Stability of supply function equilibria: Implications for daily versus hourly bids in a poolco market.

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Abstract

We consider a supply function model of a poolco electricity market where demand varies significantly over a time horizon such as a day and also has a small responsiveness to price. We show that a requirement that bids into the poolco be consistent over the time horizon has a significant influence on the market outcome. In particular, although there are many equilibria yielding prices at peak that are close to Cournot prices, such equilibria are typically unstable and consequently are unlikely to be observed in practice. The only stable equilibria involve prices that are relatively closer to competitive prices. We demonstrate this result both theoretically under somewhat restrictive assumptions and also numerically using both a three firm example system and a five firm example system having generation capacity constraints. This result contrasts with markets where bids can be changed on an hourly basis, where Cournot prices are possible outcomes. The stability analysis has important policy implications for the design of day-ahead electricity markets.

Outline

- Background,
- Formulation,
- Basic analysis of supply function equilibria,
- Wide range of supply function equilibria,
- Stability analysis,
- Numerical confirmation,
- Significance,
- Policy implications,
- Conclusion.

Background

- **Consider an electricity market where generating firms bid to supply energy,**
- **Analyze the effect of requiring that the bid applies throughout a time horizon:**
  - ∗ **for example, a day,**
  - ∗ **England and Wales, until 2001,**
  - ∗ **markets in Eastern United States such as PJM,**
- **Represent bid as a function from prices-to-quantities:**
  - ∗ **bid-based pools usually require quantity-to-price function,**
  - ∗ **graphs will be presented as quantity-to-price plots.**

Background, continued.

- Klemperer and Meyer (1989):
  * basic analysis of supply function equilibrium (SFE),
  * conditions for uniqueness of equilibrium,
- Green and Newbery (1992), Green (1996), Rudkevich (1999), Baldick and Kahn (2000), and others:
  * application to the England and Wales market,
  * fixed bids over time horizon of one day.
- SFE explicitly represents bid requirements into a pool:
  * will see that this does not necessarily mean that SFE analysis is more applicable than Cournot.

Formulation:
Standard in literature.

- **Generation marginal costs for firm $i = 1, \ldots, n$ are affine:**

$$\forall q_i, C_i'(q_i) = c_i q_i + a_i,$$

  - **total variable costs $C_i(q_i)$ for firm i are quadratic,**

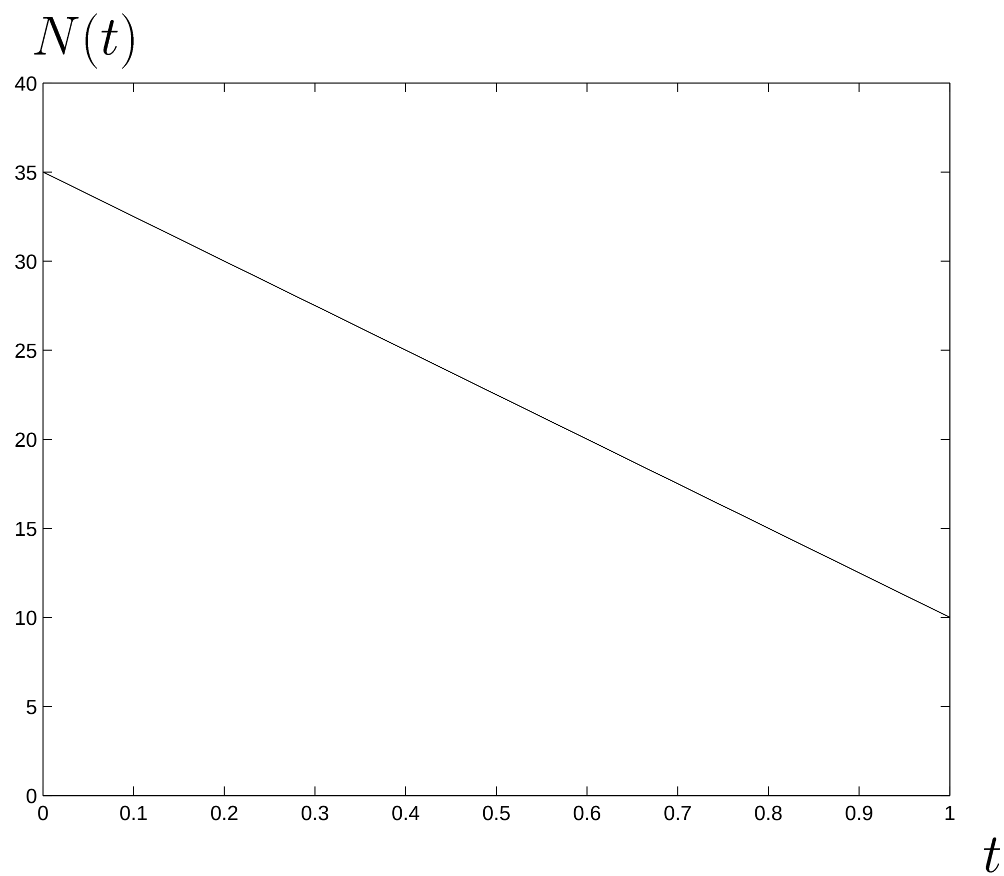
- **Bid supply function of firm i is a non-decreasing function S_i from price to quantities produced,**

