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Bidding Into Electricity Markets: A Transmission-Constrained Residual Demand Derivative Approach

Lin Xu, *Member, IEEE*, Ross Baldick, *Fellow, IEEE*, and Yohan Sutjandra, *Member, IEEE*

***Abstract*—This paper proposes a novel approach to optimize a generator's profit in an offer-based electricity market cleared by an optimal power flow (OPF) program. A generator's offer is optimized based on its transmission-constrained residual demand derivative (TCRDD), which avoids representing the full network model in the optimization. The TCRDD can be easily calculated based on a solved OPF without changing existing OPF algorithms and programs. As demonstrated with an IEEE 118-bus example, the TCRDD approach is computationally efficient. The TCRDD approach can help market participants to bid into electricity markets, and help market monitors to diagnose bidding behaviors in the presence of transmission constraints.**

***Index Terms*—Bidding strategy, electricity market, residual demand derivative.**

I. Introduction

THE uniqueness of electricity markets comes largely from the transmission network [1]. Flows on the electric transmission network connecting suppliers and consumers must obey physical laws, namely Kirchoff's Current Law (KCL) and Kirchoff's Voltage Law (KVL).

Existing electricity markets have diverse designs to handle the transmission network: some markets are cleared without considering the transmission network, and resort to specific congestion mitigation mechanisms afterwards, while other markets incorporate the power flow equations and transmission capacity limits as constraints into the market clearing process. The latter is a more systematic approach and is the typical design for new electricity markets in the U.S. In this paper, we will focus on the latter design.

More specifically, in this paper, we consider typical electricity markets cleared by an optimal power flow (OPF) program, which incorporates the power flow equations and transmission capacity limits as constraints. Electric power is priced by locational marginal prices (LMPs), which have a congestion component dependent on the shadow prices of the transmission capacity constraints. Bidding into electricity markets is very challenging because of the complexity of the network model.

There have been many studies that look into the impacts of binding transmission constraints: [1] and [2] consider a Cournot model in a two-bus radial network and a three-bus looped network, respectively; [3] analyzes the linear supply function equilibrium both in a two-bus radial network and a four-bus meshed network using an iterative approach; and [4] further analyzes how the transmission-constrained linear supply function equilibrium changes as the load varies. These studies numerically demonstrate that strategic behaviors involving the transmission constraints could lead to a market outcome that is different from the usual analysis of imperfect competition, but they are not systematic approaches aiming at practical applications.

In industry, it is common to use simulation tools to explore profitable bidding strategies. These tools simulate the clearing process of electricity markets so that different bidding strategies can be tested through simulations. Although such simulation tools can assist in decision making, they often involve tremendous *ad hoc* human judgment and intervention, and cannot guarantee quality of solution, especially in nodal electricity markets.

The first systematic way to search for the profit maximizing bidding strategies in electricity markets is the mathematical program with equilibrium constraints (MPEC) method, proposed by Hobbs *et al.* [5]. A generator's profit maximization can be formulated as a two-level optimization problem, where the lower level is the OPF problem, and the upper level is the generator's own profit maximization problem. The MPEC method adds the Karush-Kuhn-Tucker (KKT) conditions for the OPF into the upper level profit maximization as constraints, and forms an integrated optimization problem, namely an MPEC, in the following form [5]:

$$\begin{aligned} \max_{\boldsymbol{\alpha}_i} \quad & \text{profit}_i \\ \text{s.t.} \quad & \underline{\boldsymbol{\alpha}}_i \le \boldsymbol{\alpha}_i \le \overline{\boldsymbol{\alpha}}_i, \\ & \text{OPF KKT conditions} \end{aligned} \tag{1}$$

where $\boldsymbol{\alpha}_i$ is generator i's strategic variable vector, and $\underline{\boldsymbol{\alpha}}_i$ and $\overline{\boldsymbol{\alpha}}_i$ are its lower bound and upper bound vectors, respectively. The major difficulty of the MPEC approach stems from the complexity of the full network model, represented by the OPF KKT conditions, because OPF KKT conditions are nested in the generator's profit maximization problem (1). First, generally the OPF KKT conditions are nonlinear, although for a network-constrained OPF, the OPF KKT conditions can be linearized by introducing binary variables if the generator offers are step func-

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The authors are with the University of Texas at Austin, Austin, TX 78712 USA (e-mail: linxu@mail.utexas.edu; ross.baldick@engr.utexas.edu).

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tions [6]. Second, production level security constrained OPF typically models hundreds of generators, thousands of buses and lines, and hundreds to thousands of contingencies [7], which makes the MPEC problem beyond the computational capability of existing MPEC solvers [8]. In addition, although there exist advanced algorithms and programs to solve the OPF, it is difficult to reuse them in solving the MPEC due to the nested structure. The MPEC approach essentially requires power system application developers to start from scratch in order to implement such an algorithm. Due to these reasons, there is rarely any implementation of the MPEC method to calculate a generator's optimal offer in practice.

II. Maximizing Profit With Residual Demand Derivative

We propose an alternative systematic approach to overcome the computational difficulty of the MPEC approach. In the MPEC method, note that the full network model OPF greatly complicates the profit maximization problem. If we can solve the full network model OPF and the profit maximization problem separately, then it can help resolve the computational difficulty. The challenge of this idea is how to represent the impact of the network in the profit maximization in a simpler form without using the full network model. As we will demonstrate below, the transmission-constrained residual demand contains the essential information for the purpose of profit maximization, and can be used to bridge between the full network model OPF and the profit maximization.

In this paper, we make the following assumptions.

- Inputs to the OPF, including offers from all market participants, loads, and network model, are available.
- The generation firm knows its own costs and contracts.
- The generation firm only owns one "generator," which can either be a physical generation unit or a generation portfolio located at a single pricing location (node or zone) in the system.

Like the MPEC approach, we are going to explicitly model the OPF, so the inputs to the OPF are assumed to be available. In practice, a generation firms knows its own offers, costs, and contracts; loads can be predicted by a standard load forecast algorithm; the network model is typically available from the ISO; however, offers from competitors are not typically publicly available until well after the time of the market clearing and need to be estimated based on historical data or generic cost data. In this paper, we will focus on the one-generator case. However, we stress that if a multi-generator version of transmission-constrained residual demand derivative (TCRDD) can be characterized, then the TCRDD approach can be generalized to a multi-generator case, where one firm owns multiple generators located at multiple locations in the system.

