Firm-based Measurements of Market Power in Transmission-Constrained Electricity Markets: Technical Reference

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Abstract

In this report, two different approaches to analyzing firm-based market power considering transmission constraints are proposed. One is an application of the transmission-constrained residual demand Jacobian, while the other is a generalization of the "residual supply index" to the case of transmission constraints. These two approaches provide complementary evaluations of market power. Medium- and large-scale system examples are provided to demonstrate computational efficiency, and both approaches could be applied to real-world electricity markets.

Keywords: Transmission constraints, electricity markets, market power, market power index, residual supply index.

1 Introduction

Market power continues to be a problematic issue in restructured electricity markets. Analyzing market power is challenging due to the complexity of modeling strategic behavior of market participants. Transmission constraints, which complicate the interaction between the behavior of generation firms and the market outcome, pose particular difficulties for market power analysis.

In their basic forms, some current approaches to analyzing market power, such as the Herfindahl–Hirshman index (HHI) and the residual supply index (RSI) [1] [2] [3], ignore the effects of transmission constraints. In general, this type of analysis ignores the significant effects of transmission constraints on the market outcome.

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Other approaches to market power do try to incorporate transmission constraints. For example, [4] uses revenue-price sensitivities to estimate the price mark-ups above competitive levels. As another example, [5] describes a sensitivity-based approach, which utilizes shift factors of binding constraints to find clusters of generators that can perturb market clearing prices without affecting the dispatch. As a third example, the element competitiveness index (ECI) used by the electric reliability council of Texas (ERCOT) [6, §3.19.1] is based on an HHI analysis modified in an *ad hoc* manner by shift factors of generators to transmission constraints. However, these kinds of analyses fail to consider the impacts of transmission constraints on the market outcome in a principled manner because they do not represent the incentives faced by firms.

Another category of market power analysis involves modeling the generation firm's behavior as a two level optimization problem [7]: the top level is the firm's profit maximization problem and the bottom level is an optimal power flow (OPF) program that determines the market outcome, including the locational marginal prices (LMPs), given the action of the firm. While this is a principled method of modeling the strategic behavior of a firm, the two-level optimization problem is non-convex and very difficult to solve, especially for large-scale systems.

In this report, two different approaches to analyzing firm-based market power considering transmission constraints are proposed. The first one is based on "small-signal" analysis, while the other is based on "large-signal" analysis. They provide different insights about market power and can both be integrated into market power analysis flow. Both approaches analyze market power for a single market clearing interval but, due to their computational efficiency, could also be applied repeatedly over a time horizon to assess the average or typical case.

The first approach is a generalization of the transmission-constrained market power indices proposed in [8], which are based on the the transmission-constrained residual demand derivative that was described in [9]. Unlike *ad hoc* approaches, the four market power indices in [8] represent the incentives faced by a firm, including the effects of Kirchhoff's laws. That analysis is an *ex post* assessment based on offer information and market clearing results, and is computationally efficient.

A limitation of the indices in [8], however, is that each generation firm is assumed to own generation assets at only one bus. In reality, firms often own assets located at multiple buses having different LMPs. As shown by the example in [10], strategic behavior of firms with assets at multiple buses can be qualitatively different to the simpler case analyzed in [8]. In this report, we extend the transmission-constrained market power indices to the case that a firm has assets at multiple locations, based on the inverse residual demand Jacobian described in [11], and propose an analogous firm-based transmission-constrained market power index (TCMPI).

We also propose another, different approach to assess firm-based market power: the transmission-constrained residual supply index (TCRSI), which generalizes the RSI to the case of binding transmission constraints. The residual supply index (RSI) proposed by the California ISO (CAISO) is used to predict market power in the CAISO and some other markets. The RSI reflects the degree to which a firm's offers are necessary to meet demand. Empirical results indicate that the RSI is highly correlated to the exercise of market power [1][2]. However, the original RSI definition ignores the effects of transmission constraints and might be less helpful in the context of LMP markets.

The CAISO uses "competitive path assessment" to determine the competitiveness of each transmission constraint [12]. However, the CAISO approach focuses on the degree of violation of transmission capacity constraints needed to meet demand, whereas the approach proposed in this report directly generalizes the RSI to the case of transmission constraints. The TCRSI proposed here can efficiently assess the extent to which a firm is "pivotal" when transmission constraints bind.

The rest of this report is organized as follows: Section 2 introduces the TCMPI. Section 3 provides case studies of the TCMPI. Section 4 develops the TCRSI. Section 5 provides case studies of the TCRSI. Section 6 compares these two approaches and Section 7 concludes. Our discussion will not explicitly treat forward contracts, but the analysis can easily be modified to include the effect of exogenously specified forward contracts.

2 Firm-Based Transmission-constrained Market Power Index

To develop the firm-based transmission-constrained market power index, we first analyze the incentives for a firm to profitably alter the prices from competitive levels, which leads to the new index. Then we discuss how to compute the index efficiently and provide some examples.

2.1 Principles-based analysis of firm-based market power

Suppose that firm i owns generators at buses $k \in R$, where $|R| = r$. We collect the production quantities $q_k, k \in R$, at all these generators into a vector $q \in \mathbb{R}^r$. Assume that $p_{-k} : \mathbb{R}^r \to \mathbb{R}$ is the resulting market clearing price at bus k given that the firm produces the quantities q. That is, p_{-k} is the inverse residual demand function for firm i at bus k. We collect the inverse residual demands $p_{-k}, k \in R$, together into a vector function $p : \mathbb{R}^r \to \mathbb{R}^r$.

Note that the inverse residual demand faced by a generator at bus k depends on the whole vector q. That is, actions by the firm at any one of its generators may result in a change in the price at bus k.

We now assume that firms are profit maximizers and consider the condition for firm i to maximize its profit. Assume that the production cost functions of the firm are specified by the cost functions $c_k : \mathbb{R} \to \mathbb{R}, k \in R$. Ignoring forward contracts, the profit for the market participant is:

$$\forall q \in \mathbb{R}^r, \pi(q) = \sum_{k \in R} \Big(q_k p_{-k}(q) - c_k(q_k) \Big),$$

where the term $\sum_{k \in R} q_k p_{-k}(q)$ is the total revenue of firm i. (The case with exogenously specified forward contracts is similar.) We first consider the case where the capacity constraints of the generators owned by firm i are not binding and then consider the more general case.

2.1.1 Ignoring generator capacity constraints

Assuming that sufficient conditions for maximization are satisfied, that p and $c_k, k \in R$, are differentiable, and that generator capacity constraints of firm i are not binding, we can maximize the profit of firm i by setting the partial derivatives of profit with respect to quantities equal to zero. Focusing on the partial derivative with respect to q_m for a particular $m \in R$, we obtain:

$$0 = \frac{\partial \pi}{\partial q_m}(q) = p_{-m}(q) + \sum_{k \in R} q_k \frac{\partial p_{-k}}{\partial q_m}(q) - c'_m(q_m),$$

where $c'_m = \frac{\partial c_m}{\partial q_m}$ is the marginal cost of the generator owned by firm i at bus m. Re-arranging the above equation, we obtain the price-cost mark-up at bus m under the hypothesis that the firm was maximizing its profits:

$$p_{-m}(q) - c'_m(q_m) = -\sum_{k \in R} q_k \frac{\partial p_{-k}}{\partial q_m}(q). \tag{1}$$

This is a generalization of (1) in [8] to the case of firms owning multiple generators. In the firm-based analysis, the profit maximizing mark-up at generator m depends on productions at other buses q_k and cross derivatives $\frac{\partial p_{-k}}{\partial q_m}$.