- **Demand is a continuous function of the price p and of the (normalized) time t:**

$$\forall p, \forall t \in [0, 1], D(p, t) = N(t) - \gamma p,$$

  - **specified by continuous load-duration characteristic N over time horizon and by demand slope γ.**

Continuous load-duration characteristic.



Formulation, continued.

- **Price $P(t)$ at each time $t \in [0,1]$ is determined by market clearing conditions of demand equaling supply:**

$$D(t, P(t)) = N(t) - \gamma P(t) = \sum_{i=1}^{n} S_i(P(t)),$$

- **Profit per unit time to firm i is revenue minus costs:**

$$\pi_{it} = S_i(P(t))P(t) - C_i(S_i(P(t))),$$

- **Total profit π_i to firm i is the integral of profit per unit time over the time horizon:**

$$\begin{aligned} \pi_i(S_i, S_{-i}) &= \int_{t=0}^{1} \pi_{it} dt, \\ &= \int_{t=0}^{1} [S_i(P(t))P(t) - C_i(S_i(P(t)))] dt, \end{aligned}$$

where $S_{-i} = (S_j)_{j \neq i}$ are the supply functions of the other firms.

Nash supply function equilibrium (SFE).

- **A collection of supply functions $S^\star = (S_i^\star)_{i=1,\ldots,n}$ such that no firm can be made better off by unilaterally changing its bid:**

$$\forall i = 1, \ldots, n, S_i^\star \in \underset{S_i}{\operatorname{argmax}}\{\pi_i(S_i, S_{-i}^\star)\},$$

where $S_{-i}^\star = (S_j^\star)_{j \neq i}$ are the supply functions of the other firms.

Basic analysis.

- Klemperer and Meyer (1989) expressed the equilibrium conditions as coupled differential equations:
  - there is an affine solution to these equations,
  - when the load-duration characteristic is bounded there are multiple solutions,
- Equations can be transformed into a standard non-linear vector differential equation, (Baldick and Kahn, 2000),
  - solution characterizes some, but not all, equilibria,
  - does not represent capacity constraints,
  - solution not guaranteed to be non-decreasing.

Differential equation form of conditions (Baldick and Kahn, 2000).

$$S^{\star\prime}(p) = \left[\frac{1}{n-1}\mathbf{1}\mathbf{1}^{\dagger} - \mathbf{I}\right] \begin{bmatrix} \frac{S_1^\star(p)}{p - C_1'(S_1^\star(p))} \\ \vdots \\ \frac{S_n^\star(p)}{p - C_n'(S_n^\star(p))} \end{bmatrix} - \frac{\gamma}{n-1}\mathbf{1},$$