Let us consider a generator's profit maximization problem from the residual demand point of view. For simplicity, assume the generator under consideration does not have any forward bilateral contracts. As discussed in [9] and [10], handling the contracts only involves shifting the offer curve by the contracted amount. Conceptually, each generator is facing a residual demand curve. The residual demand curve specifies at each price level the maximum market share left for the generator. From generator i's point of view, the market clears at the point where its supply $S_i(\bullet)$ meets its residual demand $R_i(\bullet)$:

$$S_i(p_i) = R_i(p_i). \tag{2}$$

Generator i's profit is

$$\Pi_i(q_i) = P_i(q_i)q_i - C_i(q_i) \tag{3}$$

where $P_i(\bullet) = R_i^{-1}(\bullet)$. That is, $P_i(\bullet)$ is the inverse function of $R_i(\bullet)$, so that $(P_i(q_i), q_i)$ is a point on the residual demand curve. We assume continuous piecewise quadratic cost functions and continuous piecewise linear offer functions,[1] such that the residual demand curve is a continuous piecewise linear function, and the profit is a continuous piecewise quadratic function.

Although this discussion has been in terms of supply quantity as the strategic variable, the choice of the generator's strategic decision variable is not important. For example, the generator's strategy can involve: offer price, as in the Bertrand model; supply quantity, as in the Cournot model; or supply function, as in the supply function model.[2] Without loss of generality, we choose the supply quantity as the generator's strategic variable in this paper.

Consider supply quantity $q_i \in [q_i^{\min}, q_i^{\max}]$, where $q_i^{\min}$ and $q_i^{\max}$ are generator i's output lower and upper limit, respectively. Define $(d\Pi_i/dq_i)(q_i^{\max}+) = (d\Pi_i/dq_i)(q_i^{\min}-) = 0$. The supply quantity q_i maximizes generator i's profit if it satisfies the following first-order necessary conditions (FONCs):

$$\begin{aligned} \frac{d\Pi_i}{dq_i}(q_i+) &= P_i(q_i+) + P_i'(q_i+)(q_i+) - C_i'(q_i+) \leq 0 \\ \frac{d\Pi_i}{dq_i}(q_i-) &= P_i(q_i-) + P_i'(q_i-)(q_i-) - C_i'(q_i-) \geq 0. \end{aligned} \tag{4}$$

By (2), the point on the residual demand curve with quantity and price satisfying (4) maximizes the generator's profit.

Profit maximizing bidding strategies have been widely studied in the absence of transmission constraints. For example, [10] and [12]–[16] are empirical analyses: [10] analyzed the impact of bilateral contracts using the 1997 Australian market data, [12] measured the unilateral market power in the California market from 1998 to 2000, [13] analyzed the strategic behaviors in New Zealand market from 2001 to 2007, [14] benchmarked the supply function equilibrium model for the ERCOT zonal balancing market form 2002 to 2003, [15] analyzed the bidding behaviors in the ERCOT zonal balancing market from 2001 to 2003, and [16] compared the Cournot model and supply function equilibrium model for the German electricity market. Reference [17] optimizes a generators' expected profit based on historical residual demand curves. Reference [18] characterizes the residual demand in a transportation network.

[1]Electricity markets in the U.S. that accept piecewise linear offers include the ERCOT zonal and nodal markets, the Midwest ISO market, and the Southwest Power Pool market.

[2]Although the generator's strategic decision variable is not important in calculating the generator's maximum profit, it will make a huge difference for a Nash Equilibrium model. In other words, the Nash Equilibrium, if it exists, largely depends on the strategy space used in the model as has been observed in many references, such as [11]. Characterizing the Nash Equilibrium is beyond the scope of this paper.

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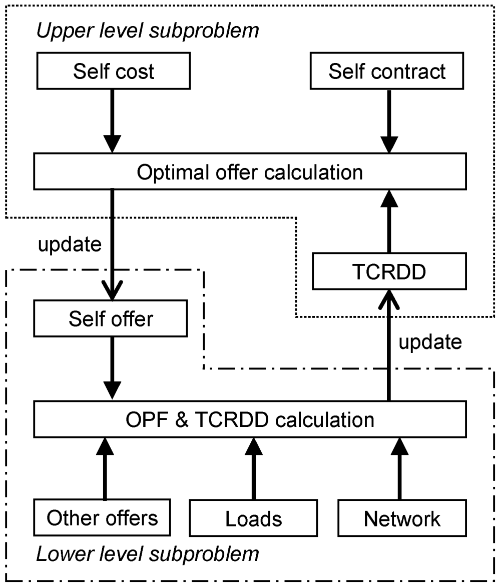

Fig. 1. Maximizing generation profit based on TCRDD.

Characterizing a transmission-constrained residual demand curve requires more rigorous calculation as discussed in [19]. Nevertheless, given a residual demand curve, whether transmission-constrained or not, the calculation of the profit maximizer can be carried out in the same way.

The fact that the profit maximizer can be calculated based on the given residual demand curve suggests that the generator's profit maximization problem can be decoupled into two subproblems: the upper subproblem of maximizing the generator's profit based on TCRDD, and the lower subproblem of calculating the TCRDD, as illustrated in Fig. 1. This gives rise to the new generator profit maximization approach in this paper, and we will refer it as the TCRDD approach hereafter. In the TCRDD approach, the upper problem and lower problem are not in nested structure, and can be solved separately. The whole problem can be solved by iteratively solving the two subproblems. The advantage of the TCRDD method over the MPEC approach is that the full network model is represented in the OPF, but not in the profit optimization, which greatly simplifies the profit maximization problem. In addition, the OPF is a subroutine in the lower level subproblem, so existing advanced OPF algorithms and solvers can be incorporated into the algorithm, i.e., "reused."

The organization of the rest of the paper is as follows. Section III improves the TCRDD calculation by aligning with commonly used OPF models. Section IV proposes an algorithm to maximize a generator's profit based on the TCRDD. Section V applies the proposed approach to the IEEE 118-bus system, and compares the results with the MPEC method. Section VI concludes.

III. Solving the Lower Subproblem: TCRDD Calculation

As illustrated in Fig. 1, the TCRDD bridges between the full-network model OPF and profit maximization. An efficient TCRDD calculation is crucial for the success of the TCRDD method. The concept of transmission-constrained TCRDD was introduced in [19] as a sensitivity analysis of the OPF solution. From the implementation point of view, the TCRDD formula [19, equation (29)] can be improved towards more practicality and efficiency.

To present the nodal TCRDD concept clearly, [19] used a simplified nodal injection OPF model. Although the model captures the network characteristics, it is not perfectly aligned with most production level OPF formulation and design, and thus may not be suitable for practical implementation. In this section, we will re-derive the TCRDD using a commonly used production level OPF formulation, which will improve the practicality of the TCRDD calculation.