To aggregate all the estimated mark-ups into one index for firm i, a natural approach is to evaluate the quantity-weighted average mark-up of the firm:

$$\frac{\left(-\sum_{m \in R} q_m \sum_{k \in R} q_k \frac{\partial p_{-k}}{\partial q_m}(q)\right)}{\left(\mathbf{1}^\dagger q\right)} = \frac{\left(-q^\dagger \frac{\partial p}{\partial q}(q) q\right)}{\left(\mathbf{1}^\dagger q\right)}, \tag{2}$$

where: $\frac{\partial p}{\partial q}(q)$ is the inverse residual demand Jacobian evaluated at q, superscript $\dagger$ means transpose, and $\mathbf{1} \in \mathbb{R}^r$ is the vector of all ones. Note that the matrix $\frac{\partial p}{\partial q}(q)$ is symmetric and negative semi-definite, as proved in [11] and [13]. Therefore, the estimated average mark-up is always non-negative.

Since cross-derivatives $\frac{\partial p_{-k}}{\partial q_m}$ for $k \neq m$ can be positive, it may be the case that, at some buses, profit maximization corresponds to a *mark-down* rather than a mark-up. That is, for some buses m, the estimated mark-up in (1) may be negative. While this seems to be counter-intuitive, Hogan [14] and Cardell *et al.* [10] describe just such a case where a firm offers below marginal cost at bus e on the exporting side of a constraint in order to congest the line and consequently be able to offer well above marginal cost at a bus m on the importing side. That is, $-\sum_{k\in R} q_k \frac{\partial p_{-k}}{\partial q_e}(q)$ is negative, while $-\sum_{k\in R} q_k \frac{\partial p_{-k}}{\partial q_m}(q)$ is significantly positive. The mark-up at each bus considered separately does not, in this case, give a full picture of the situation faced by a firm. However, the index (2) combines the effect of mark-up at all buses.

To summarize, the TCMPI proposed here is the quantity-weighted average of estimated price-cost mark-ups of firm i in (2). This estimate assumes that the firm is maximizing its profits and that the firm can evaluate the residual demand it faces. As discussed in [8], even given these assumptions, these must be viewed as only approximate estimates of mark-up over competitive prices, since competitive prices at each bus m may deviate from the marginal costs $c'_m(q_m)$ at the market clearing conditions. To estimate the mark-ups above competitive prices more precisely, it is necessary to consider the generator capacity constraints, which will be discussed in the following two sections, 2.1.2 and 2.1.3.

2.1.2 Considering binding generator capacity constraints at market clearing conditions

To consider the case of generators at their maximum capacity at market clearing conditions, partition the generators owned by firm i into generators that are:

- at their maximum capacity, denoted by subscript f for fixed, and
- marginal, denoted by subscript v for variable.

The mark-up of each generator m owned by firm i that is marginal is still given by (1). Collect the mark-ups of the marginal generators owned by firm i into the vector Δp_{v}. Writing p_{v} and p_{f} for the vector functions representing the prices at the marginal and maximum capacity generators and using the symmetry of $\frac{\partial p}{\partial q}$, we have:

$$\Delta p_{\mathrm{v}} = -\frac{\partial p_{\mathrm{v}}}{\partial q}(q)q. \tag{3}$$

In contrast, the mark-up, Δp_{f}, of the generators at their maximum capacity will not satisfy the condition (1). Moreover, there would be non-zero "mark-up" of prices for such generators even with competitive offers since generators at their maximum capacity receive infra-marginal rents. This "mark-up" with competitive offers does not represent excess transfer above competitive levels. Nevertheless, above-competitive offers by the marginal generators owned by firm i would result in even higher mark-ups at the other generators and we will estimate this effect to estimate the mark-up above competitive prices for the generators at maximum capacity. We will not attempt to estimate the higher mark-up of firm i's generators at maximum capacity that is due to other firms' actions.

In particular, for the generators owned by firm i that are at maximum capacity, we will estimate the mark-up Δp_{f} of prices at their buses that is due to the mark-ups at the marginal generators owned by firm i. We will estimate Δp_{f} by considering the change Δq_{v} at the marginal generators of firm i that would result in changing the prices at those buses by Δp_{v}. By definition of the derivative, we have that:

$$\begin{bmatrix} \Delta p_{\mathrm{f}} \\ \Delta p_{\mathrm{v}} \end{bmatrix} \approx \begin{bmatrix} \dfrac{\partial p_{\mathrm{f}}}{\partial q_{\mathrm{v}}}(q) \\ \dfrac{\partial p_{\mathrm{v}}}{\partial q_{\mathrm{v}}}(q) \end{bmatrix} \Delta q_{\mathrm{v}}. \tag{4}$$

To eliminate Δq_{v} from this expression, first assume that $\dfrac{\partial p_{\mathrm{v}}}{\partial q_{\mathrm{v}}}(q)$ invertible and ignore capacity constraints of generators that are marginal at the market clearing conditions. We have that:

$$\Delta p_{\mathrm{f}} \approx \frac{\partial p_{\mathrm{f}}}{\partial q_{\mathrm{v}}}(q) \left[\frac{\partial p_{\mathrm{v}}}{\partial q_{\mathrm{v}}}(q) \right]^{-1} \Delta p_{\mathrm{v}}.$$

Combining with (3), we obtain:

$$\begin{bmatrix} \Delta p_{\mathrm{f}} \\ \Delta p_{\mathrm{v}} \end{bmatrix} \approx - \begin{bmatrix} \dfrac{\partial p_{\mathrm{f}}}{\partial q_{\mathrm{v}}}(q) \left[\dfrac{\partial p_{\mathrm{v}}}{\partial q_{\mathrm{v}}}(q) \right]^{-1} \dfrac{\partial p_{\mathrm{v}}}{\partial q_{\mathrm{f}}}(q) & \dfrac{\partial p_{\mathrm{f}}}{\partial q_{\mathrm{v}}}(q) \\ \dfrac{\partial p_{\mathrm{v}}}{\partial q_{\mathrm{f}}}(q) & \dfrac{\partial p_{\mathrm{v}}}{\partial q_{\mathrm{v}}}(q) \end{bmatrix} q.$$

The estimated quantity-weighted average mark-up above competitive prices is then:

$$\frac{-q^{\dagger} \begin{bmatrix} \dfrac{\partial p_{\mathrm{f}}}{\partial q_{\mathrm{v}}}(q) \left[\dfrac{\partial p_{\mathrm{v}}}{\partial q_{\mathrm{v}}}(q) \right]^{-1} \dfrac{\partial p_{\mathrm{v}}}{\partial q_{\mathrm{f}}}(q) & \dfrac{\partial p_{\mathrm{f}}}{\partial q_{\mathrm{v}}}(q) \\ \dfrac{\partial p_{\mathrm{v}}}{\partial q_{\mathrm{f}}}(q) & \dfrac{\partial p_{\mathrm{v}}}{\partial q_{\mathrm{v}}}(q) \end{bmatrix} q}{\mathbf{1}^{\dagger} q}. \tag{5}$$

However, $\dfrac{\partial p_{\mathrm{v}}}{\partial q_{\mathrm{v}}}(q)$ can be singular. This occurs, for example, if multiple marginal generators are located at the same bus and therefore have the same shift factors with respect to all binding constraints, or if the number of marginal generators owned by firm i is greater than the number of binding constraints. In this case, we use the analogous expression to (5) that utilizes the pseudo-inverse of $\dfrac{\partial p_{\mathrm{v}}}{\partial q_{\mathrm{v}}}(q)$. Using the pseudo-inverse results in a proxy to

Δq_{v} that has the least Euclidean norm, which acts to under-estimate the profit maximizing generation weighted mark-up.

