Computing the Electricity Market Equilibrium: Uses of market equilibrium models

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Abstract

We discuss the formulation of electricity market equilibrium models, distinguishing the physical, commercial, and economic models. We outline the uses of such models, qualified in the light of the many assumptions that must be made for them to be tractable. We discuss the types of questions that can be sensibly answered by such models.

Outline

1. Introduction,
2. Model formulation,
3. Market operation and price formation,
4. Equilibrium and solution,
5. Validity, uses, and limitations of equilibrium models,
6. Conclusion.

1. Introduction

- Electricity market equilibrium modelling has progressed significantly in the last two decades.
- Basic unifying principle is the **Nash equilibrium**.
- Four difficulties of applying Nash equilibrium to electricity markets:
  1. non-convexity of generator feasible operating region or of operating costs, non-concavity of generator profit function,
  2. inelastic demand,
  3. complexity of electricity market rules, and
  4. representation of regulatory intervention.

2. Model formulation

Consider the modelling of:

1. Transmission network,
2. Generator cost function and operating characteristics,
3. Offer function,
4. Demand, and
5. Uncertainty.

For each, we will distinguish the:

Physical model: a (notionally) exact model of the physical characteristics.

Commercial model: the model used in the actual market.

Economic model: the model used in the equilibrium formulation.

2.1. Transmission network

2.1.1. Physical model

- Kirchhoff’s laws (equality constraints),
- Thermal, voltage, and stability constraints (inequality constraints).

2.1.2. Commercial network model

- Simplified transmission model used in market.
- Examples for Kirchhoff's laws:
  - DC power flow,
  - buses aggregated into zones joined by equivalent lines ("commercially significant constraints" in ERCOT).
- Examples for inequality constraints:
  - limits on real power flow in DC power flow model,
  - limits on flow on commercially significant constraints.

Transmission network model, continued

2.1.3. Economic model

- Further simplified model used in equilibrium analysis,
- Examples:
  - ignore transmission constraints,
  - only consider pricing intervals when transmission constraints are not binding,
  - simplified network model,
  - assume that market participants ignore the effect of their actions on transmission congestion or on congestion prices.
- Simplifies the profit maximization problem faced by generators.
  - For example, assuming that participants ignore the effect of their actions on congestion removes potential non-concavities from participant profit functions.

2.2. Generator cost function and operating characteristics

2.2.1. Physical model

- Thermal generators have energy costs, unit commitment issues, reserves and reactive power capability, ramp and other constraints on operation.
- Energy cost functions for thermal generation are non-linear functions of production.
- Hydro generators have low, roughly constant, marginal costs, but are energy limited.

2.2.2. Economic model

- Portfolio models abstract from unit commitment and other issues.
- May ignore the joint production of energy and ancillary services by a generator.

2.3. Offer function

2.3.1. Commercial model

- Complex versus simple offer functions,
- Requirements to hold offers constant over multiple intervals,
- Uncertainty managed through day-ahead markets, real-time markets, and ancillary services.
- Installed capacity markets.

2.3.2. Economic model

- Choice of strategic variable may abstract from the commercial model:
  - quantity, as in Cournot model, does not literally represent market rules,
  - supply functions are closer in form to requirements of market rules.
- Bilateral contract representation.

2.4. Demand

2.4.1. Physical model

- Temporal variation and uncertainty,
- Usually small (possibly zero) short-term price elasticity.

2.4.2. Commercial model

- Forecast of temporal variation,
- Uncertainty managed through day-ahead markets, real-time markets, and ancillary services.

2.4.3. Economic model

- Forecast of temporal variation,
- Estimate of elasticity:
  1. May be calibration to observed behavior,
  2. May be representation of "competitive" market participants.

2.5. Uncertainty

2.5.1. Physical model

- Demand, residual demand, fuel costs and availability, and equipment capacity are stochastic.

2.5.2. Commercial model

- Uncertainty in generator capacity and of demand is represented through:
  - day-ahead and real-time markets,
  - reserves and other ancillary services.

Uncertainty, continued

2.5.3. Economic model

- Many stochastic issues could be incorporated into the models.
- Uncertainty in demand is typically represented, but most other stochastic issues are typically not explicitly represented.
  - Consequently, effect on prices of stochastic issues may be absent.
- Real-time markets may not be explicitly modeled or may be modeled separately, ignoring the joint equilibrium between the markets.

3. Market operation and price formation

3.1. Physical model

- Lack of storage and limited elasticity of demand mean that action by ISO is necessary to match supply and demand through utilization of ancillary services.
- For example, real-time market deals with deviations from day-ahead market positions.

3.2. Commercial model

- The role of ancillary services in matching supply and demand is not explicitly represented in, for example, day-ahead energy market model,
- Ancillary services become critical under scarcity.

Market operation and price formation, continued

3.3. Economic model

- Models crossing of supply and demand.
- Typically ignores ancillary services:
  - Typically require elastic demand at each node to obtain well-defined prices.
  - Model results may be extremely dependent on the specification of demand elasticity.
- Typically ignores unit commitment and installed capacity markets.

