New Indices of Market Power in Transmission-Constrained Electricity Markets

Yen-Yu Lee, *Student Member, IEEE*, Jin Hur, *Student Member, IEEE*, Ross Baldick, *Fellow, IEEE*, and Salvador Pineda, *Student Member, IEEE*

***Abstract*—In this paper, we introduce four new indices of market power in transmission-constrained electricity markets that are based on economic principles together with faithful representation of the effects of Kirchhoff's laws.**

***Index Terms*—Electricity markets, market power, transmission constraints.**

I. Introduction

MARKET power continues to be a problematic issue in restructured electricity markets, with the interaction between tight supply conditions and transmission limits posing serious difficulties for market operations, particularly in the context of "energy-only" markets such as the Electric Reliability Council of Texas (ERCOT) balancing market and the upcoming ERCOT "nodal"—that is, "locational marginal pricing," (LMP) market—to be introduced in late 2010 [1]. In energy-only markets, since there is no installed capacity payment mechanism as in the Northeast U.S. restructured markets [2], competitive energy prices must occasionally rise above the highest typical marginal costs in the system in order for generation asset owners to recover their investments and to encourage new investment to ensure resource adequacy to cope with demand growth.

Because the occasions of high competitive prices are also times when market participants have market power—defined as the ability to profitably shift market prices away from competitive prices [3, Section 4-1.1]—it can be extremely difficult to distinguish high competitive prices from the exercise of market power. Normatively, market power mitigation rules should be aimed at restoring prices towards competitive levels; however, when supply is tight, the competitive level of electricity prices depends on demand willingness-to-pay for electricity, in principle exhibited to the market by price-responsive demands or, under curtailment conditions, specified through a proxy to the "value of lost load" [3].

In restructured electricity markets today, the paucity of price-responsive demands and the lack of experience in North America with energy-only markets makes the determination of competitive price levels problematic under tight supply conditions. Moreover, the very lack of price-responsive demand contributes to the market power of market participants. This issue is problematic in both energy-only markets such as ERCOT and in the markets in the Northeast U.S. that have installed capacity markets.

Consequently, in the face of market power issues, restructured markets have adopted *ad hoc* market power mitigation rules that have only tenuous connection to:

- the fundamental economic drivers of the exercise of market power, such as the "residual demand" faced by market participants [4]; and
- the fundamental goal of making prices more competitive.

The significance of this issue is that in an energy-only market, such as in ERCOT, deviation from competitive prices is problematic. On the one hand, "under-mitigation," allowing market prices to rise excessively above competitive prices, would cause unacceptable transfers of wealth from consumers to generators. Market power mitigation should be aimed at preventing such unacceptable transfers; however, on the other hand, *ad hoc* approaches to market power assessment and mitigation also run the risk of "over-mitigating," driving prices below competitive levels. Prices below competitive levels will result in inadequate investment in generation in an energy-only market.

To summarize, in an energy-only market, it is critical to avoid both under- and over-mitigation of market power. *Ad hoc* approaches to analyzing market power are unlikely to result in appropriate mitigation of market power in an energy-only market, but such approaches are prevalent in North American markets today.

In this paper, we develop four related indices of market power in a transmission-constrained electricity market that are based on economic fundamentals as well as a representation of the transmission network that is faithful to Kirchhoff's laws. The indices involve the transmission-constrained residual demand faced by a market participant as derived in [5]. Calculation and visualization of the indices developed in this paper provides the basis for a screening tool for market power that can be of particular use to regulators and independent system operators (ISOs) in their assessment of market power in LMP markets.

Manuscript received September 15, 2009; revised September 21, 2009, February 11, 2010, May 30, 2010, and June 17, 2010; accepted June 21, 2010. This work was supported by the National Science Foundation under grant number ECCS-0801511. Paper no. TPWRS-00735-2009.

Y.-Y. Lee, J. Hur, and R. Baldick are with the Department of Electrical and Computer Engineering, The University of Texas at Austin, Austin, TX 78712 USA (e-mail: yenyu@mail.utexas.edu; jinhur@mail.utexas.edu; ross.baldick@engr.utexas.edu).

S. Pineda is with the Universidad de Castilla, La Mancha, Ciudad Real 13071, Spain (e-mail: salvador.pineda@uclm.es).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TPWRS.2010.2060738

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

The rest of this paper is organized as follows. In Section II, there is a literature survey of current approaches to analyzing market power. Then, in Section III, the indices are developed. Implementation of the indices is discussed in Section IV. Section V presents case studies to illustrate the indices. Section VI concludes and describes future work.

II. Current Approaches to Analyzing Market Power

Current market power assessment approaches and screening tools fall into two broad categories:

1) principles-based approaches that explicitly examine incentives to deviate from competitive prices; and
2) *ad hoc* approaches based on indices such as the Herfindahl-Hirschman Index (HHI).

As an example of the first category, [3] and [4] use the derivative of the residual demand faced by a generator to assess the incentives of a hypothetical profit-maximizing firm to mark-up prices above marginal; however, the analysis does not explicitly treat transmission constraints. An example of this kind of assessment is outlined in Section II-A in order to provide background and basis for extension to the case of transmission constraints.

As another example of the first category, [6] considers two zones joined by a single radial transmission line in a Cournot framework. While a two-zone model is appropriate for some particular cases, such as modeling of California represented as two zones, LMP markets have more complicated interactions due to the meshed nature of transmission. To summarize, the approaches to assessing market power that are rooted in economic analysis have not been extended to consider the implications of Kirchhoff's laws in a realistic meshed network. The limitations of a model of transmission that is limited to a single radial transmission line will be explored in Section II-B.