Again, the TCMPI in (5) may also act to over-estimate the mark-up above competitive prices to the extent that competitive prices deviate from the marginal costs at the market clearing conditions. Note that all generators are assumed to reveal their true capacity. In the particular case where all of firm i's generators are at maximum capacity, we define the index to be zero. As mentioned above, this ignores the effect of other firms on the mark-up of firm i, which acts to under-estimate the mark-up above competitive prices in this case. On the other hand, by ignoring capacity constraint of generators that are marginal at the market clearing condition, the index over-estimate the actual mark-up over competitive conditions.

2.1.3 Considering binding generator capacity constraints at competitive conditions

In (4), Δq_{v}, the estimated generation output deviation from the case that firm i behaves competitively, does not consider the generation capacity constraints at the competitive condition:

$$\underline{q}_{\mathrm{v}} \leq q_{\mathrm{v}} - \Delta q_{\mathrm{v}} \leq \overline{q}_{\mathrm{v}}, \tag{6}$$

where $q_{\mathrm{v}} - \Delta q_{\mathrm{v}}$ represents the estimated production quantities at the competitive condition, and $\overline{q}_{\mathrm{v}}, \underline{q}_{\mathrm{v}}$ are the vectors of maximum and minimum generation capacities. Let V be the set of generators owned by firm i that are marginal at the market clearing condition. Also define V^{f} and V^{v} as the sets of binding and marginal generators at the competitive condition, respectively. Note that $V^{\mathrm{f}}, V^{\mathrm{v}} \subseteq V$ and $V^{\mathrm{f}} = V \setminus V^{\mathrm{v}}$. Assuming sets $V^{\mathrm{f}}, V^{\mathrm{v}}$ are known, we partition generators in V into generators that are:

- in set V^{f}, denoted by subscript vf,
- in set V^{v}, denoted by subscript vv.

Collect the mark-ups of the generators in sets V^{f} and V^{v} into vectors Δp_{vf} and Δp_{vv}, respectively. Also, collect the production quantities deviated from the competitive condition of generators in set V^{f} and V^{v} into vectors Δq_{vf} and Δq_{vv}, respectively. By definition of the derivative, we have that:

$$\begin{bmatrix} \Delta p_{\mathrm{vf}} \\ \Delta p_{\mathrm{vv}} \end{bmatrix} \approx \begin{bmatrix} \dfrac{\partial p_{\mathrm{vf}}}{\partial q_{\mathrm{vf}}}(q) & \dfrac{\partial p_{\mathrm{vf}}}{\partial q_{\mathrm{vv}}}(q) \\ \dfrac{\partial p_{\mathrm{vv}}}{\partial q_{\mathrm{vf}}}(q) & \dfrac{\partial p_{\mathrm{vv}}}{\partial q_{\mathrm{vv}}}(q) \end{bmatrix} \begin{bmatrix} \Delta q_{\mathrm{vf}} \\ \Delta q_{\mathrm{vv}} \end{bmatrix} \tag{7}$$

By definition of set V^{f}, we have

$$\Delta q_{\mathrm{vf}} = q_{\mathrm{vf}} - \overline{q}_{\mathrm{vf}}. \tag{8}$$

Combining with (7) and re-arranging, we obtain:

$$\Delta q_{\mathrm{vv}} \approx \left[\frac{\partial p_{\mathrm{vv}}}{\partial q_{\mathrm{vv}}}(q)\right]^{-1} \left(\Delta p_{\mathrm{vv}} - \frac{\partial p_{\mathrm{vv}}}{\partial q_{\mathrm{vf}}}(q)(q_{\mathrm{vf}} - \overline{q}_{\mathrm{vf}})\right), \tag{9}$$

where Δp_{vv} is the estimated price-cost mark-up given by (1). Using the symmetry of $\dfrac{\partial p}{\partial q}$, we have

$$\Delta p_{\text{vv}} = -\frac{\partial p_{\text{vv}}}{\partial q}(q)q. \tag{10}$$

Again, the pseudo-inverse can be utilized in the case that $\dfrac{\partial p_{\text{vv}}}{\partial q_{\text{vv}}}(q)$ is singular, as discussed in 2.1.2. Combining (7) and (9), the estimated mark-ups above competitive prices for generators in set V^{f} are:

$$\Delta p_{\text{vf}} \approx \frac{\partial p_{\text{vf}}}{\partial q_{\text{vf}}}(q)\Delta q_{\text{vf}} + \frac{\partial p_{\text{vf}}}{\partial q_{\text{vv}}}(q)\left[\frac{\partial p_{\text{vv}}}{\partial q_{\text{vv}}}(q)\right]^{-1}\left(\Delta p_{\text{vv}} - \frac{\partial p_{\text{vv}}}{\partial q_{\text{vf}}}(q)(q_{\text{vf}} - \overline{q}_{\text{vf}})\right). \tag{11}$$

Note that for generators in set V^{f}, we no longer use the price-cost mark-ups given by (1), since the price-cost mark-ups might significantly over-estimate the mark-ups above competitive prices for the generators that would be binding at competitive conditions. Furthermore, according to (4), the estimated mark-ups above competitive prices for generators that are binding at the market clearing condition are:

$$\Delta p_{\text{f}} \approx \begin{bmatrix} \dfrac{\partial p_{\text{f}}}{\partial q_{\text{vf}}}(q) & \dfrac{\partial p_{\text{f}}}{\partial q_{\text{vv}}}(q) \end{bmatrix} \begin{bmatrix} \Delta q_{\text{vf}} \\ \Delta q_{\text{vv}} \end{bmatrix}, \tag{12}$$

where Δq_{vf} and Δq_{vv} are given by (8) and (9), respectively. The estimated quantity-weighted average mark-up above competitive price is then:

$$\frac{\begin{bmatrix} q_{\text{vf}}{}^{\dagger} & q_{\text{vv}}{}^{\dagger} & q_{\text{f}}{}^{\dagger} \end{bmatrix} \begin{bmatrix} \Delta p_{\text{vf}} \\ \Delta p_{\text{vv}} \\ \Delta p_{\text{f}} \end{bmatrix}}{\mathbf{1}^{\dagger} q} \tag{13}$$

The analysis above is based on the assumption that the set V^{f} is known. However, this information is usually not available as a priori. Therefore, a procedure is developed to conjecture the set V^{f}. The basic idea is to iteratively calculate Δq_{vv} based on (9) and move generators violating capacity constraints from V^{v} to V^{f}, until that capacity constraints (6) are satisfied for all generators in V. Algorithm 1 describes the main steps of this procedure.