We consider the following DC OPF model:

$$\min_{\mathbf{q}_{\mathsf{G}}} \quad \sum_{g\in\mathsf{G}} O_g(q_g) \tag{5}$$

$$\text{s.t.} \quad \mathbf{H}_{\mathsf{G}}\mathbf{q}_{\mathsf{G}} - \mathbf{H}_{\mathsf{L}}\mathbf{q}_{\mathsf{L}} \le \mathbf{Z} \tag{6}$$

$$\mathbf{q}_{\mathsf{G}}^{\min} \le \mathbf{q}_{\mathsf{G}} \le \mathbf{q}_{\mathsf{G}}^{\max} \tag{7}$$

$$\sum_{l\in\mathsf{L}} q_l - \sum_{g\in\mathsf{G}} q_g = 0 \tag{8}$$

where

- G is the set of generators, with each segment of an offer represented by a distinct element;
- L is the set of loads;
- $\mathbf{q}_{\mathsf{G}}$ is the generator output variable vector;
- $\mathbf{q}_{\mathsf{L}}$ is the load vector;
- $O_g(q_g)$ is generator g's total offer cost function, whose derivative, $O'_g(q_g)$, is generator g's offer function;
- $\mathbf{H}_{\mathsf{G}}$ is the generator shift factor matrix corresponding to the transmission constraints;
- $\mathbf{H}_{\mathsf{L}}$ is the load shift factor matrix corresponding to the transmission constraints;
- $\mathbf{q}_{\mathsf{G}}^{\min}$ consists of generator operating lower limits;
- $\mathbf{q}_{\mathsf{G}}^{\max}$ consists of generator operating upper limits;
- $\mathbf{Z}$ consists of the transmission capacity limits;
- (6) consists of the transmission constraints;
- (7) is the generator capacity constraint;
- (8) is the energy balance constraint.

Although this OPF model seems very simple, it is general enough to capture most of the advanced features in production level OPF programs.[3] Currently, most production level OPF programs in existing and proposed nodal electricity markets either are DC, or solve AC by successive linearization, such as in the CAISO nodal market [20] and the ERCOT nodal market. A typical successive linearization scheme is described in [7], where the AC OPF is decoupled into two subproblems, namely the optimization problem and the network assessment problem. The network assessment problem is to solve the AC power flow and contingency analysis, and generate a list of overloaded (and/or nearly overloaded) lines and nomograms to be passed to the optimization problem as constraints. The optimization problem

[3]This is a single interval OPF model, which is applicable to most the real-time electricity markets in the U.S., such as PJM, ISO-NE, and ERCOT. Very few electricity markets, such as the California nodal market, solve a multi-interval look-ahead OPF in the real-time market, where they include the inter-temporal constraints, such as the ramp constraints. Handling these multi-interval inter-temporal constraints is beyond the scope of this paper.

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models these constraints linearly by using sensitivities, such as the shift factors, outage compensated shift factors [21], and generalized nomogram shift factors.

The optimization problem solves the OPF program using the linearized constraints passed from the network assessment problem, so it is much easier to solve than the original problem with AC power flow constraints, and is able to solve large scale problems. The optimization problem and network assessment problem will be solved iteratively, so if the process converges, it converges to a solution to the original OPF problem. The linearized OPF optimization problem in the production level OPF program is essentially the same as the DC OPF problem we consider in this paper. Even for "pure" AC OPF solved by nonlinear programming, as long as the transmission constraints can be linearized in a post-processing step, the results of this paper are still applicable.

The subsequent derivation largely follows [19]. As discussed in [19], the TCRDD is a post-OPF solution calculation. An OPF variable with a hat represents the OPF solution, and the binding transmission constraints set is denoted by b. Denote the LMP at the slack bus by λ. The OPF solves with a slack bus price $\lambda = \hat{\lambda}$. Following [19], without loss of generality, we calculate the TCRDD for a generator s located at the slack bus n. Because the TCRDD is a post-OPF solution calculation, any bus can be designated as the "slack" bus for TCRDD calculation purpose, and it is not necessary to use the same slack bus that is used in the OPF.

Partition the generator segments other than generator s into three subsets: the generator segments with binding output quantities, denoted by f; the generator segments with constant price offers, denoted by z; and the generator segments with offers having nonzero slopes, denoted by v, so that

$$\mathbf{q}_{\mathsf{G}} = [\mathbf{q}_{\mathsf{v}} \quad \mathbf{q}_{\mathsf{f}} \quad \mathbf{q}_{\mathsf{z}} \quad q_s]^{\mathrm{T}}.$$

By doing this, we can explicitly handle constant price offers and binding generation capacity constraints, which make the matrix $\bar{\mathbf{O}}''_{\mathsf{G}}$ singular in [19]. Accordingly, partition $\mathbf{H}_{\mathsf{Gb}}$ into

$$\mathbf{H}_{\mathsf{Gb}} = [\mathbf{H}_{\mathsf{vb}} \quad \mathbf{H}_{\mathsf{fb}} \quad \mathbf{H}_{\mathsf{zb}} \quad 0].$$

Similarly to [19], we consider the OPF solution to be parameterized by the price at the slack bus λ, with the TCRDD defined by

$$\begin{aligned}\frac{dR_G}{d\lambda}(\hat{\lambda}) &= \frac{d\left(\sum_{l\in\mathsf{L}} q_l - \sum_{g\in\mathsf{G},g\neq s} q_g\right)}{d\lambda}\\ &= -\sum_{g\in\mathsf{G},g\neq s}\frac{dq_g}{d\lambda}(\hat{\lambda})\\ &= -\mathbf{1}_{\mathsf{v}}^{\mathrm{T}}\frac{d\mathbf{q}_{\mathsf{v}}}{d\lambda}(\hat{\lambda}) - \mathbf{1}_{\mathsf{f}}^{\mathrm{T}}\frac{d\mathbf{q}_{\mathsf{f}}}{d\lambda}(\hat{\lambda}) - \mathbf{1}_{\mathsf{z}}^{\mathrm{T}}\frac{d\mathbf{q}_{\mathsf{z}}}{d\lambda}(\hat{\lambda})\end{aligned}$$

where $\mathbf{1}_{\mathsf{v}}$, $\mathbf{1}_{\mathsf{f}}$, and $\mathbf{1}_{\mathsf{z}}$ are column vectors of 1 s whose dimensions equal the number of generators in sets v, f, and z, respectively. By definition

$$\frac{d\mathbf{q}_{\mathsf{f}}}{d\lambda}(\hat{\lambda}) = \mathbf{0}_{\mathrm{f}} \tag{9}$$

so

$$\frac{dR_G}{d\lambda}(\hat{\lambda}) = -\mathbf{1}_{\mathsf{v}}^{\mathrm{T}}\frac{d\mathbf{q}_{\mathsf{v}}}{d\lambda}(\hat{\lambda}) - \mathbf{1}_{\mathsf{z}}^{\mathrm{T}}\frac{d\mathbf{q}_{\mathsf{z}}}{d\lambda}(\hat{\lambda}). \tag{10}$$