The second category, of *ad hoc* approaches, includes application to transmission-constrained markets with meshed networks. However, because the foundation is *ad hoc*, the results can be unreliable or even misleading. (See, for example, the discussion in [3, Ch. 4–5].) An example of such an *ad hoc* screening tool, based on the ERCOT nodal market design, is outlined and critically evaluated in Section II-C.

A. Principles-Based Analysis of Market Power in the Absence of Transmission Constraints

As discussed above, [3] and [4] describe an approach to assessing the incentives for a generator to mark-up its offer price above marginal costs in the absence of transmission constraints. The basic analysis considers the residual demand, that is, actual demand minus the supply of all the other participants in the market, and asks how a hypothetical profit-maximizing market participant would have offered in response to this residual demand. If that offer would involve significant mark-up of price above marginal cost and the generator is not operating at its maximum capacity, then the market participant has market power. Such a finding could be used in a subsequent market power mitigation process.

To understand the residual demand, suppose that the demand in a particular pricing interval was D. (We ignore price-responsiveness of demand here, but it can be incorporated into the analysis.) Furthermore, consider a particular market participant k and suppose that the total offered generation of all the *other* market participants besides k was specified by the function $q_{-k} : \mathbb{R} \rightarrow \mathbb{R}$. In particular, at the price P, the total offered generation of all the other market participants is $q_{-k}(P)$. (We will follow the "economics" convention of using the symbol P for price and the symbol q for quantity produced, in this case quantity of electricity.) The *residual demand* faced by market participant k is simply $D - q_{-k}(P)$. The inverse of the function $(D - q_{-k})$ is the inverse residual demand function faced by participant k, $p_{-k} : \mathbb{R} \rightarrow \mathbb{R}$.

Following, for example, [3] and [4], to analyze the incentives faced by market participant k, we consider the conditions for market participant k to maximize its profit. In this context, profit, $\pi_k : \mathbb{R} \rightarrow \mathbb{R}$, is defined as operating profit, meaning revenue minus costs. Ignoring forward contracts, revenue equals the product of:

- quantity, Q_k, of electricity produced by the generator, multiplied by
- the resulting price $p_{-k}(Q_k)$

noting that the definition of inverse residual demand is the resulting market clearing price in the market given that the generator produces the quantity Q_k. We assume that the production cost function of participant k is specified by the function $c_k : \mathbb{R} \rightarrow \mathbb{R}$.

Summarizing, profit for market participant k is

$$\forall Q_k \in \mathbb{R}, \quad \pi_k(Q_k) = Q_k p_{-k}(Q_k) - c_k(Q_k).$$

Assuming that sufficient conditions for maximization are satisfied, that p_{-k} and c_k are differentiable, and that generation capacity constraints are not binding at the profit maximizing condition, we can find the maximum of profit by setting its derivative to zero:

$$0 = \frac{\partial \pi_k}{\partial Q_k}(Q_k) = p_{-k}(Q_k) + Q_k \frac{\partial p_{-k}}{\partial Q_k}(Q_k) - c'_k(Q_k)$$

where $c'_k = \frac{\partial c_k}{\partial Q_k}$ is the marginal costs. Re-arranging, we obtain the price-cost mark-up of price above marginal cost under the above assumptions and the hypothesis that the generator was maximizing its profits:

$$p_{-k}(Q_k) - c'_k(Q_k) = -Q_k \frac{\partial p_{-k}}{\partial Q_k}(Q_k). \tag{1}$$

(Since the market clearing price is non-increasing in increasing generation by market participant k, we have that $\frac{\partial p_{-k}}{\partial Q_k}(Q_k) \leq 0$ and the right-hand side of (1) is non-negative.) As a basic measure of market power, if the right-hand side of (1) is "large," then a profit-maximizing generator has incentives to drive up prices by withholding (at least in the absence of a forward contract, to be discussed below).

The estimate of mark-up (1) provides one basic index of market power that could be utilized by a market monitor. For example, a threshold could be established of, say, 10% above marginal cost or \$20/MWh above marginal cost. Any generator that is not at full production but such that the right-hand side of (1) is above the threshold would be flagged as of concern. Such

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

generators might then be subject to market power mitigation such as limits on offer prices.

Equation (1) is sometimes re-arranged to obtain an expression for the Lerner index $(p_{-k}(Q_k) - c'_k(Q_k))/p_{-k}(Q_k)$ in terms of the price elasticity [3, Section 4-3.4]. However, as argued in [3, Section 4-5.2], particularly in the context of average measures over time, mark-up above competitive prices (and mark-up above marginal costs) is more useful than the Lerner index. In any case, the subsequent analysis in this paper is most easily written in terms of the price mark-up, so the rest of the analysis will focus on mark-up rather than Lerner index.

To the extent that marginal costs roughly represent the level of competitive prices, the estimated mark-up approximates the excess transfer of wealth, over and above competitive levels, from consumers to producers, per MW of production. Multiplying by production Q_k, the following can be used as an index to estimate the excess wealth transfer to participant k:

$$-(Q_k)^2 \frac{\partial p_{-k}}{\partial Q_k}(Q_k). \tag{2}$$

However, this index of excess wealth transfer should be used with caution since the marginal cost $c'_k(Q_k)$ of participant k at its production level may be below the competitive price [3] and, consequently, the estimated excess wealth transfer may be less than implied by (2).