In this algorithm, at least one generator is added to set V^{f} at each iteration, or otherwise the algorithm terminates. Therefore, the above algorithm is guaranteed to be terminated in no more than $|V|$ iterations. Minimum production constraints can also be included in the same manner to deal with the case that the estimated generation outputs at competitive conditions are lower than their minimum production levels. Note that this procedure only provides a conjecture of the set V^{f}, which might differ from the actual set of binding generators at the competitive condition. Also, since the estimated price-cost mark-ups for generators in V^{v} are used to approximate the mark-ups above competitive prices, the index might still over-estimate the mark-ups above competitive levels, particularly under the case that the derivatives of marginal costs are greater than zero.

Algorithm 1 Find binding generators at competitive conditions

Input: $\dfrac{\partial p_{\text{v}}}{\partial q_{\text{v}}}(q), q_{\text{v}}, \overline{q}_{\text{v}}$
Output: V^{f}
$V^{\text{f}} \leftarrow \emptyset$, $V^{\text{v}} \leftarrow V$
repeat
  $T \leftarrow \emptyset$
  Calculate Δq_{vv} using equation (9)
  for each i in V^{v} **do**
    if $q_i - \Delta q_i > \overline{q}_i$ **then**
      $T \leftarrow T \bigcup \{i\}$
    end if
  end for
  $V^{\text{f}} \leftarrow V^{\text{f}} \bigcup T$, $V^{\text{v}} \leftarrow V^{\text{v}} \setminus T$
until $V^{\text{f}} = V$ or $T = \emptyset$

2.2 Evaluation of market power index

The key to evaluating the TCMPI is to efficiently evaluate the inverse residual demand Jacobian $\dfrac{\partial p}{\partial q}$. Reference [9] describes the calculation of residual demand derivative when there are binding transmission constraints, using sensitivity analysis of market clearing conditions, and [11] extends the methodology to calculate the inverse residual demand Jacobian matrix faced by a firm. Given the following information, the residual demand Jacobian can be evaluated efficiently:[1]

- Binding transmission constraints and their shift factors for injection at each generator,
- Offer information, and
- Market clearing quantities.

Once the residual demand Jacobian is evaluated, we can substitute it into (5) to evaluate the TCMPI.

2.3 Implementation

We implemented the calculation of the TCMPI using the same framework as in [8]. We used PowerWorld as the OPF solver and visualization tool, and implemented the index calculation in MATLAB. The flow chart is shown in Figure 1. We have successfully calculated and

[1] In many markets, offers are piecewise constant. The residual demand derivative implicitly assumes that the offer is differentiable. From a practical perspective, the transmission constrained inverse residual demand derivative is estimating the change in prices for a small change in injection. Consequently, the average slope of offers over a suitable small change in production is utilized, as discussed in [8, §IV-D].

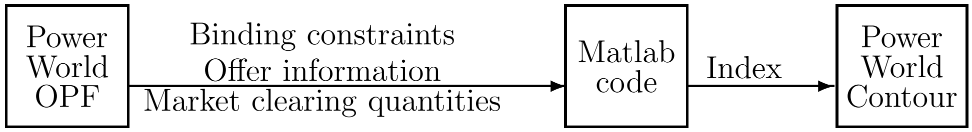


Figure 1: Flow chart of tool design. (Source: Figure 2 of [8].)

visualized the index for medium- and large-scale systems, as will be shown in Sections 3.3 and 3.4.

3 Case Studies of Firm-Based Transmission-Constrained Market Power Index

3.1 Single Firm, no transmission constraints

This example is aimed to show the importance of considering binding capacity constraints at competitive conditions as discussed in 2.1.3. A single firm owning two generators, a and b, faces a residual demand $D - p$ MW, where D is a parameter characterizing overall demand level and p is the market clearing price. The marginal costs of generator a and b are $10 + 0.1q_a$ \$/MWh and $10 + 0.2q_b$ \$/MWh, respectively, where q_a and q_b are the production quantities. For various values of D, the market clearing prices are evaluated for two cases:

- The generation firm maximizes its profit by selecting optimal production quantities, represented by the blue line in Figure 2, and
- The generation firm behaves competitively by using marginal costs as offers, represented by the green dashed line in Figure 2.

Also, two indices are evaluated for each D based on the profit-maximizing case:

- The quantity-weighted average mark-ups considering binding capacity constraints at competitive conditions based on (13), represented by the red dashed line in Figure 3, and
- The quantity-weighted average mark-ups ignoring capacity constraints at competitive conditions based on (5), represented by the blue dashed line in Figure 3.

Note that in this particular example, the estimated quantity-weighted mark-up is equal to the estimated mark-up of either generator since no transmission constraints are presented. When $D > 230$, both generators become binding at the competitive condition. However, generator b is still marginal in the profit-maximizing case for $D > 230$. In this scenario, as shown in Figure 3, the estimated mark-up given by (5) significantly over-estimates the mark-up above the competitive price, while the estimated mark-up according to (13) corrects this inaccuracy and produces an estimation that is much more closer to the actual mark-up above the competitive price.

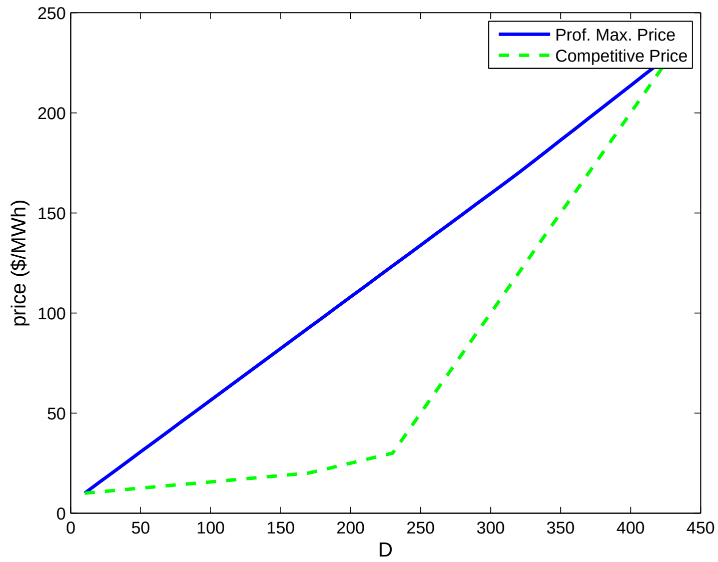


Figure 2: Profit maximization and competitive price for single firm, no transmission constraint example.

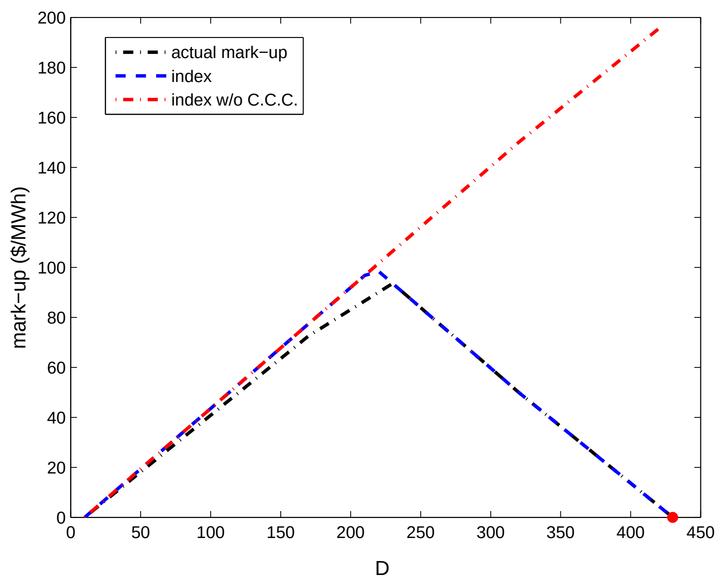


Figure 3: Actual mark-ups and estimated mark-ups for single firm, no transmission constraint example.

