167 \end{bmatrix} .$$

- That is, the prices of the affine SFE are higher than competitive prices for any demand condition.
- We have also seen that the prices of the affine SFE are lower than the Cournot prices.