So far the analysis has not considered forward contracts. Forward contracts change the competitive situation somewhat [3, Section 4-4.3]. In the presence of a forward sale of quantity Q_k^{f}, a similar derivation results in the following estimate of price-cost mark-up:

$$p_{-k}(Q_k) - c'_k(Q_k) = -\left(Q_k - Q_k^{\mathrm{f}}\right) \frac{\partial p_{-k}}{\partial Q_k}(Q_k). \tag{3}$$

Similarly, excess wealth transfer can be estimated by

$$-\left(Q_k - Q_k^{\mathrm{f}}\right)^2 \frac{\partial p_{-k}}{\partial Q_k}(Q_k). \tag{4}$$

To the extent that a significant fraction of the production is forward contracted, these expressions show that the short-term incentives for mark-up are reduced [3, Section 4-4.3]. (However, in general, a more complicated analysis involving the interaction between incentives in the forward and "spot" markets is necessary to fully elucidate incentives [7]–[10]. Although forward contracts do contribute to reducing incentives for exercising market power, the results are not completely understood in the context of supply offers in an electricity market, however, and we will ignore this interaction in the discussion below by simply assuming that the expression on the right-hand side of (3) is appropriate as an index for assessing market power.)

Paralleling the previous argument, the right-hand side of (3) provides an index for assessing the incentives to exercise market power. However, it relies on knowledge of forward market positions. In some cases, forward contract positions are available to the market monitor. For example, in the context of a real-time market, the positions from the day-ahead market constitute forward financial positions.

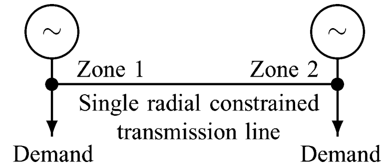

Fig. 1. Two-zone network joined by radial transmission.

To summarize, given the derivative of the inverse residual demand faced by a market participant, it is possible to evaluate the incentives to mark-up price above marginal cost. This incentive can be used as an index of market power that, together with a threshold value of "large" for the index, constitutes a screening tool for market power. Unlike other indices such as HHI as applied to electricity markets (particularly "HHI" based on *capacity* rather than market share [3, Section 4-3.3]) this index is based on a principled analysis of the underlying economic incentives.

In addition to estimating the price-cost mark-up for generator k, three further indices of market power that can be useful are:

- the inverse residual demand derivative itself faced by generator k, $\frac{\partial p_{-k}}{\partial Q_k}(Q_k)$;
- the derivative of price with respect to injection at bus k, which can be calculated from the inverse residual demand derivative by adding to it the slope of the offer by generator k; and
- the estimated excess wealth transfer to generator k, (2) or (4).

The analysis so far does not consider the issue of transmission constraints. To the best of the authors' knowledge, no extensions of first principles analysis to the case of transmission constraints in *meshed* systems have been reported in the literature. In Section III, an approach will be proposed to extend this principled analysis to a meshed transmission system.

Before discussing the proposed approach to meshed systems, however, we will first discuss the case of a single *radial* transmission constraint in Section II-B. Although a single radial transmission constraint is an unrealistic situation in the context of an LMP market, it will help to explain and motivate the more complex and realistic context of a meshed system. Then, in Section II-C, we will discuss an example of an *ad hoc* approach to treating market power in the presence of transmission constraints in a meshed system.

B. Principled Analysis of Radial Transmission Constraints

In a system with a single radial transmission constraint joining two zones, as shown in Fig. 1, whenever there is transmission congestion between the zones, the two zones are separated into two markets. In this case, analysis of residual demand involves considering each zone separately. Consequently, it is straightforward to extend the approach in [3] and [4] to analyze the residual demand in each zone when the single constraint is binding. The basic approach simply divides the market participants into the two zones and considers each zone as a separate market.

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

Such analysis is possible in radial systems because of a particular feature of the "shift factors," that is, the fraction of power flowing on a line due to injection at one zone and withdrawal at another zone. For a radial line, the absolute value of shift factors are always either zero or one. Consequently, given that the constraint is binding, the two-zone system can be validly analyzed as two separate markets.

However, in meshed systems, the shift factors are typically between minus and plus one, so that market participants *cannot* be unequivocally divided into being in one "zone" or another. The market participants interact in more complex ways due to the shift factors having values other than zero and one, requiring additional analysis. The complexity of meshed systems has prompted *ad hoc* approaches that will be discussed in the next section.

C. Ad Hoc Analyses of Market Power With Transmission Constraints

In contrast to the discussion in Section II-A, which started with an underlying economic model of profit maximization to define an index of market power, in this section we will consider an example of an *ad hoc* analysis that attempts to analyze market power in the presence of transmission constraints in a meshed system.

In the upcoming ERCOT nodal market, there are several *ad hoc* methods to assess market power. One of them, the "Element Competitiveness Index" (ECI) is styled as an *ex ante* test of competitiveness in the face of transmission constraints. In particular, it is performed to "[d]etermine if there is sufficient competition to resolve the [transmission] constraint on the import and export side" [1, Section 3.19.1]. There are several parts to this complicated test involving capacity shares and DC shift factors for potentially binding transmission constraints.

At its heart, the ECI test is an HHI test based on capacity. As Stoft points out, despite the use of capacity-based HHI measures by various regulatory authorities, there is no theoretical justification for capacity-based HHIs as a measure of market power [3, Section 4-3.3]. When HHIs are based on market shares instead of capacities, there is a connection to the Cournot model; however, even such share-based HHIs are unreliable as a measure of market power since they omit consideration of supply and demand elasticity and of forward contract positions, which are essential determinants of market power [3, Section 4-5.1], [11].

Although the ECI test incorporates, through the shift factors, a proxy for the geographical extent of the market, it fundamentally omits the drivers of market power: the "residual demand" faced by market participants and the forward contract positions. In particular, the ECI test is not based on any offer information [1, Section 3.19].