Table 1: Actual and Estimated Mark-ups

Gen#	q	q_{comp}	$\overline{q}$	mark-up		
				actual	estimation [a]	estimation w/o c.c.c [b]
1	77.75	105.23	162.5	6.69	9.98	9.98
2	58.52	78	78	0.32	-0.08	14.22
3	65	65	65	-0.21	-1.02	20.81
4	45.5	45.5	45.5	-0.38	-1.32	22.89
5	97.5	97.5	97.5	0.61	0.41	9.94
6	45.5	45.5	45.5	1.57	4.83	5.95
7	65	65	65	1.61	5.04	5.86
8	59.91	80.43	97.5	1.38	3.83	6.30
9	36.57	71.81	162.5	1.61	5.04	5.83
quantity-weighted average index				1.60	2.99	11.61

[a] Estimated mark-ups considering capacity constraints at competitive conditions.

[b] Estimated mark-ups ignoring capacity constraints at competitive conditions.

3.2 Single firm in 118 Bus Reliability Test System

This example uses the IEEE 118 Bus Reliability Test System to demonstrate that the market power index can produce good estimation to the actual mark-ups above competitive prices. Assume there is one firm which maximizes its profit by choosing the optimal production quantities given all other suppliers' offers. The profit-maximizing strategies and corresponding market-clearing prices are computed using a mathematical program with equilibrium constraints (MPEC) [10]. Given the profit-maximizing production quantities of the firm and the corresponding market-clearing information, we compute the estimated mark-ups using (5) and (13). Table 1 shows the results, where q represents the profit-maximizing production quantities for the firm, q_{comp} represents the production quantities in the competitive condition, and $\overline{q}$ represents the maximum capacities of each generator. The minimum capacities are all zero. The actual mark-ups are the profit-maximizing prices minus the competitive prices and are shown in the fifth column, with two estimates in the last two columns. As shown in the Table 1, the estimated mark-ups are reasonably good estimations of the actual mark-ups. The discrepancies in this example are mainly due to non-zero derivatives of marginal costs. That is, for marginal generators, the price-cost mark-ups over-estimate the mark-ups above competitive prices, as discussed in 2.1.3. Note that the estimated mark-ups given by (5), which ignores capacity constraints at the competitive condition, significantly over-estimate the actual mark-ups. Again, this example shows that it is necessary to consider capacity constraints at the competitive condition to produce more plausible estimations of mark-ups.

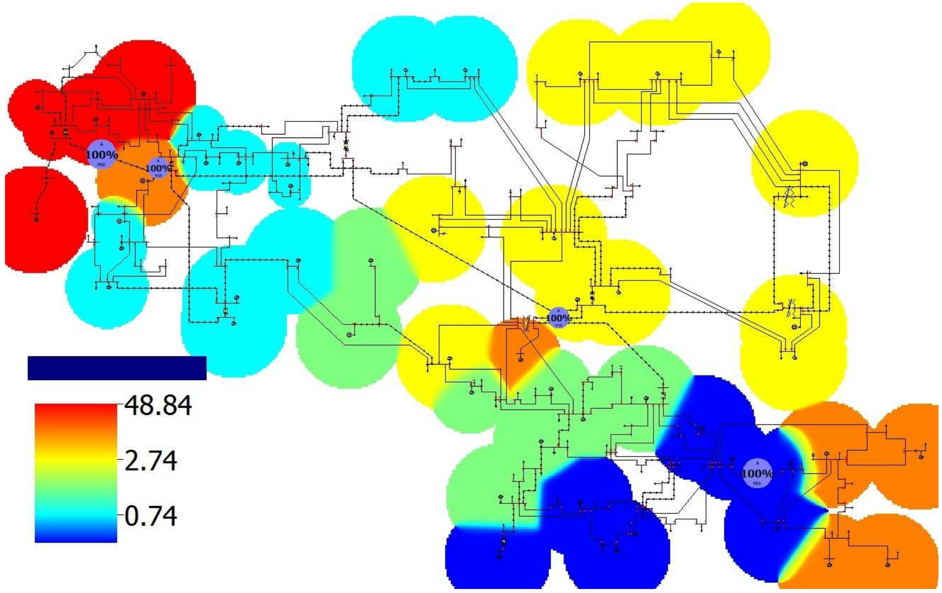


Figure 4: Contour map of transmission-constrained market power index for the 118 bus Reliability Test System. The units are \$/MWh.

3.3 Multiple firms in 118 Bus Reliability Test System

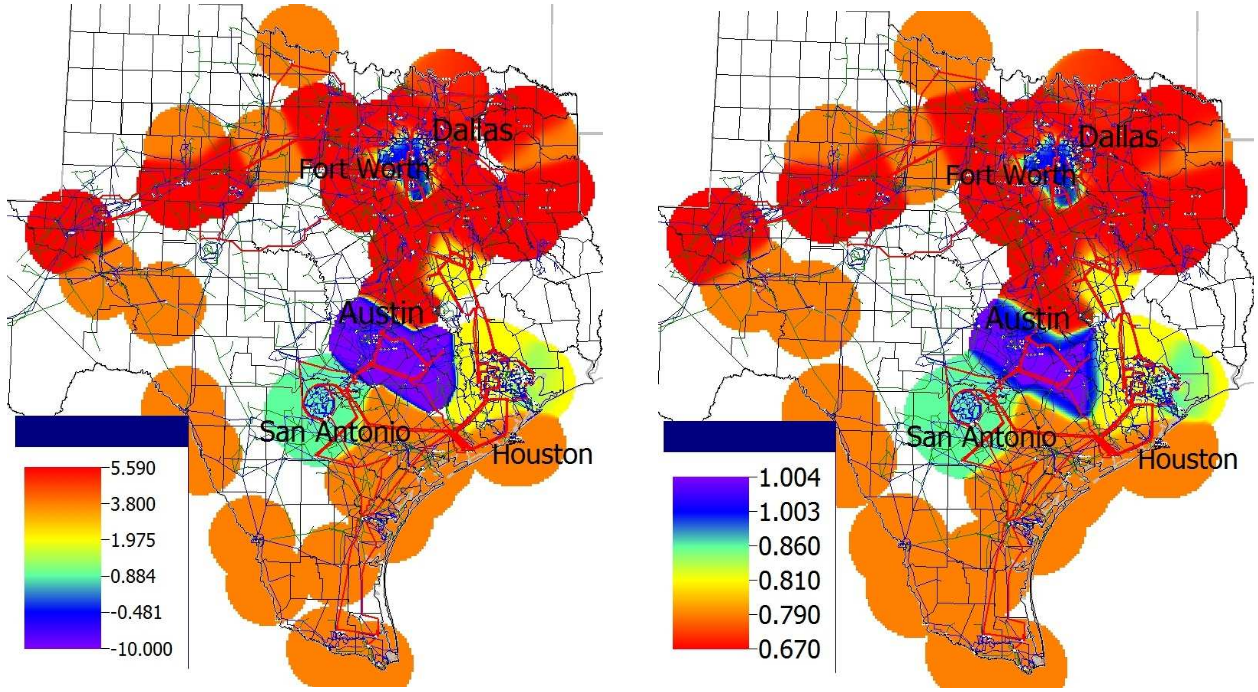
Power flow data together with generic generator offers and arbitrarily defined ownership data for the 118 bus Reliability Test System [15] are used for calculating the index. We assume the generators belong to six different owners, firms A,...,F, and each firm owns 5 to 13 generators. Note that the power flow data and generator offers are the same set of data as used in [8, §V.A].