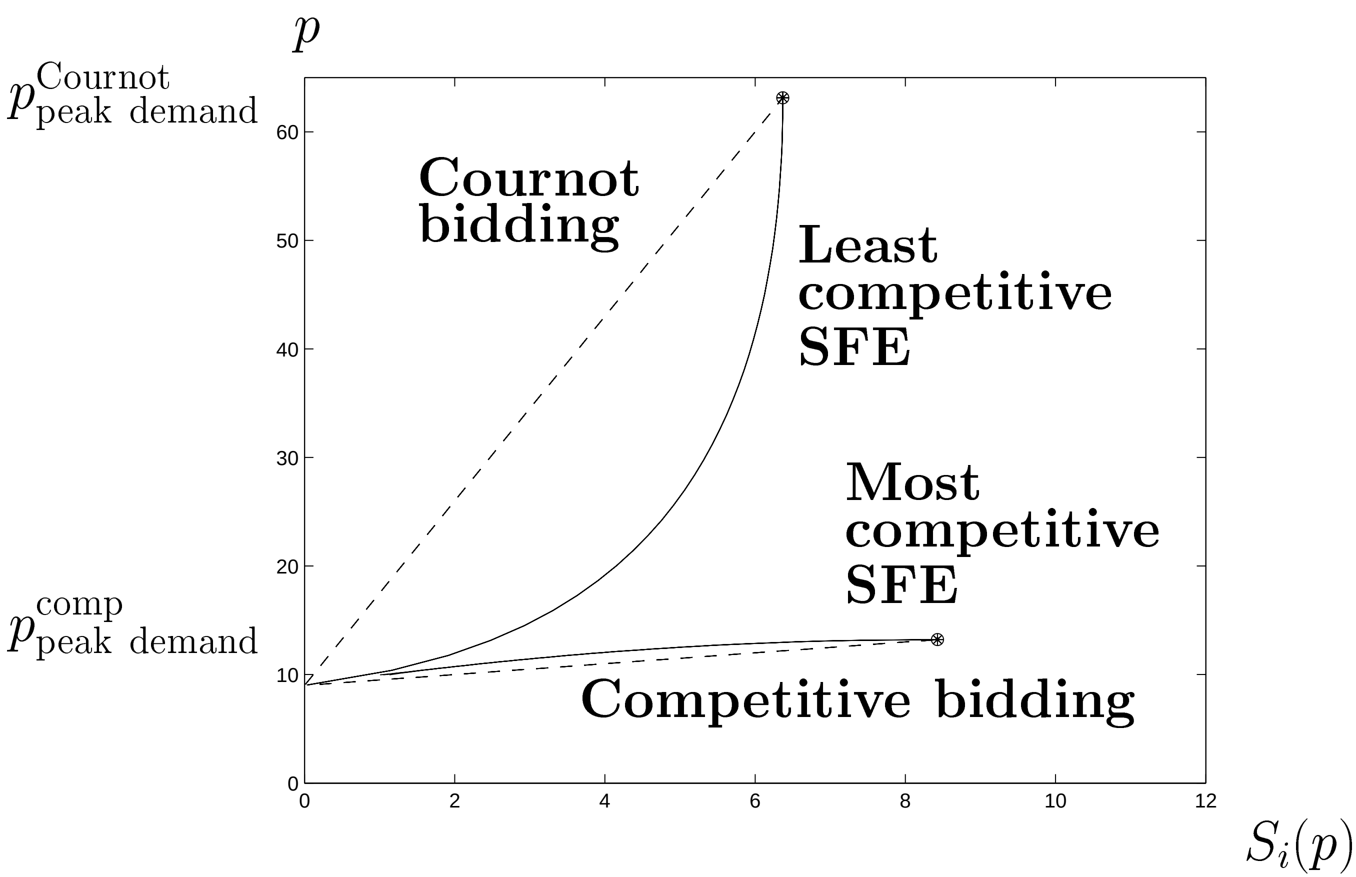
where:

- $S^\star = (S_i^\star)_{i=1,\ldots,n}$ **is the vector of supply functions and** $S^{\star\prime}$ **is the derivative of this vector,**
- $\mathbf{1}$ **is a vector of all ones of length** n**,**
- **superscript † means transpose, and**
- $\mathbf{I}$ **is the identity matrix.**

Wide range of solutions and wide range of economic equilibria.