Construct the Lagrangian of the OPF with only the binding constraints:

$$\begin{aligned}\mathcal{L} = &\sum_{g\in\mathsf{G}} O_g(q_g) + \lambda\left(\sum_{l\in\mathsf{L}} q_l - \sum_{g\in\mathsf{G}} q_g\right)\\ &- \boldsymbol{\mu}_{\mathsf{b}}^T(\mathbf{H}_{\mathsf{vb}}\mathbf{q}_{\mathsf{v}} + \mathbf{H}_{\mathsf{fb}}\mathbf{q}_{\mathsf{f}} + \mathbf{H}_{\mathsf{zb}}\mathbf{q}_{\mathsf{z}} - \mathbf{H}_{\mathsf{Lb}}\mathbf{q}_{\mathsf{L}} - \mathbf{Z}_{\mathsf{b}})\\ &+ \boldsymbol{\rho}_{\max}^{\mathrm{T}}(\mathbf{q}_{\mathsf{f}} - \mathbf{q}_{\mathsf{f}}^{\max}) + \boldsymbol{\rho}_{\min}^{\mathrm{T}}\left(\mathbf{q}_{\mathsf{f}} - \mathbf{q}_{\mathsf{f}}^{\min}\right).\end{aligned}$$

Similarly to [19], we calculate $(d\mathbf{q}_{\mathsf{v}}/d\lambda)(\hat{\lambda})$, $(d\mathbf{q}_{\mathsf{z}}/d\lambda)(\hat{\lambda})$, and $(d\boldsymbol{\mu}_{\mathsf{b}}/d\lambda)(\hat{\lambda})$ from a sensitivity analysis of the following OPF FONCs:

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial\mathbf{q}_{\mathsf{v}}} &= \mathbf{0}_{\mathsf{v}}\\ \frac{\partial\mathcal{L}}{\partial\mathbf{q}_{\mathsf{z}}} &= \mathbf{0}_{\mathsf{z}}\\ \frac{\partial\mathcal{L}}{\partial\boldsymbol{\mu}_{\mathrm{b}}} &= \mathbf{0}_{\mathrm{b}}\end{aligned}$$

i.e.

$$\begin{aligned}\mathbf{O}'_{\mathsf{v}}(\mathbf{q}_{\mathsf{v}}) - \lambda\mathbf{1}_{\mathsf{v}} - \mathbf{H}_{\mathsf{vb}}^{\mathrm{T}}\boldsymbol{\mu}_{\mathrm{b}} &= \mathbf{0}_{\mathsf{v}}\\ \mathbf{O}'_{\mathsf{z}}(\mathbf{q}_{\mathsf{z}}) - \lambda\mathbf{1}_{\mathsf{z}} - \mathbf{H}_{\mathsf{zb}}^{\mathrm{T}}\boldsymbol{\mu}_{\mathrm{b}} &= \mathbf{0}_{\mathsf{z}}\\ \mathbf{H}_{\mathsf{vb}}\mathbf{q}_{\mathsf{v}} + \mathbf{H}_{\mathsf{fb}}\mathbf{q}_{\mathsf{f}} + \mathbf{H}_{\mathsf{zb}}\mathbf{q}_{\mathsf{z}} - \mathbf{H}_{\mathsf{Lb}}\mathbf{q}_{\mathsf{L}} &= \mathbf{Z}_{\mathrm{b}}\end{aligned} \tag{11}$$

where

- $\mathbf{O}'_{\mathsf{v}}(\mathbf{q}_{\mathsf{v}}) = \nabla_{\mathbf{q}_{\mathsf{v}}}(\sum_{g\in\mathsf{G}} O_g(q_g))$;
- $\mathbf{O}'_{\mathsf{z}}(\mathbf{q}_{\mathsf{z}}) = \nabla_{\mathbf{q}_{\mathsf{z}}}(\sum_{g\in\mathsf{G}} O_g(q_g)) = \mathbf{p}_{\mathsf{z}}$, i.e., constant price offer segments;
- $\mathbf{0}_{\mathsf{v}}$, $\mathbf{0}_{\mathsf{f}}$, and $\mathbf{0}_{\mathsf{z}}$ are column vectors of 0 s whose dimensions equal the number of generators in sets v, f, and z, respectively.

Differentiating both sides of (11) with respect to λ, we get

$$\begin{aligned}\mathbf{O}''_{\mathsf{v}}\frac{d\mathbf{q}_{\mathsf{v}}}{d\lambda} - \mathbf{1}_{\mathsf{v}} - \mathbf{H}_{\mathsf{vb}}^{\mathrm{T}}\frac{d\boldsymbol{\mu}_{\mathrm{b}}}{d\lambda} &= \mathbf{0}_{\mathsf{v}}\\ -\mathbf{1}_{\mathsf{z}} - \mathbf{H}_{\mathsf{zb}}^{\mathrm{T}}\frac{d\boldsymbol{\mu}_{\mathrm{b}}}{d\lambda} &= \mathbf{0}_{\mathsf{z}}\\ \mathbf{H}_{\mathsf{vb}}\frac{d\mathbf{q}_{\mathsf{v}}}{d\lambda} + \mathbf{H}_{\mathsf{zb}}\frac{d\mathbf{q}_{\mathsf{z}}}{d\lambda} &= \mathbf{0}_{\mathrm{b}}\end{aligned} \tag{12}$$

where

$$\mathbf{O}''_{\mathsf{v}} = \nabla^2_{\mathbf{q}_{\mathsf{v}}\mathbf{q}_{\mathsf{v}}}\left(\sum_{g\in\mathsf{G}} O_g(q_g)\right).$$

Assuming the regularity conditions hold, the TCRDD can be calculated by solving (12), and substituting $(d\mathbf{q}_{\mathsf{v}}/d\lambda)(\hat{\lambda})$ and $(d\mathbf{q}_{\mathsf{z}}/d\lambda)(\hat{\lambda})$ into (10).

Compared with the model in [19], the new model can explicitly represent either a physical generation unit or a generation portfolio located at a single pricing location (node or zone). The matrix $\mathbf{O}''_{\mathsf{v}}$ is always invertible by definition, and thus, this TCRDD calculation is more generally applicable than

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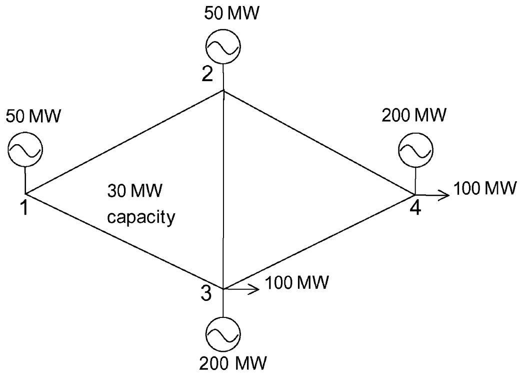

Fig. 2. Four-bus system.