Figure 4 shows the contours of the quantity-weighted average index according to (13). All of the generators owned by the same firm are assigned the estimated quantity-weighted average for the firm and are displayed in the same color. Note that there are four binding transmission constraints in the system, which are shown as the light blue pie charts.

The generators in the red contour areas are all owned by firm A, which has the largest estimated mark-up. This is due to one of the binding transmission constraints limiting power imports to the red contour areas and making it less competitive. Interestingly, we will show in Section 5.2 that this firm is actually a pivotal firm considering the effects of transmission constraints. All the other five firms have fairly small mark-ups and are all displayed in different colors.[2]

Comparing Figure 4 with Figure 5 in [8], it is clear that the generator-based market power indices proposed in [8] are not sufficient to analyze the behavior of a firm that owns generators at more than one location. In this case study, firm A, a pivotal firm as suggested

[2]There are several narrow stripes of yellow contours, such as the one between the aqua and orange contours in the middle of the figure. They are due to blending of colors and do not represent another firm.



(a) Common log of transmission-constrained market power index in $/MWh, except for purple contour, which has index of zero.

(b) Transmission-constrained residual supply index.

Figure 5: Contour maps of two different market power analyses of the ERCOT system.

in Section 5.2, has great potential to drive up the prices above the competitive levels. With the firm-based analysis proposed in this report, the estimated average mark-up of firm A is 48.84 $/MWh, which indicates that firm A should be flagged as being of concern regarding exercising market power.

In contrast, the individual generator analysis in [8] suggests that all generators owned by firm A have estimated mark-ups less than 3 $/MWh, which are fairly small. In [8], the residual demand is based on all generation offers other than the generator to be analyzed. However, to analyze the profit-maximizing incentives for a firm, the offers from all the generators owned by the firm should be excluded from the residual demand calculation, since the firm can control the behavior of all its generators to maximize its profit. Therefore, the firm-based TCMPI is a more suitable tool to analyze market power of a firm.

3.4 ERCOT system

Power flow data together with generic generator offers and ownership and control data for the ERCOT system for summer peak demand are used for calculating the index. Note that the power flow data and generator offers are the same set of data used in [8, §V.B]. We calculated the index based on security-constrained OPF (SCOPF) solution, assuming all generators in-service. There are 19 binding transmission constraints in the system, with three of them being pre-contingency constraints. Figure 5(a) shows the contours of estimated quantity-weighted average index according to (13). The contours represent the common log of the index. Again, all generators owned by a given firm are assigned the estimated quantity-weighted average for the firm and are displayed in the same color. For clarity, only the six largest firms in terms of capacity share are shown in the figure. The firm with the largest index is also the biggest firm in terms of capacity share, which is about 20%. The generators owned or controlled by this firm are shown in red contours in the figure and are distributed in several areas in ERCOT. Note that the firm represented by purple contours has estimated mark-up of 0, since all of its generators are at full capacity. In the figure, we use a small number, 10^{-10}, to represent this mark-up, since common log of 0 is not defined.

The result shown in Figure 5(a) suggests that the market is much less competitive compared to the case in [8, §V.B], where all generators are assumed to be owned by different firms. In [8], only five generators have estimated mark-ups higher than 1000 \$/MWh using generator-based market power indices. However, with the firm-based analysis, four firms, with 103 generators, have average estimated mark-ups higher than 1000 \$/MWh.

4 Transmission-Constrained Residual Supply Index

In the absence of binding transmission constraints, the RSI of firm i is defined as follows:

$$\text{RSI} = \frac{\text{total available supply} - \text{available supply from firm } i}{\text{demand}}$$

The RSI measures the maximum available supply divided by the demand, without any supply from firm i. When the RSI is less than 1, firm i is said to be "pivotal" and has significant potential market power. On the other hand, a larger RSI value implies that firm i has less potential market power. In [1], empirical evidence of negative correlation between the RSI and price-cost mark-up was provided, which indicates that the RSI might be a useful index to predict market power. Newbery [16] concludes that the RSI is particularly useful for the case of a single dominant firm or symmetric oligopoly.

However, the basic RSI definition ignores the impact of transmission constraints. In the presence of transmission constraints, a firm might be pivotal in particular geographic areas even if from a region-wide perspective ignoring transmission constraints there is no pivotal firm. CAISO uses an RSI-like approach [17] to assess the competitiveness of transmission constraints: it evaluates the extent to which a firm's supply is necessary to produce counter-flow that relieves congestion on a particular constraint. In addition, CAISO applies "Competitive Path Assessment" [12] to identify pivotal suppliers under transmission constraints by repeatedly solving multiple OPFs that successively omit each given firm's supply. This approach evaluates the shortfall in transmission capacity to meet demand and might be computationally expensive because of the need to solve an OPF for each firm.

In order to generalize the original definition of the RSI into the context of transmission constraints, but also provide a computationally efficient procedure, we developed a new index, the transmission-constrained residual supply index (TCRSI). Unlike the approaches adopted by CAISO, the TCRSI directly generalizes the original definition of the RSI while considering all constraints simultaneously. The TCRSI can measure the extent to which a firm's supply is necessary to meet demand including the effects of transmission constraints. Generalizing the RSI, the TCRSI of firm i evaluates the residual supply at each load bus, in the absence of supply from firm i. As will be discussed in the next section, the TCRSI is represented as a single parameter that scales each load throughout the system conformally, and can be evaluated by solving a linear programming (LP) problem if the transmission constraints are represented using DC power flow. The TCRSI can be computed efficiently by utilizing the dual simplex method, which will be further discussed in section 4.3.

4.1 Definition of transmission-constrained residual supply index

Suppose that there are n generators, ℓ loads, and m transmission constraints. Variables $q \in \mathbb{R}^n$ represent the output quantity of each generator while $d \in \mathbb{R}^\ell$ is the vector composed of load quantities, which is given as problem data. The matrix $H_g \in \mathbb{R}^{m\times n}$ consists of the shift factors for injection at each generator corresponding to each transmission constraint; similarly, $H_d \in \mathbb{R}^{m\times \ell}$ is the matrix of shift factors for injection at each load bus corresponding to each transmission constraint; and $b \in \mathbb{R}^m$ represents the limits for the transmission constraints. We assume that $\overline{q}_j, j = 1, \ldots, n$, is the maximum available supply of each generator j. We again assume that R is the set of generators owned by firm i.