Moreover, the ECI test considers each line separately and therefore does not consider the effect of *interactions* between constraints on market power. As will be illustrated in the case studies in Section V, multiple constraints are typically binding in a large system.

The focus in the ECI and other tests such as PJM's "three pivotal supplier test" [12, Appendix J] is on particular *lines*. However, in fact, the key economic issue is the incentives to market participants at particular *buses* due to potentially multiple interacting constraints. The ECI test obscures the locus of the fundamental economic incentives.

Another approach to monitoring market power is proposed in [13], and is based on the sensitivity of the dispatch of generators to prices considering transmission constraints. The calculation of sensitivities in [13] does not represent the variation of price with generation as specified by the offers. However, representing the offers is essential to characterizing the incentives for a generator to exercise market power.

To summarize, the approaches that consider Kirchhoff's laws in the assessment of market power have not considered the fundamental economic incentives. As will be discussed in the next section, the indices we develop are based on analysis of incentives to market participants at each bus.

III. Transmission-Constrained Market Power Indices

In previous work, [5] describes calculation of the inverse residual demand faced by a given generator at a single bus, assuming that offers are differentiable. Information about the binding constraints and injections at the market clearing conditions based on solution of an optimal power flow (OPF), together with the generator offers, provide the information needed to evaluate the derivative. The calculation is computationally efficient given the market clearing results.

To be concrete, consider generator k located at bus k. (The case where there are multiple generators owned by different firms at a single bus is similar. We briefly mention the case where a firm owns multiple generators at different buses in Section VI.) The calculation of inverse residual demand derivative at bus k considers the effects of binding transmission constraints and quantitatively evaluates the decrease in residual demand elasticity faced by generator k when transmission constraints are binding.

Suppose that $H \in \mathbb{R}^{b\times w}$ is the matrix of shift factors for injection at marginal generators $j \neq k$ for the b binding constraints in the system at the market clearing conditions, with bus k chosen to be the (price) reference bus and there are w marginal generators $j \neq k$. That is, $\forall \ell, H_{\ell j}$ is the amount of power flowing on constrained line ℓ given a unit injection at bus j and withdrawal at bus k. Note that the generators $g \neq k$ at their full capacities are excluded from the calculation, since their injections are assumed to be fixed in the sensitivity analysis. Furthermore, let $\Lambda \in \mathbb{R}^{w\times w}$ be a diagonal matrix whose diagonal entries are the inverses of the derivatives of the offers at buses other than bus k, evaluated at the market clearing conditions. Then, from [5], the inverse residual demand derivative faced by generator k evaluated at the market clearing injection $Q_k^\star$ is

$$\frac{\partial p_{-k}}{\partial Q_k}(Q_k^\star) = \left[-\mathbf{1}^\dagger \Lambda \mathbf{1} + \mathbf{1}^\dagger \Lambda H^\dagger \left(H \Lambda H^\dagger\right)^{-1} H \Lambda \mathbf{1}\right]^{-1} \quad (5)$$

where $\mathbf{1} \in \mathbb{R}^w$ is the vector of all ones and superscript $\dagger$ means transpose. That is, to calculate the inverse residual demand derivative for generator k, we need the following:

- slope of the offer of each generator $j \neq k$ evaluated at the market clearing quantity; and

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

- shift factor to the binding transmission lines for injection at each generator $j \neq k$ and withdrawal at generator k.

The inverse residual demand derivative provides the first of four indices of transmission-constrained market power and shows the level of price responsiveness faced by the generator at bus k due to the combination of offers and transmission constraints in the rest of the system. A second index is the derivative of price with respect to injection. This second index shows the level of price responsiveness at bus k including all offers and transmission constraints in the system, including the offer at bus k. It shows the level of price responsiveness faced by, for example, a new entrant at bus k.

We develop two further indices of the market conditions faced by the generator at bus k that depend on the transmission-constrained inverse residual demand derivative. These indices are conceptually straightforward extensions of the development in Section II-A to the transmission-constrained case. That is, they are based on analyzing the incentives to a profit maximizing market participant, assuming that it owns generation at only a single bus.

In particular, substitute into the right-hand side of (3):

- the market clearing injection at bus k, $Q_k = Q_k^\star$;
- the forward contract quantity, Q_k^{f}; and
- the inverse residual demand derivative evaluated at the market clearing injection, $\frac{\partial p_{-k}}{\partial Q_k}(Q_k^\star)$.

This expression provides the estimated price-cost mark-up, again assuming that sufficient conditions for profit maximization are satisfied, that p_{-k} and the cost c_k are differentiable, and that generator capacity constraints are not binding at the profit maximum. As a fourth index of transmission-constrained market power, excess wealth transfer can be estimated using (4).

In contrast to, for example, ECI, the index (3) has a concrete interpretation in terms of market power: it estimates the mark-up of price above marginal cost for a hypothetical profit maximizing generator. If forward contract information was not available, then $-Q_k \frac{\partial p_{-k}}{\partial Q_k}(Q_k)$ could be used as an index instead; however, any subsequent market power mitigation should be sensitive to the implications of forward contracting on market power. The index (4) also has a concrete interpretation: it estimates the excess transfer above competitive levels.

To summarize, four indices of transmission-constrained market power developed in this section are:

- transmission-constrained inverse residual demand derivative;
- derivative of price with respect to injection;
- estimated mark-up; and
- estimated wealth transfer over competitive levels.

Calculation and visualization of these indices is described in the next section.