4. Equilibrium and solution

- Equilibrium is set of participant offers such that no participant can improve its profit by unilaterally deviating from the offer.
- Suppose strategic variables are s_k for participants $k = 1, \ldots, n$.
- Suppose that profit to participant k is $\pi_k(s_k, s_{-k})$, where $s_{-k} = (s_\ell)_{\ell \neq k}$ is the collection of strategic variables of all the participants besides participant k.
- Then $(s_k^\star)_{k=1,\ldots,n}$ is a **pure strategy Nash equilibrium** if:

$$s_k^\star \in \arg\max_{s_k} \pi_k(s_k, s_{-k}^\star),$$

  where $s_{-k}^\star = (s_\ell^\star)_{\ell \neq k}$.
- “Single-shot” versus “repeated game.”

Equilibrium solution methods

4.1. Analytical models

- Solve for equilibria analytically.
- Possible for some simple cases:
  - Single pricing interval with certain demand,
  - Cournot model (strategic variables are quantities).

4.2. Fictitious play

- For complex models, a natural approach is to successively update the strategic variables.
- Each participant may find its profit maximizing response to the other participants' strategic variables and use that to update its own strategic variables.
- In principle, converges to "single-shot" pure strategy equilibrium if it exists.

Equilibrium solution methods, continued

4.3. Mathematical program with equilibrium constraints

- Model the market clearing mechanism by its optimality conditions,
- Incorporate optimality conditions into the optimization problems faced by each participant.
- May deliberately simplify the profit maximization problems to avoid non-concave profit functions for participants.

5. Validity, uses, and limitations of equilibrium models

- Are equilibrium models reasonable?
  - In the ERCOT balancing market, some smaller market participants' behavior is not consistent with a model of profit maximization.
  - Sometimes, there are only "mixed strategy" equilibria, but there is little evidence of randomized offers in actual electricity markets:
    * Simplifications of representation of transmission are typically aimed at ensuring concavity of generator profit function to help assure that pure strategy equilibria exist,
    * Not clear whether this simplification is an appropriate model of participant behavior.
  - There may be multiple equilibria, particularly for supply function equilibria, reducing the predictive value.
  - There are a large number of modelling assumptions.
- Cannot expect to predict outcomes and prices accurately!

Principled analysis of the effect of changes

- Evaluate alternative market rules such as:
  - allowing offers to change from interval to interval versus requiring offers to remain constant over multiple intervals, and
  - single clearing price versus pay-as-bid prices,
- Evaluate changes in market structure such as mandated divestitures,
- Effect of transmission constraints.

Principled analysis of the effect of changes

- Evaluate the effect of the level of contracts, such as:
  - physical and financial bilateral energy contracts, and
  - financial transmission rights,
- Evaluate modelling assumptions, such as:
  - the assumed form of cost functions or offer functions,
  - the use of portfolio-based versus unit-specific costs or offers, and
  - the representation of unit commitment.

Strategy to evaluate changes

- Hold most market rules and features constant.
- Vary one particular issue for a qualitative “sensitivity” analysis.
- Estimate the *change* due to the modeled variation.
- Allows the potential for policy conclusions to be made from studies even in the absence of absolute accuracy.
- Case studies:
  1. Market rules regarding changing of offers.
  2. Single clearing price versus pay-as-bid prices.
  3. Divestitures.

5.1. Market rules regarding changing of offers

- Single set of energy offers that must apply across all intervals in the day versus offers that can vary from hour to hour.
- A supply function equilibrium model can represent both cases.
- Many of the detailed features of electricity markets, including transmission constraints, might be ignored.
- Such an analysis was performed by Baldick and Hogan (2002, 2006).
- A rule requiring consistent offers can help to mitigate market power.

5.2. Single clearing price versus pay-as-bid prices

- Proposals for pay-as-bid markets usually neglect to realize that offers will change in response to changes in market rules.
- The "revenue equivalence theorem" suggests that prices should be the same.
- Not all of the assumptions required for the revenue equivalence theorem actually hold in electricity markets.
- A simplified model of an electricity market can be used to obtain a sensitivity result for the change between single clearing price and pay-as-bid prices.
- In some models of electricity markets, pay-as-bid pricing can result in lower equilibrium prices than in single clearing price markets (Fabria, 2000, and Son, Baldick, and Lee, 2004).
  - Effect is small and unlikely to compensate for downsides of pay-as-bid such as poor dispatch decisions.

5.3. Divestitures

- Market structure has been changed by mandated divestitures in the England and Wales market in the late 1990s.
- A supply function equilibrium model reproduced the change in prices from before to after the divestitures, given calibration to observed demand pre-divestiture (Baldick, Grant, and Kahn, 2004, and Day and Bunn, 2001).

6. Conclusion

- Discussed equilibrium models, their solution, and uses.
- There has been considerable effort in recent years in developing the theory and application of these models.
- There are strong prospects for improving such models, although their application should be tempered with the understanding that the actual market is likely to include a host of details that remain unmodeled.
- Qualitative sensitivity analysis can be useful, even in the absence of quantitative accuracy.