- **Least competitive supply function equilibrium (SFE):**
  - ∗ **Cournot price** $p_{\text{peak demand}}^{\text{Cournot}}$ **at time of peak demand,**
  - ∗ **off-peak prices are lower than Cournot prices, (Green and Newbery, 1992),**
- **Most competitive SFE:**
  - ∗ **competitive price** $p_{\text{peak demand}}^{\text{comp}}$ **at time of peak demand,**
  - ∗ **off-peak prices are higher than competitive prices, (Green and Newbery, 1992),**
- **Affine SFE:**
  - ∗ **intermediate between most and least competitive.**

Wide range of supply function equilibria.



Wide range of supply function equilibria, continued.

- **Weakens usefulness of results:**
  - ∗ **Which equilibrium will occur?**
  - ∗ **Will the market converge to any equilibrium?**
- **Green and Newbery (1992) use least competitive SFE:**
  - ∗ **using "reasonable" values of demand slope,**
  - ∗ **calculated prices well above observed,**
- **Green (1996), Rudkevich (1999), Baldick and Kahn (2000), and others use affine SFE:**
  - ∗ **prices typically closer to observed,**
  - ∗ **results still depend on assumed demand slope.**

Wide range of supply function equilibria, continued.

- **Under some conditions, range of equilibria is smaller:**
  - ∗ **binding generator capacity constraints at peak, (Green and Newbery, 1992),**
- **What other issues might limit the range of the observed equilibria?**
  - ∗ **price caps,**
  - ∗ **instability of equilibria,**
- **First consider stability,**
- **Then return to generator capacity constraints and price caps using numerical framework.**

Stability analysis.

- There are various timescales in an electric power system (and in an electricity market) and corresponding notions of stability:
  - ∗ responses of automatic controls and stability of the electromechanical system,
  - ∗ stability of interaction between electromechanical system and short-term electricity markets, (Alvarado, Meng, DeMarco, Mota, 2001),
  - ∗ stability of economic equilibria:
    - – Quantity bids, (Alvarado, 1999),
    - – Supply function bids, (Baldick and Hogan, 2001).

Stability analysis.

- We analyze the stability of the SFE,
- Show that all SFEs between the least and most competitive, except for the affine SFE, are unstable:
  - ∗ unstable equilibria unlikely to be observed in practice,
  - ∗ conditions are somewhat restrictive, (equilibrium that is solution of differential equations),
- Use numerical framework to investigate range of observed equilibria under less restrictive assumptions:
  - ∗ generator capacity constraints,
  - ∗ price caps.

Unstable equilibrium.

- **An SFE $S^\star = (S_i^\star)_{i=1,\ldots,n}$ is unstable if a small perturbation $S^\epsilon = (S_i^\epsilon)_{i=1,\ldots,n}$ to $S^\star$ results in responses $\tilde{S} = (\tilde{S}_i)_{i=1,\ldots,n}$ by firms that deviate even more from $S^\star$.**