IV. Implementation

We implemented the prototype calculation of the indices in a convenient graphical tool using PowerWorld and Matlab. The flow chart of the tool design is shown in Fig. 2. Given generation offer cost data, load data, and transmission data, the OPF solution is generated by PowerWorld. Using results from the OPF solution, the market power index is evaluated using Matlab. We ignore forward contracts. The calculated market power index is then visualized with PowerWorld. The following section describes these issues in detail.

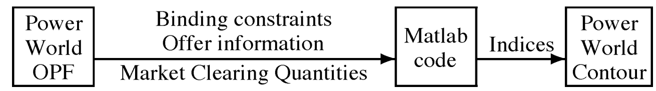

Fig. 2. Flow chart of tool design.

A. PowerWorld OPF

The PowerWorld OPF is capable of considering various representations of cost curves. In the context of an electricity market, the *cost* data used by the OPF correspond to the integral of the offers. We utilized the representation in PowerWorld that allows for cubic (or lower degree) cost curves (corresponding to quadratic or lower degree offers). This guarantees that the offers are differentiable, consistent with the assumption in the analysis of residual demand as developed in [5], but requires that piecewise constant and other non-smooth offers be smoothed for use in the OPF.[1]

Generator data including cost data, transmission data, and load data are utilized by PowerWorld to calculate the OPF. From the solution of the OPF, several other files are produced in order to calculate the market power index as described in more detail in the next section.

B. Market Power Indices Calculation in Matlab

The files produced by PowerWorld are passed to Matlab and the calculation of the indices is implemented with Matlab. To calculate the residual demand derivative faced by each generator, we evaluate (5). However, using this formula literally would involve matrix inversion, which has various drawbacks. In order to avoid matrix inversion and to avoid ill-conditioning issues, QR factorization and forwards and backwards substitution is utilized. The residual demand derivative calculation is performed for each generator.

As well as the residual demand derivative, the three other indices are also calculated:

- derivative of price with respect to injection;
- estimated price-cost mark-up;
- estimated wealth transfer.

For each index, the calculated values for each generator are then visualized in PowerWorld, as discussed in the next section.

C. Visualization

In order to better illustrate the market power indices, the contouring function of the PowerWorld simulator is utilized. The contouring visualization function can significantly improve understanding of the calculated market power indices and help market monitors to evaluate market power of a large interconnected system. As indicated in Fig. 2, the market power indices are passed to the contour module of the simulator as a customized market power index field.

[1]A DC OPF based on linear programming (LP) is used in PowerWorld, implying that the polynomial cost curves are themselves piecewise linearized for calculations within PowerWorld.

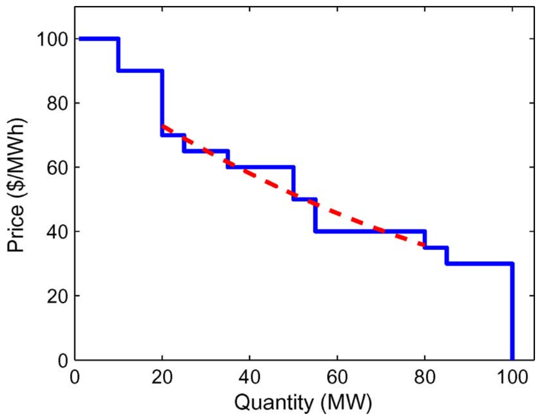

Fig. 3. Piece-wise constant residual demand and its fitted quadratic curve near 50 MW.

D. Piece-Wise Constant Offer Functions

In some electricity markets such as ERCOT, Midwest ISO, and the Southwest Power Pool, piece-wise linear offers are allowed [1, Sections 4.4.9.3 and 6.4.3], [14, Section 4.2.2.2.1], [15, Section 5.4]. However, in many other markets, the offers can only be specified by piece-wise constant functions, which poses difficulties for calculating the indices since the offer slopes at the market clearing quantities might be 0, causing the matrix Λ to be ill-defined. As discussed in [16], due to the large number of price increments allowed for each generator, plus the large number of generators in a market, the number of steps in the residual demand curve faced by any market participant is typically large, especially in the neighborhood of the market clearing quantity. Therefore, this issue can be dealt with by fitting a smooth curve to the piece-wise constant offer function in the neighborhood of the market clearing quantity, so that the average slope in the vicinity of the market clearing quantity is estimated.

Although smoothing out the piece-wise constant offers creates some inaccuracy, this approximation works well given that the number of steps in the residual demand curve is large. The following example shows how smoothing out the offer does not necessarily introduce serious errors into the estimation of the mark-up. Consider generator k facing a residual demand curve as shown in Fig. 3: $100/MWh for [0 10) MW, $90/MWh for [10 20) MW, $70/MWh for [20 25) MW, $65/MWh for [25 35) MW, $60/MWh for [35 50) MW, $50/MWh for [50 55) MW, $40/MWh for [55 80) MW, $35/MWh for [80 85) MW, and $30/MWh for [85 100) MW. Assume that the operating cost of generator k is $0.1Q^2 + 20Q$.

We claim that the profit maximizing quantity of the generator would be (just less than) 50 MW, with a clearing price of approximately $60/MWh. To see, this note that:

- if generator k produced more than 50 MW then the price would drop by $10/MWh from $60/MWh to $50/MWh and profit would drop by around $10/MWh $\times$ 50 MW;
- if generator k produced somewhat less than 50 MW then the price would stay the same, and the price would be above marginal cost, but the generator would sell less, lowering its profit; and
- if generator k produced significantly less than 50 MW in order to drive prices up to $65/MWh, then the production would be reduced by around 15 MW and profit would decrease by around $300/h or more.

Given the profit maximizing condition for generator k, its actual mark-up is $30/MWh.