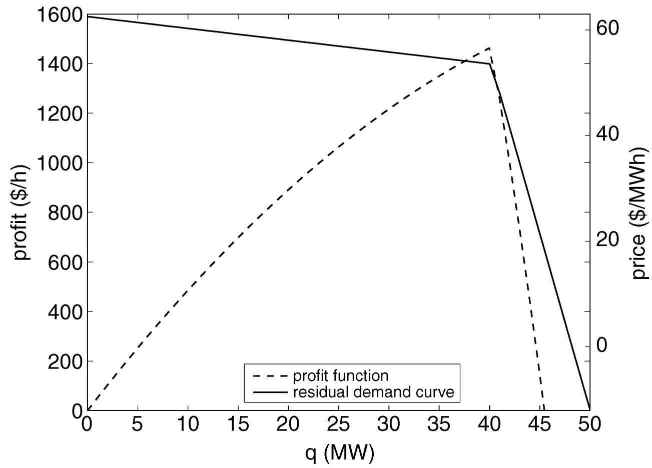

Fig. 3. Profit function and residual demand curve for generator 1.

[19]. In addition, the TCRDD formula derived here only depends on binding transmission constraints and the offers of the generators having nonzero slopes, and thus can be calculated more efficiently than [19], which depends on both transmission constraints and all generation capacity constraints. With these improvements, the TCRDD can be calculated very efficiently post-OPF solution.

IV. Solving the Upper Subproblem: Maximizing Profit Based on TCRDD

In this section, we propose an algorithm to maximize a generator's profit utilizing the TCRDD information that can be calculated as discussed in Section III.

To make the idea easy to understand, we will illustrate the algorithm in a very simple four-bus system example. A larger scale 118-bus system will be solved and discussed in Section V. The four-bus example is very similar to the one in [19]. As illustrated in Fig. 2, there are four generators in the system, and each one is located at a different bus. Generators 1 and 2 each have 50-MW capacity. Generators 3 and 4 each have 200-MW capacity. There are two loads, each 100 MW, located at bus 3 and bus 4, respectively. Branch 1-3 has a capacity limit of 30 MW. All other branches have capacities of 200 MW. All branches have the same impedance. The generators' cost functions are

$$C_1(q_1) = 0.175q_1^2 + 10q_1, \quad C_2(q_2) = 0.497q_2^2 + 10q_2$$
$$C_3(q_3) = 0.260q_3^2 + 20q_3, \quad C_4(q_4) = 0.325q_4^2 + 20q_4.$$

Assume all generators offer at their true marginal cost.

From now on, to simplify notation, we drop the subscript i in all the variables and functions, meaning that all the variables and functions are relevant to the generator under consideration. In this example, we consider generator 1. We plot generator 1's profit function and residual demand in Fig. 3. Each point on the residual demand or the profit function is produced by solving an OPF at the corresponding output level. While this is useful for understanding profit maximization for this example, optimizing the profit by plotting the profit function is not a practical approach for large scale problems.

Because the decision variable for a generator is its output level, the problem to find a local optimum is basically a line search. There are various existing standard line search algorithms, such as the Wolfe condition and the Armijo-Goldstein condition [22]. However, they typically require a relatively large number of function evaluations, which may not be efficient for this specific problem, because evaluating a generator's profit involves solving the OPF, which is computationally intense for large scale systems.

In order to improve the performance, we developed a special algorithm aimed at requiring less profit function evaluations. The algorithm can be viewed as a line search applied to each segment of the residual demand curve. Each segment of the piecewise linear residual demand curve corresponds to one specific combination of $(\mathsf{v}, \mathsf{f}, \mathsf{z}, \mathsf{b})$. The slope of that segment, i.e., the TCRDD, is determined by solving (12). As shown in Fig. 3, generator 1's residual demand curve has two segments with the kink at 43 MW resulting from a binding constraint change. Branch 1-3 is a nonbinding constraint with generator 1's output q less than 43 MW, and becomes a binding constraint with q greater than 43 MW.

The proposed algorithm will search over the linear segments on the residual demand curve to look for the profit maximizer, so the number of iterations will be bounded by the number of segments on the residual demand curve. The algorithm is expected to have better performance than a standard line search, because possible combinations of $(\mathsf{v}, \mathsf{f}, \mathsf{z}, \mathsf{b})$, are typically very limited in an actual power system. On the one hand, the transmission network is typically adequately planned, so that very few transmission lines get congested. On the other hand, there are typically a large volume of self-schedules in an electricity market due to forward contracts, which greatly reduces the dispatching range of units, and thus the number of possible combinations of $(\mathsf{v}, \mathsf{f}, \mathsf{z})$.

Similar to most local search algorithms, such as the MPEC approach [5], this algorithm is designed to find a local maximum, and cannot guarantee that the local maximum is also the global maximum. However, the approach in this paper can be provided with different initial points to explore a broader region in order to seek the global optimizer.

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Now we define some notations. Suppose we have an OPF solved at (p_0, q_0) for the generator. Denote the TCRDD evaluated at q_0 by $TCRDD(q_0)$, and the tangent of the inverse residual demand curve by

$$\overline{P}(q; q_0) = \frac{1}{TCRDD(q_0)}(q - q_0) + p_0. \tag{13}$$

$\overline{P}(q; q_0)$ matches the linear segment on the residual demand curve containing q_0. In the four-bus example, for the particular value $q_0 = 5$, $\overline{P}(q; 5)$ matches the residual demand curve for $q \in [0, 43]$.

A corresponding approximation to the generator's profit $\Pi(q)$ is

$$\overline{\Pi}(q; q_0) = \overline{P}(q; q_0)q - C(q) \tag{14}$$

by replacing $P(q)$ with $\overline{P}(q; q_0)$ in (3). If the generator cost function is quadratic, this approximated profit function is also quadratic. Define

$$\overline{q}(q_0) = \arg\max\left\{\overline{\Pi}(q; q_0)\right\}$$

which will be used later.[4]

Generally, a one-piece approximation cannot accurately model the possibly kinked residual demand curve. It is necessary to add another piece to the approximation so that it could handle kinks. For example, in addition to a solved OPF at $q_0 = 5$, if we have another solved OPF at $q_0 = 48$, then the two-piece function $\hat{P}(q; 5, 48)$ defined as

$$\hat{P}(q; 5, 48) = \overline{P}(q, 5), \quad q \in [0, 43],$$
$$\hat{P}(q; 5, 48) = \overline{P}(q, 48), \quad q \in (43, 50]$$

matches the residual demand curve exactly. Similarly, we can define the corresponding two-piece profit approximation function as

$$\hat{\Pi}(q; 5, 48) = \hat{P}(q; 5, 48)q - C(q)$$

which also exactly matches the true profit function in this case.