We define the TCRSI LP, which is used to calculate TCRSI for firm i, as follows:

$$\max_{q,t} \quad t \tag{14a}$$

$$\text{subject to} \quad \mathbf{1}^\dagger q - (\mathbf{1}^\dagger d)t = 0 \tag{14b}$$

$$H_g q - H_d(dt) \leq b \tag{14c}$$

$$0 \leq q_j \leq \overline{q}_j, j \notin R \tag{14d}$$

$$q_j = 0, j \in R \tag{14e}$$

where (14b) is the power balance constraint, (14c) are transmission constraints, (14d) are maximum supply constraints while (14e) remove the supply of firm i. With transmission constraints and maximum supply constraints, the above LP finds the maximum load $(\mathbf{1}^\dagger d)t$ that the system could meet after removing all available supply from firm i; in other words, the LP maximizes the residual supply in the absence of firm i assuming demand is scaled conformally.

Definition 1 *The TCRSI for the firm i is the optimal value of the linear program TCRSI LP, as specified in* (14)*.*

If the TCRSI is less than 1, it implies that there is insufficient supply to satisfy all the loads in the system without the supply from firm i; that is to say, firm i is pivotal. When the TCRSI is greater than 1, the system has more than enough generation resources to satisfy all demand even without any supply from firm i. This observation is analogous to the interpretation of RSI.

Note that without the transmission constraints (14c), the optimal value of TCRSI LP evaluates the RSI. Therefore, the RSI can be interpreted as the optimal value of the relaxed TCRSI LP. Thus for a given firm i, the value of TCRSI is always less than or equal to the RSI. If a market monitor uses the TCRSI and RSI as screening tools for market power to flag the firms whose index is less than a given threshold, the firms that are flagged using the TCRSI would be a superset of firms flagged by the RSI. In other words, the TCRSI is a more comprehensive market screening tool since it flags all the firms that the RSI detects as pivotal, plus some firms that are pivotal due to transmission constraints that the RSI fails to discover.

The reason for using a single parameter to scale each individual load conformally is that loads are distributed throughout the system with certain geographical patterns. For example, it is usually the case that metropolitan areas have significantly higher demand than rural areas. This pattern should be kept while we search for the maximum demand that the system could provide. On the contrary, if each load is modeled as a free variable, the optimal solution might not represent the geographical distribution of the load in the system. This issue will be further discussed in Section 5.1 with reference to a small four bus example system.

4.2 Implications of transmission-constrained residual supply index

Unlike the TCMPI analysis, the TCRSI does not directly model the incentives for a firm to drive the prices above competitive levels. Nevertheless, as discussed in the first paragraph in Section 4, this type of *ex ante* analysis, which measures the extent to which a firm's supply is necessary to meet demand, might provide some useful insights concerning market power.

If the TCRSI of firm i is less than 1, it implies that the system cannot meet all demand without at least some supply from firm i. Assuming offers from other market participants are fixed, firm i could submit its generation offers at any arbitrarily high prices and at least some of its offered supply is guaranteed to be accepted by the ISO, given that the system has to satisfy all demand, all transmission constraints must be satisfied, and there exists no demand elasticity. As a result, a TCRSI value that is less than 1 implies an absolute potential of exercising market power and should be flagged for further analysis.

In the case that the TCRSI of firm i is equal to or slightly greater than 1, the system can just meet all demand without the supply from firm i. However, the operator might need to dispatch some expensive generation resources. In this situation, firm i can offer strategically, resulting in its generators being dispatched at prices much higher than competitive levels.

4.3 Implementation

Section 4.1 defined the TCRSI of a given firm, which is derived from solving a LP problem. To calculate the TCRSIs for all the market participants, multiple TCRSI LPs have to be

Table 2: Computational performance.

Test case	Method	CPU Time[a]	Iterations[b]
118 Bus RTS[c]	Cold start	0.07	419
	Warm start	0.06	417
ERCOT[d]	Cold start	180.3	38983
	Warm start	41.5	11310

[a]The units are second. Based on 3.0 GHz CPU, 16GB RAM Linux workstation. Note that for the ERCOT case, the CPU time does not include the time for solving SCOPF.

[b]Number of total simplex iterations.

[c]The Reliability Test System has 118 buses, 54 generators, 91 loads, and 194 transmission constraints.

[d]The ERCOT system has 5526 buses, 3518 loads, 564 generators, and 6836 transmission constraints.

solved. The TCRSI LP problem (14) has a similar structure to an optimal power flow problem (OPF), except for the difference in the objective function. Solving an OPF is computationally expensive if the system size is large. Therefore, an efficient implementation is desired.

The only changes between the TCRSI LP formulation (14) for different firms are changes in the generators' maximum available supply. More specifically, a different set of generators' maximum available supplies are suppressed to 0 when formulating the TCRSI LP for each different market participant. This characteristic can be exploited by using the dual simplex method to solve all the TCRSI LPs starting from a dual feasible solution. Once the TCRSI LP is solved for a given firm, we keep the basis of the current LP, change the right-hand sides of the constraints and continue the dual simplex method until optimality conditions are reached. This technique is called "warm start" and is supported by most LP solvers[3].

We used Gurobi 2.0 as the LP solver and wrote the code in the Python language to calculate the TCRSI. A small four bus example is first developed to provide a comparison between RSI and TCRSI. Then the TCRSI is also evaluated for the two test cases from Sections 3.3 and 3.4 and Table 2 shows the results of computational performance for these two test cases. We observe that the computational efficiency for solving a large system is significantly improved by using warm start, compared to solving TCRSI LP from scratch for each firm, labeled "Cold start" in Table 2.

[3]In principle, the same approach could also be used to speed up calculations in the CAISO "Competitive Path Assessment" approach.

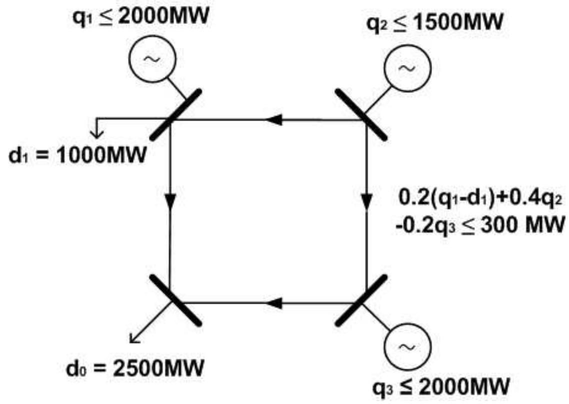


Figure 6: One-line diagram of the four bus system.

Table 3: TCRSI and RSI of the four bus system.

Firm	TCRSI	RSI
g_1	1	1
g_2	1.14	1.14
g_3	0.58	1

5 Case Studies of the TCRSI

5.1 4 Bus System

We use a simple four bus system to illustrate why the TCRSI is a more suitable index than the RSI in the context of transmission constraints. Figure 6 shows the one-line diagram of the four bus system. There are three generators and two loads in the network. Suppose that each generator is owned by a different firm and the subscripts of generators and loads also represent the bus where they are located. Assume that the transmission line from bus 2 to 3 has a thermal limit of 300MW, while the other lines have sufficiently large thermal limits so that the associated transmission constraints are never binding.

Table 3 shows the TCRSI and RSI of each generator. The RSI analysis implies that there is no pivotal firm, since the RSI of each firm is greater than or equal to 1. However, the TCRSI of generator g_3 is 0.58 and indicates that g_3 is a pivotal supplier. Without the supply from g_3, the maximum conformal loads the system could serve are 1458.3MW at d_0 and 583.3MW at d_1, due to the congested line from bus 2 to bus 3.