By fitting a quadratic curve to the offer in the neighborhood of 50 MW (from 20 MW to 80 MW; see dashed curve in Fig. 3) and estimating the slope of the fitted curve at the market clearing quantity 50 MW, the approximated slope of the offer is -0.6196($/MWh)/MW. Based on this slope, the estimated price-cost mark-up is $30.98/MWh, which is very close to the actual mark-up. This example shows that smoothing out the offer gives a good approximation to the actual mark-up in this case. The approximation of the derivative does not produce a large error in the estimate of the mark-up in this example.

The reason that the approximation of the derivative does not necessarily produce a large error is related to the assumption of profit maximization. Profit maximization when facing a piece-wise constant residual demand implies that the prices and quantities in adjacent segments of the residual demand curve would not result in higher profits to generator k. For this to be the case, the change in price and change in quantity to adjacent segments must be such as to involve a decrease in profits. This effect is well approximated by the average slope over several adjacent segments.

In our indices calculator, non-decreasing quadratic curves are utilized to fit the piece-wise constant offers near the market clearing quantities and the offer slopes are evaluated based on the quadratic curves. Note that this post-processing step for the offer functions is only required for calculating the residual demand derivatives, while the OPF solver still uses piece-wise constant offers to clear the market.

V. Case Studies

In this section, we will demonstrate the indices in the context of the 118-bus Reliability Test System and the ERCOT system.

A. Reliability Test System

Power flow data for the 118-bus Reliability Test System [17] together with generic cost data were used to create a data set for PowerWorld. An OPF was solved for this system with all pre-contingency thermal limits imposed. Fig. 4 shows the contours of the inverse residual demand derivative for this system, given offers equal to generic marginal costs. Note that four transmission constraints (shown by the four light blue “pie charts” labelled as 100% flow) are binding in this system.

Fig. 4 indicates the competitive situation faced by generators at the buses, as determined by the shift factors to binding constraints, market clearing quantities, and the offers. Many buses in the system are such that the residual demand faced at that bus is not significantly affected by the transmission constraints. These buses are shown in deep to light blue. Other buses are somewhat affected by the transmission constraints, as shown by the green contours.

However, one bus in this system has a significantly greater magnitude for its inverse residual demand derivative, as shown by the red contours in the top left of the system near to two of the

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

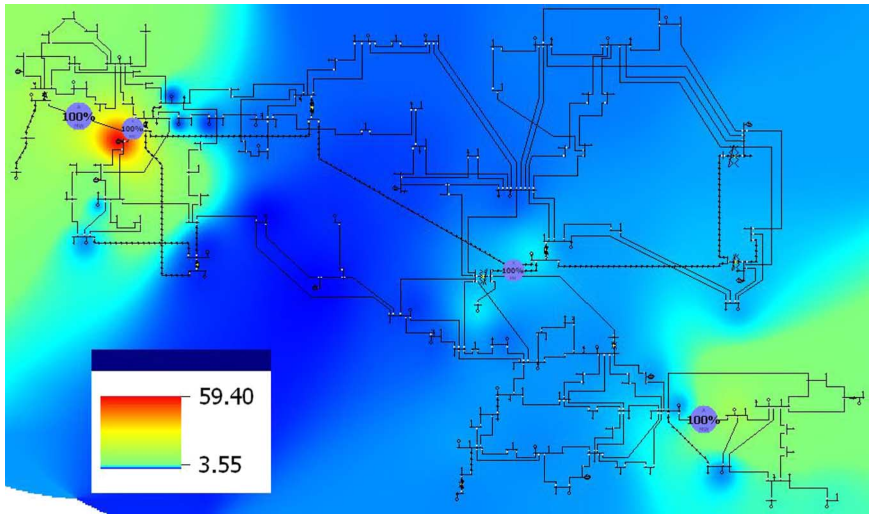

Fig. 4. Contour map of absolute value of inverse residual demand derivative for the 118-bus Reliability Test System. The units are ($/MWh)/GW.

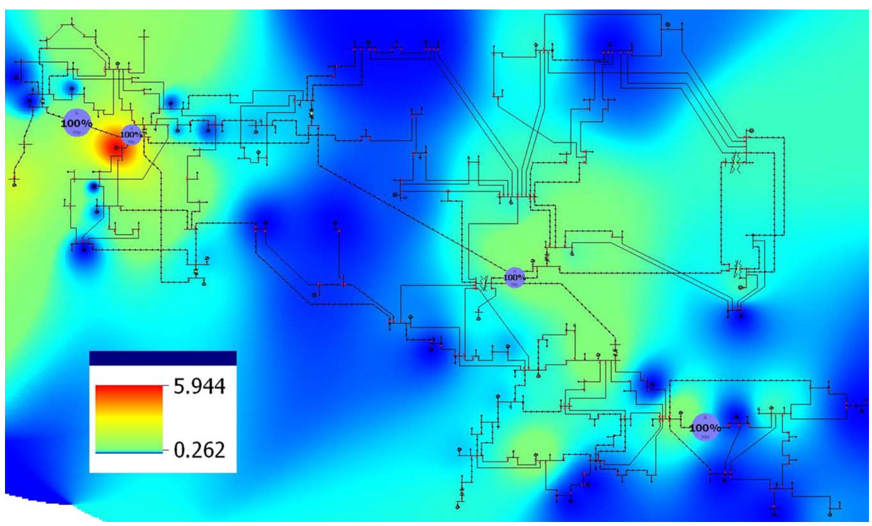

Fig. 5. Contour map of estimated price-cost mark-up for the 118-bus Reliability Test System. The units are $/MWh.

binding transmission constraints. This indicates that changes in injection at this bus would significantly affect prices. Generation at this bus would potentially be flagged for further market power analysis.