Generally, suppose we have two solved OPFs at q^{lo} and q^{hi}, respectively, and $\overline{P}(q; q^{\text{lo}})$ and $\overline{P}(q; q^{\text{hi}})$ intersect at q^x, where $q^x \in [q^{\text{lo}}, q^{\text{hi}}]$. We will approximate $P(\bullet)$ by the two-piece function $\hat{P}(\bullet; q^{\text{lo}}, q^{\text{hi}})$ defined as follows:

$$\hat{P}(q; q^{\text{lo}}, q^{\text{hi}}) = \overline{P}(q; q^{\text{lo}}), \quad \forall q \le q^x$$
$$\hat{P}(q; q^{\text{lo}}, q^{\text{hi}}) = \overline{P}(q; q^{\text{hi}}), \quad \forall q > q^x. \tag{15}$$

Accordingly, we approximate the profit function by

$$\hat{\Pi}(q; q^{\text{lo}}, q^{\text{hi}}) = \hat{P}(q; q^{\text{lo}}, q^{\text{hi}})q - C(q)$$

which has the same kink at q^x as $\hat{P}(q; q^{\text{lo}}, q^{\text{hi}})$.

The whole algorithm consists of two parts: the Local Optimum Screening Loop and the Bisection Loop. The Local Optimum Screening Loop is to find two output levels q^{lo} and q^{hi} that satisfy

$$q^{\text{lo}} \le \overline{q}(q^{\text{lo}}), \quad q^{\text{hi}} \ge \overline{q}(q^{\text{hi}}). \tag{16}$$

[4]Strictly speaking, $\arg\max\{\bullet\}$ represents the set of maximizers. Because $\overline{\Pi}(q; q_0)$ is a quadratic function with a unique maximizer, we can use $\arg\max\{\overline{\Pi}(q; q_0)\}$ to denote the maximizer itself.

Condition (16) means profit increases in opposite directions at q^{lo} and q^{hi}, which implies there is a local optimum in $[q^{\text{lo}}, q^{\text{hi}}]$. The Local Optimum Screening Loop is designed to start at some q^0, and check whether profit increases in opposite directions at q^0 and $\overline{q}(q^0)$. If profit increases in the same direction from these points, then reset q^0 to $\overline{q}(q^0)$. If $\overline{q}(q^0)$ is outside the interval $[q^{\min}, q^{\max}]$, replace $\overline{q}(q^0)$ with the corresponding boundary $q^{\min}$ or $q^{\max}$ (step 4).

Local Optimum Screening Loop:

1) Start with an initial point $q = q^0$.
2) Solve OPF with $q = q^0$, and calculate $TCRDD(q^0)$ and $\overline{q}(q^0)$.
3) If $\overline{q}(q^0) = q^0$, local optimal solution found with $q^* = q^0$, stop.
   Or if $q^0 = q^{\max}$ and $\overline{q}(q^0) > q^{\max}$, local optimal solution found with $q^* = q^{\max}$, stop.
   Or if $q^0 = q^{\min}$ and $\overline{q}(q^0) < q^{\min}$, local optimal solution found with $q^* = q^{\min}$, stop.
4) If $\overline{q}(q^0) > q^0$, let $q^1 = \min\{\overline{q}(q^0), q^{\max}\}$, else let $q^1 = \max\{\overline{q}(q^0), q^{\min}\}$. Solve OPF with $q = q^1$, and calculate $TCRDD(q^1)$ and $\overline{q}(q^1)$.
5) If $\overline{q}(q^1) = q^1$, local optimal solution found with $q^* = q^1$, stop.
   Or if $q^1 = q^{\max}$ and $\overline{q}(q^1) > q^{\max}$, local optimal solution found with $q^* = q^{\max}$, stop.
   Or if $q^1 = q^{\min}$ and $\overline{q}(q^1) < q^{\min}$, local optimal solution found with $q^* = q^{\min}$, stop.
6) If $(\overline{q}(q^1) - q^1)(\overline{q}(q^0) - q^0) < 0$, local optimum exists in

   $$\left[\min\{q^0, q^1\}, \max\{q^0, q^1\}\right].$$

   Let $q^{\text{lo}} = \min\{q^0, q^1\}$, and $q^{\text{hi}} = \max\{q^0, q^1\}$, stop.
   Otherwise, $q^0 = q^1$, and continue with step 4).

After the Local Optimum Screening Loop, we either have found a local optimum, with $\overline{q}(q^0) = q^0$ in step 3) or $\overline{q}(q^1) = q^1$ in step 5); or we end up with two output levels q^{lo} and q^{hi} that satisfy (16), in which case we can start the bisection process. In the four-bus example, for generator 1, the Local Optimum Screening Loop will find two output levels q^{lo} and q^{hi} that satisfy (16) after one iteration starting from any $q^0 \neq 43$.

The Bisection Loop searches for a local profit maximizer in the segment $[q^{\text{lo}}, q^{\text{hi}}]$ resulting from the Local Optimum Screening Loop. It is a special bisection process in that it bisects at the maximizer of $\hat{\Pi}(q; q^{\text{lo}}, q^{\text{hi}})$ if $\overline{P}(q; q^{\text{lo}})$ and $\overline{P}(q; q^{\text{hi}})$ intersect at a unique point $q^x \in [q^{\text{lo}}, q^{\text{hi}}]$. In this case, because $\hat{\Pi}(q; q^{\text{lo}}; q^{\text{hi}})$ is a two-piece quadratic function, its maximizer can be determined as specified in Table I and illustrated in Fig. 4. If $\overline{P}(q; q^{\text{lo}})$ and $\overline{P}(q; q^{\text{hi}})$ do not intersect, or they intersect at (q^x, p^x) with $q^x \notin [q^{\text{lo}}, q^{\text{hi}}]$, $\hat{\Pi}(q; q^{\text{lo}}; q^{\text{hi}})$ cannot suggest a meaningful bisection point, so we bisect at $0.5(q^{\text{lo}} + q^{\text{hi}})$ instead. The bisection choice is determined in step 2). A local maximum can fall into one of the two cases: a differentiable maximum as in the "left hump,", "right hump," or "double hump" case, or a nondifferentiable maximum as in the "no hump" case. A differentiable maximum can be determined