To further motivate the TCRSI index as defined in (14), consider the following alternative formulation. Instead of scaling load conformally as in (14b), suppose that we had modeled each load as an independent variable and then maximized the total demand without the supply of each given firm. In the case of omitting the supply from generator g_3, the resulting loads at buses 0 and 1 would be $d_0 = 0$MW and $d_1 = 3500$MW, respectively. This

Table 4: TCRSI and RSI of the 118 Bus Reliability Test system.

Firm	TCRSI	RSI	Capacity Share[a]
A	0.61	3.30	0.09
B	1.00	3.10	0.14
C	0.85	2.83	0.22
D	1.48	3.17	0.13
E	1.23	3.06	0.15
F	1.39	2.76	0.24

[a]capacity share = firm's total capacity/total capacity in the system

alternative modeling distorts the original geographical pattern of demand and would give the false impression that g_3 were not pivotal. In this example, generator g_3 is in fact pivotal due to its particular situation with respect to transmission constraints. The TCRSI reveals that g_3 has great potential for exercising market power since it can offer its production at arbitrarily high prices and consequently drive up the market clearing prices.

5.2 118 Bus Reliability Test System

We evaluated the TCRSI for the 118 bus Reliability Test System with the same data set used in Section 3.3. Table 4 shows the TCRSI, the RSI, and the capacity share of each firm. We assume the available supply of each generator is equal to its capacity. Ignoring transmission constraints, the system has an abundance of capacity to satisfy demand since the RSI of all firms are higher than 2.5. According to the RSI, there are no pivotal firms in the system. However, the TCRSI indicates that firms A and C are actually pivotal considering transmission constraints. Interestingly, firm A, which has the smallest TCRSI value, also has the largest RSI value and the least capacity share. This example again demonstrates that RSI analysis is insufficient to indicate the pivotal firms in the context of transmission constraints. Note that firm A also has largest estimated mark-up according to the analysis in Section 3.3.

5.3 ERCOT system

We evaluated the TCRSI for the ERCOT system with the same data set used in Section 3.4 and assuming that the capacity of each generator represents its available supply. In addition to all the pre-contingency transmission constraints, the binding and near binding post-contingency constraints from the SCOPF results are also incorporated into the analysis.[4] Table 5 lists the six firms with the largest capacity share. The RSI analysis indicates that only one firm is pivotal, while the TCRSI reveals that four market participants are pivotal

[4]That is, we potentially omit some actually binding post-contingency constraints for some firms. This means that the values we obtain may over-estimate the actual TCRSI.

Table 5: TCRSI and RSI for selected firms in the ERCOT System.

Firm	TCRSI	RSI	Capacity Share
A	0.668	0.976	0.199
B	0.793	1.151	0.056
C	0.809	1.077	0.116
D	0.859	1.158	0.050
E	1.003	1.184	0.028
F	1.004	1.184	0.029

firms due to the effect of transmission constraints. Note that firms B and D have a relatively small capacity share, compared to firms A and C. However, the location of generators owned by firms B and D causes them to be pivotal suppliers.

Figure 5(b) shows the contour map of the TCRSI for the ERCOT system. In both Figures 5(a) and 5(b), the generators owned by firm A are shown in red contours: firm A has the smallest TCRSI and also the largest estimated mark-up among all the firms. We can also observe that among the six selected firms, the firms with larger index according to (5) also have lower TCRSI. However, this observation is not always true for other firms in the system, which will be further discussed in section 6.

6 Comparison of Indices

Two different categories of firm-based transmission-constrained market power assessment tools have been proposed in this report. The TCMPI involves *ex post* analysis, that is, it depends on offer information and the OPF results. Based on this information, the index estimates the quantity-weighted average mark-up above competitive levels for a given firm. A firm should be flagged as of potential concern if its index value is high.

In contrast, the TCRSI is an *ex ante* analysis, which does not require generation offers nor the OPF results, and estimates the extent to which a firm's supply is necessary to meet the demand under transmission constraints. Note that the TCRSI does not depend on the behavior of the market participants and is aimed at predicting the potential for market power beforehand.

As a further distinction between these approaches, note that the transmission-constrained market power index is "small signal" analysis; that is, it is based on the derivative of market clearing price with respect to demand. On the other hand, the TCRSI is a "large signal" analysis, since all the available supply from a firm is withdrawn during the analysis.

Although these two market power analysis schemes are different from several perspectives, they have one common characteristic: they both focus on what a firm could do, given the behavior and characteristics of the others. The two different approaches have different applications in evaluating market power. The TCMPI is suitable for helping to detect the exercise of market power, since it is based on the hypothesis that a firm is maximizing its profit. In other words, if a firm is maximizing its profit and significantly raising the prices

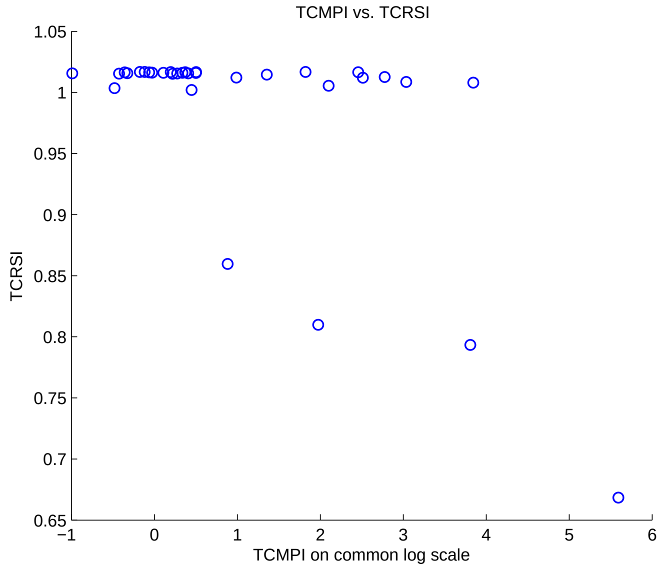


Figure 7: TCRSI versus TCMPI for the ERCOT system.

above competitive levels, the index would reflect this behavior.

On the other hand, the TCRSI is an appropriate tool for predicting the possession of market power. It does not require generation offers and market clearing results but could identify the firms that have absolute market power. Therefore, both tools can be integrated into the market power analysis flow due to their different implications for market power. Figure 7 shows a scatter plot of the TCRSI versus the TCMPI (on a common log scale) for the ERCOT system. From the figure, we can observe that the TCRSI and TCMPI are complementary in some cases: some firms are pivotal but their estimated mark-ups are just moderately large, while some firms have very large estimated mark-ups even without being pivotal. The numerical results are consistent with our claim that these two approaches are complementary in assessment of market power and can both be utilized to analyze the behavior of the firms.

7 Conclusion

In this report, we propose two different approaches to firm-based market power analysis and related indices: the transmission-constrained market power index (TCMPI), and the transmission-constrained residual supply index (TCRSI). We not only show the rationale of these approaches but also develop computationally efficient implementations for them, which can cope with large-scale systems such as the ERCOT system. We also compare the two indices with qualitative analysis together with experimental results, and conclude that both methods could be incorporated into market power analysis flow due to their different insights about market power.

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