The contours of the absolute value of derivative of price with respect to injection for the Reliability Test System are qualitatively similar to those in Fig. 4 and, for brevity, we do not show this plot. The main difference compared to Fig. 4 is that the magnitudes of the entries are smaller, reflecting the inclusion of the effect of the offers at each bus in the derivative of price with respect to injection.

Fig. 5 shows the estimate of the price-cost mark-up according to (1). Many buses have a very low estimate, as shown by the dark to light blue contours. However, it is interesting to note that even in the regions of low magnitude of inverse residual demand derivative, the estimated mark-ups can still be moderately high, as shown by the green contours. Not surprisingly, the bus that has a high value of magnitude for its inverse residual demand derivative also has a high estimate for the price-cost mark-up, as indicated by the red contours.

The contours of the estimate of wealth transfer according to (2) are qualitatively similar to those in Fig. 5, although there are some detailed differences. For brevity, we do not show this plot.

B. ERCOT System

Power flow data for the ERCOT system together with generic cost data were used to create a data set for PowerWorld. A security-constrained OPF was solved for this system with all thermal pre- and single post-contingency constraints enforced except for those post-contingency constraints that could not be satisfied, such as those involving:

- radially connected generators;
- radially connected loads;
- certain radial loops; and
- nearby normally open circuit breakers that could be closed in the event of the contingency to mitigate an overload.

We did not enforce voltage constraints.

There were three binding pre-contingency constraints and 16 binding post-contingency constraints at the OPF solution, indicating the importance of considering multiple interacting

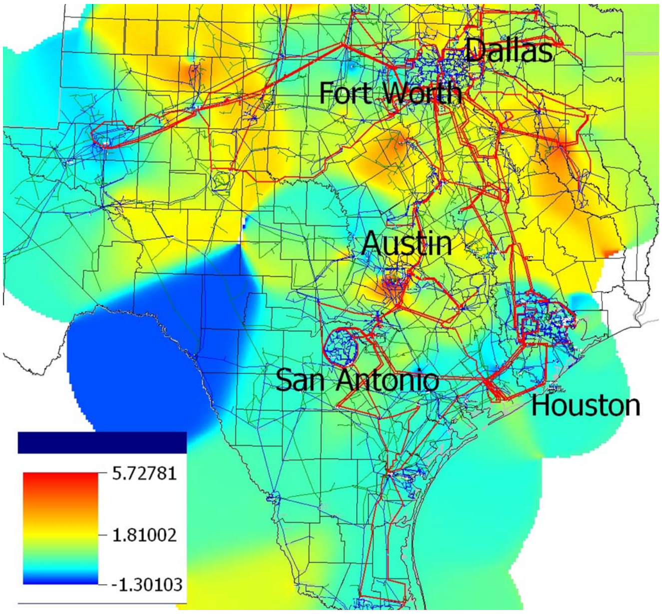

Fig. 6. Contour map of common log of estimated price-cost mark-up (in $/MWh) for the ERCOT system.

constraints. The inverse residual demand was calculated from the solution of the security-constrained OPF according to (5). For brevity, only the index of estimated price-cost mark-up according to (1) will be shown.

Fig. 6 shows the estimate of the price-cost mark-up (on a common log scale) according to (1) for buses in the ERCOT system. Most of ERCOT has relatively low values for the estimates of the price-cost mark-up. However there are several regions of moderate estimated mark-up and one region with very large estimated mark-up nearby to Austin due to several generators having large shift factors to a binding transmission constraint, which results in large inverse residual demand derivatives. Estimated price-cost mark-ups of more than 5 million $/MWh are unrealistic given the (unmodeled) levels of forward contract cover and offer caps. If this information was known, more realistic estimates could be obtained that would more faithfully reveal the incentives for generators to drive up the prices.

VI. Conclusion and Future Work

In this paper, we have developed four indices of market power in the context of transmission-constrained electricity markets and shown an effective visualization of the indices. The indices could be evaluated in several contexts, such as:

- part of an *ex ante* simulation of market operation over pricing intervals in a time horizon using a production cost simulator, either based on competitive offers or based on some assumed strategic offers;
- alongside the clearing of the actual market based on actual offers; and
- part of an *ex post* analysis by a market monitor, based on historical offer information.

In future work, we plan to consider a number of additional issues. For example, the profit maximizing condition in Section III was derived under the assumption that the cost function and, more importantly, the inverse residual demand function is differentiable. In fact, this assumption does not always hold. In particular, if a transmission or generator capacity constraint is only "just" binding at the market clearing conditions (or if a marginal offer is at a point of non-differentiability of the offer), then the left-hand and right-hand derivatives of the inverse residual demand are not equal. We will consider this issue in more detail in future work.

As another example, the development in this paper has assumed that any particular firm owns a generator or generators at only one bus, bus k. However, in reality, in LMP markets, assets will be owned at multiple buses and the LMPs at these different buses are related. We will consider the issue of ownership at multiple buses in future work using the concept of the residual demand *Jacobian*.

As a third example, the analysis in this paper considers the "small signal" issue of whether the slope of the residual demand is such that the first-order necessary conditions for profit maximization imply "economic" withholding that would significantly increase price over marginal costs. At high levels of demand, particularly, another concern is that the "large signal" action of "physically" withholding capacity would lead to infeasibility. In this case, the generator is sometimes said to be "pivotal" [18]–[21]. If a generator is pivotal then, in principle, it can increase the price arbitrarily.