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TABLE I
DETERMINE MAXIMIZER OF $\hat{\Pi}(q; q^{\mathrm{lo}}; q^{\mathrm{hi}})$

condition	characteristic	maximizer of $\hat{\Pi}\left(q; q^{\mathrm{lo}}; q^{\mathrm{hi}}\right)$
$\overline{q}(q^{\mathrm{lo}}) \notin [q^{\mathrm{lo}}, q^x]$, $\overline{q}(q^{\mathrm{hi}}) \notin [q^x, q^{\mathrm{hi}}]$	no hump	q^x
$\overline{q}(q^{\mathrm{lo}}) \in [q^{\mathrm{lo}}, q^x]$, $\overline{q}(q^{\mathrm{hi}}) \notin [q^x, q^{\mathrm{hi}}]$	left hump	$\overline{q}(q^{\mathrm{lo}})$
$\overline{q}(q^{\mathrm{lo}}) \notin [q^{\mathrm{lo}}, q^x]$, $\overline{q}(q^{\mathrm{hi}}) \in [q^x, q^{\mathrm{hi}}]$	right hump	$\overline{q}(q^{\mathrm{hi}})$
$\overline{q}(q^{\mathrm{lo}}) \in [q^{\mathrm{lo}}, q^x]$, $\overline{q}(q^{\mathrm{hi}}) \in [q^x, q^{\mathrm{hi}}]$	double hump	$\overline{q}(q^{\mathrm{lo}})$ or $\overline{q}(q^{\mathrm{hi}})$

in step 4) when $\overline{q}(q^{\mathrm{mid}}) = q^{\mathrm{mid}}$. A nondifferentiable maximum can be determined in step 5) when at q^x satisfies (4). At the kink, the TCRDD calculation can only provide one-sided derivative, depending on which set of constraints are binding in OPF solution. To get the TCRDD on the other side, we do the incremental test in step 5), where we change q^x by a small amount ϵ so that it goes to the other side of the kink.

Bisection Loop:

1) If $q^{\mathrm{hi}} - q^{\mathrm{lo}} < \epsilon$, where $\epsilon > 0$ is the tolerance threshold, optimal solution found with $q^* = 0.5(q^{\mathrm{lo}} + q^{\mathrm{hi}})$. Stop.
2) Calculate the bisection point q^{mid} as follows.
   - If $\overline{P}(q; q^{\mathrm{lo}})$ and $\overline{P}(q; q^{\mathrm{hi}})$ specify the same function, $q^{\mathrm{mid}} = \overline{q}(q^{\mathrm{lo}})$.
   - If $\overline{P}(q; q^{\mathrm{lo}})$ and $\overline{P}(q; q^{\mathrm{hi}})$ do not intersect, or they intersect at (q^x, p^x) with $q^x \notin [q^{\mathrm{lo}}, q^{\mathrm{hi}}]$, then $q^{\mathrm{mid}} = 0.5(q^{\mathrm{lo}} + q^{\mathrm{hi}})$.
   - Otherwise, determine the bisection point q^{mid} by the maximizer of $\hat{\Pi}(q; q^{\mathrm{lo}}; q^{\mathrm{hi}})$ as specified in Table I.
3) Solve OPF with $q = q^{\mathrm{mid}}$, and calculate $TCRDD(q^{\mathrm{mid}})$ and $\overline{q}(q^{\mathrm{mid}})$.
4) If $\overline{q}(q^{\mathrm{mid}}) = q^{\mathrm{mid}}$, optimal solution reached with $q^* = q^{\mathrm{mid}}$. Stop.
5) If $q^{\mathrm{mid}} = q^x$ and $p^{\mathrm{mid}} = p^x$, do an incremental test as follows.
   - If $TCRDD(q^{\mathrm{mid}}) = TCRDD(q^{\mathrm{lo}})$, run OPF with $q = q^x + \epsilon$, and calculate $TCRDD(q^x + \epsilon)$, $\overline{q}(q^x + \epsilon)$. If $\overline{q}(q^x + \epsilon) \leq q^x + \epsilon$, then optimal solution reached with $q^* = q^x$ because q^x satisfies (4). Stop.
   - If $TCRDD(q^{\mathrm{mid}}) = TCRDD(q^{\mathrm{hi}})$, run OPF with $q = q^x - \epsilon$, and calculate $TCRDD(q^x - \epsilon)$, $\overline{q}(q^x - \epsilon)$. If $\overline{q}(q^x - \epsilon) \geq q^x - \epsilon$, then optimal solution reached with $q^* = q^x$ because q^x satisfies (4). Stop.
6) If $\overline{q}(q^{\mathrm{mid}}) > q^{\mathrm{mid}}$, then $q^{\mathrm{lo}} = q^{\mathrm{mid}}$, else $q^{\mathrm{hi}} = q^{\mathrm{mid}}$. Continue with step 1).

The Bisection Loop will update $\hat{P}(q; q^{\mathrm{lo}}, q^{\mathrm{hi}})$ and $\hat{\Pi}(q; q^{\mathrm{lo}}, q^{\mathrm{hi}})$ whenever the combination of $(\mathsf{v}, \mathsf{f}, \mathsf{z}, \mathsf{b})$ changes. In the four-bus system, the bisection loop will find the maximizer for generator 1 in step 5) after one iteration.

The Screening Loop and Bisection Loop always stop in a finite number of iterations bounded by the number of segments on the piecewise linear residual demand curve for $q \in [q^{\mathrm{min}}, q^{\mathrm{max}}]$. This is because in each iteration, the algorithm either finds a local optimum, or finds a new segment on the residual demand curve, and proceeds to the next iteration.

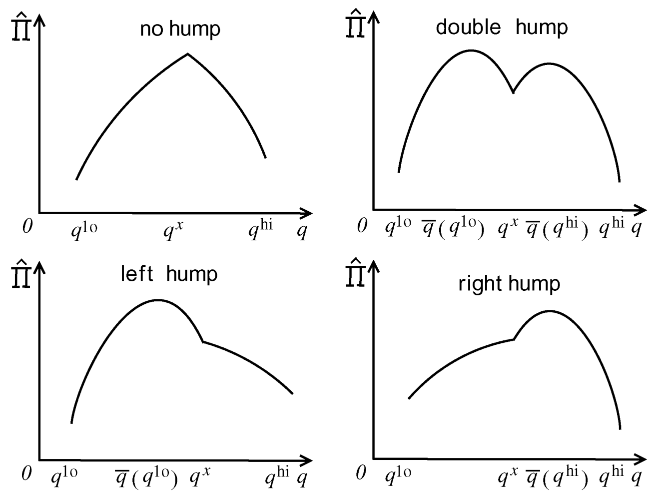

Fig. 4. Determine maximizer of $\hat{\Pi}(q; q^{\mathrm{lo}}; q^{\mathrm{hi}})$.

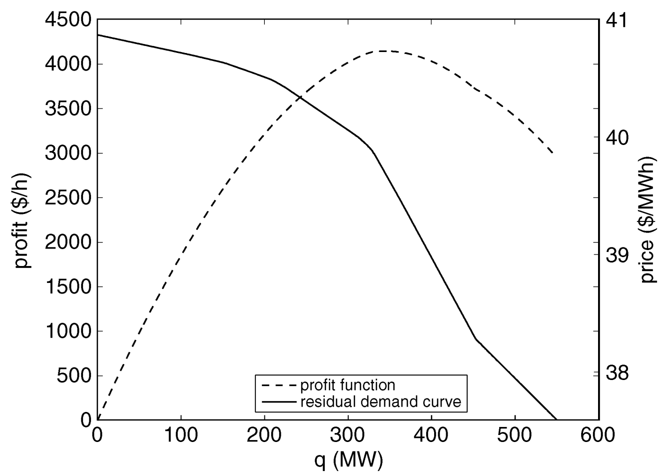

Fig. 5. Profit function and residual demand curve for generator 5.

The process does not revisit any segments on the residual demand curve, so it is finitely terminating.

V. COMPUTATIONAL EXAMPLE

In this section, we apply the algorithm to the IEEE 118-bus test system distributed in the MATPOWER package [23]. There are 186 branches and 54 generators in the system. The total load in the system is 4242 MW. We optimize the profit for generator 5 located at bus 10 with 550-MW capacity. Branches 30-17, 26-30, and 38-37 all have capacities of 200 MW, so they are likely to be binding transmission constraints. All other branches have capacities large enough such that the flows will be within their limits. We plot the residual demand curve and the profit function in Fig. 5.

From the profit curve in Fig. 5, one can tell the maximizer is between 344 MW and 345 MW. We start with an initial point $q^0 = 40$ MW, which is far away from the optimizer. The local optimum screening loop terminates after one iteration with $q^{\mathrm{lo}} = 40$ and $q^{\mathrm{hi}} = 439.4$. The bisection terminated after two iterations with the optimal solution $q^* = 344.76$. The first bisection iteration is a right hump case, and the second bisection iteration is a left hump case, as specified in Table 1.

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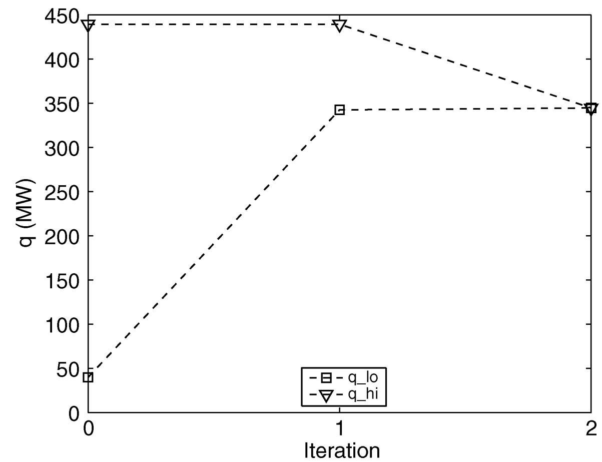

Fig. 6. Bisection loop iterations.

The bisection trajectory is illustrated in Fig. 6. The execution time on a 2.2-GHz Intel Core 2 Duo CPU laptop is 2.8 s.

We also solve the same problem using the MPEC method with the NLPEC solver [24]. The NLPEC solves an MPEC problem as an equivalent sequence of nonlinear programming problems parameterized by a scalar μ via reformulation of the complementarity constraints. The execution time is dependent on the initial value of μ. We have tried multiple initial values for μ in the set:

$$\{10^{-6}, 10^{-5}, \cdots, 1, \cdots, 10^{4}, 10^{5}\}.$$

They are all able to find exactly the same optimal solution, and the execution time ranges from 9.2 s with $\mu = 10^{-3}$ to 49.5 s with $\mu = 10^{2}$, which suggests even in the best scenario, the MPEC method solves about three times slower than the TCRDD method, and on average it is about six times slower.

VI. Conclusion

This paper proposed a new method to calculate profit maximizing offers for generators based on the TCRDD. The TCRDD method decouples the overall problem into two subproblems: the OPF and TCRDD calculation and the profit maximization based on TCRDD. The two subproblems will be solved iteratively, which captures the network characteristics accurately. The advantage of the TCRDD approach is that the profit maximization problem can be solved separately using the TCRDD instead of being bundled with the complicated full network model, which makes the MPEC method notoriously difficult to solve. In addition, the existing advanced OPF solvers could be reused in the TCRDD calculation. Due to these advantages, the TCRDD method is suitable for solving large scale problems. Through our experiments, the new algorithm has been performing well in terms of efficiency and robustness as demonstrated in the IEEE 118-bus example. However, more testing, on various systems (especially larger systems) is needed to reliably assess the performance of the algorithm. In addition, the generalization to the multi-generator firm case and consideration of inter-temporal constraints are under development.

The TCRDD approach can help market participants to bid into electricity markets and market monitors to diagnose bidding behaviors in the presence of transmission constraints.

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Lin Xu (M’07) received the B.S. and M.S. degrees in electrical engineering from Tianjin University, Tianjin, China, in 2000 and 2003, respectively, and the Ph.D. degree in electrical engineering from the University of Texas at Austin in 2009.

In 2006, he worked as an intern at the Lower Colorado River Authority. From 2007 to 2008, he was a Power Engineer at The Electric Reliability Council of Texas. He is currently a Power Engineer at the California Independent System Operator, Folsom, CA.

Ross Baldick (F’07) received the B.Sc. degree in mathematics and physics and the B.E. degree in electrical engineering from the University of Sydney, Sydney, Australia, and the M.S. and Ph.D. degrees in electrical engineering and computer sciences from the University of California, Berkeley, in 1988 and 1990, respectively.

From 1991 to 1992, he was a Postdoctoral Fellow at the Lawrence Berkeley Laboratory. In 1992 and 1993, he was an Assistant Professor at Worcester Polytechnic Institute, Worcester, MA. He is currently a Professor in the Department of Electrical and Computer Engineering at The University of Texas at Austin.

Yohan Sutjandra (M’06) received B.S. and M.S. degrees in electrical engineering from The University of Texas at Austin in 2000 and 2009, respectively.

From 2000 to 2006, he was a Software Engineer at the Cadence Design Systems. He currently works at The Energy Authority as an Energy Analyst, Jacksonville, FL.