In the absence of transmission constraints, [18] and [19] discuss an index, the "residual supply index" (RSI), that reflects the degree to which a firm's offer is necessary to meet demand. However, binding transmission constraints may make firms pivotal in particular geographic areas even if from a region-wide perspective there is no pivotal firm. In future work, we will consider generalizations of RSI that apply in the transmission-constrained case, generalizing tests such as the "three pivotal supplier test" used in PJM [12, Appendix J].

As a final example, in future work we also plan to include contract cover in the estimates.

References

[1] Electric Reliability Council of Texas, ERCOT Nodal Protocols, 2009. [Online]. Available: http://nodal.ercot.com/protocols/index.html.

[2] PJM Members Committee, Reliability Pricing Model Overview, 2004. [Online]. Available: http://www.pjm.com.

[3] S. Stoft, *Power System Economics: Designing Markets for Electricity*. Piscataway, NJ: IEEE Press/Wiley Interscience, 2002.

[4] F. A. Wolak, "Measuring unilateral market power in wholesale electricity markets: The California market 1998–2000," *Amer. Econ. Rev.*, vol. 93, no. 2, pp. 425–430, May 2003.

[5] L. Xu and R. Baldick, "Transmission-constrained residual demand derivative in electricity markets," *IEEE Trans. Power Syst.*, vol. 22, no. 4, pp. 1563–1573, Nov. 2007.

[6] S. Borenstein, J. Bushnell, and S. Stoft, "The competitive effects of transmission capacity in a deregulated electricity industry," *RAND J. Econ.*, vol. 31, no. 2, pp. 294–325, Summer, 2000.

[7] B. Allaz and J.-L. Vila, "Cournot competition, forward markets, and efficiency," *J. Econ. Theory*, vol. 59, pp. 1–17, 1993.

[8] D. M. Newbery, "Competition, contracts, and entry in the electricity spot market," *RAND J. Econ.*, vol. 29, no. 4, pp. 726–749, Winter, 1998.

[9] R. Baldick, Joint Equilibrium of Day-Ahead and Real-Time Markets, August 2002, unpublished manuscript, Dept. Elect. Comput. Eng., Univ. Texas at Austin.

[10] J. Bushnell, "Oligopoly equilibria in electricity contract markets," *J. Regulat. Econ.*, vol. 31, pp. 225–245, 2007.

[11] S. Borenstein, J. Bushnell, and C. R. Knittel, "Market power in electricity markets: Beyond concentration measures," *Energy J.*, vol. 20, no. 4, pp. 68–88, 1999.

[12] PJM Interconnection, 2006 State of the Market Report, Volume II: Detailed Analysis, Mar. 2007. [Online]. Available: http://www2.pjm.com/markets/market-monitor/downloads/mmu-reports/2006-som-volume-ii.pdf.

[13] C. Murillo-Sanchez, S. Ede, T. Mount, R. Thomas, and R. Zimmerman, “An engineering approach to monitoring market power in restructured markets for electricity,” in *Proc. 24th Annu. Int. Conf., International Association for Energy Economics*, Apr. 2001.

[14] Midwest ISO, MISO Business Practices Manual: Energy and Operating Reserve Markets, 2010. [Online]. Available: http://www.midwestiso.org.

[15] Southwest Power Pool Electric Energy Network, SPP Market Protocols, 2010. [Online]. Available: http://www.spp.org.

[16] F. A. Wolak, Identification and Estimation of Cost Functions Using Observed Bid Data. [Online]. Available: http://www.stanford.edu/wolak.

[17] Reliability Test System Task Force, “The IEEE reliability test system—1996,” *IEEE Trans. Power Syst.*, vol. 14, no. 3, pp. 1010–1020, Aug. 1999.

[18] A. Sheffrin and J. Chen, “Predicting market power in wholesale electricity markets,” in *Proc. 15th Annu. Western Conf. Advanced Workshop in Regulation and Competition*, South Lake Tahoe, CA, 2002.

[19] A. Sheffrin, J. Chen, and B. Hobbs, “Watching watts to prevent abuse of power,” *IEEE Power Energy Mag.*, vol. 2, no. 4, pp. 58–65, Jul./Aug. 2004.

[20] T. Genc and S. S. Reynolds, Supply Function Equilibria With Pivotal Suppliers, Mar. 2005, unpublished manuscript.

[21] D. M. Newbery, Predicting Market Power in Wholesale Electricity Markets, Sep. 2008, Cambridge Univ. EPRG working paper 0821.

Yen-Yu Lee (S’09) received the B.Sc. degree in electrical engineering from the National Taiwan University, Taipei, Taiwan, in 2006. He is currently pursuing the Ph.D. degree in the Department of Electrical and Computer Engineering at The University of Texas at Austin.

Jin Hur (S’08) received the B.S. and M.S. degrees in electrical engineering from Korea University, Seoul, Korea, in 1997 and 1999, respectively. Since 2006, he has been pursuing the Ph.D. degree in the Department of Electrical and Computer Engineering at The University of Texas at Austin.

He was a researcher at Korea Electro-Technology Research Institute (KERI) from 1999 to 2006.

Ross Baldick (F’07) received the B.Sc. degree in mathematics and physics and the B.E. degree in electrical engineering from the University of Sydney, Sydney, Australia and the M.S. and Ph.D. degrees in electrical engineering and computer sciences from the University of California, Berkeley, in 1988 and 1990, respectively.

He is currently a Professor in the Department of Electrical and Computer Engineering at The University of Texas at Austin.

Salvador Pineda (S’07) received the Ingeniero Industrial degree from the Universidad de Malaga, Malaga, Spain, in 2006. He is currently pursuing the Ph.D. degree at the Universidad de Castilla-La Mancha, Ciudad Real, Spain.

His research interests include stochastic programming and electricity markets.