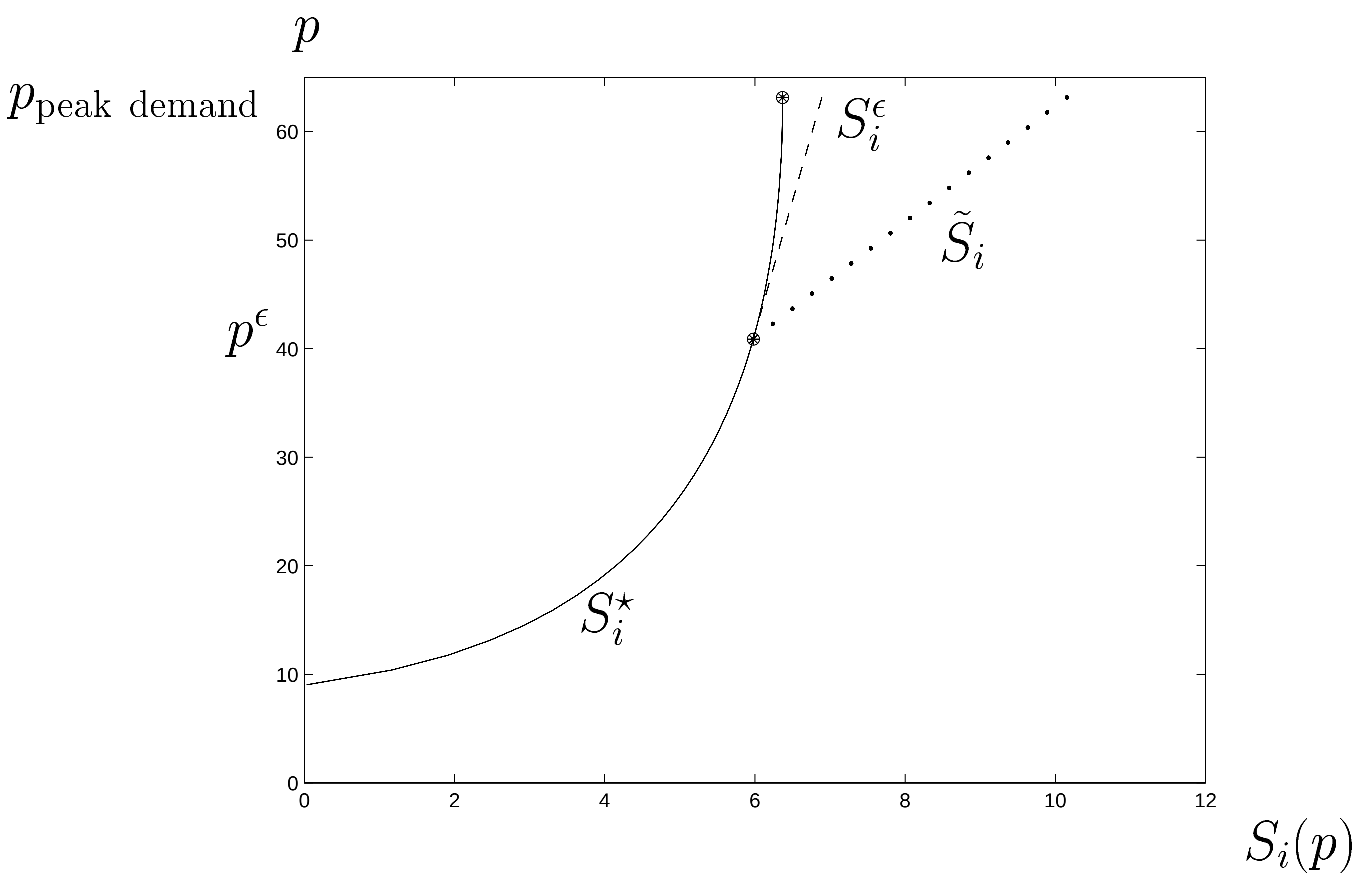
- **Let $\|\bullet\|$ be a norm on equivalence classes of SFEs such that if $\|S - S^\star\| = 0$ then the resulting prices for S are the same as the resulting prices for $S^\star$.**

- **$S^\star$ is an unstable equilibrium if for every $\epsilon > 0$ there exists another supply function $S^\epsilon = (S_i^\epsilon)_{i=1,\ldots,n}$ such that:**
  - $\|S^\epsilon - S^\star\| < \epsilon$ **and**
  - **if, for each i, $\tilde{S}_i$ is any optimal response to $S_j^\epsilon, j \neq i$ and we define $\tilde{S} = (\tilde{S}_i)_{i=1,\ldots,n}$ then $\left\|\tilde{S} - S^\star\right\| > \|S^\epsilon - S^\star\|$.**

Stability theorem.

- **Suppose that $S^\star$ is an SFE given by the solution of the differential equation form of the equilibrium conditions:**
  - $*$ **no capacity constraints,**
  - $*$ **solution of differential equation is non-decreasing,**
- **Suppose that the resulting supply functions are all strictly concave or are all strictly convex (as functions of price,)**
- **Then the SFE $S^\star$ is unstable.**

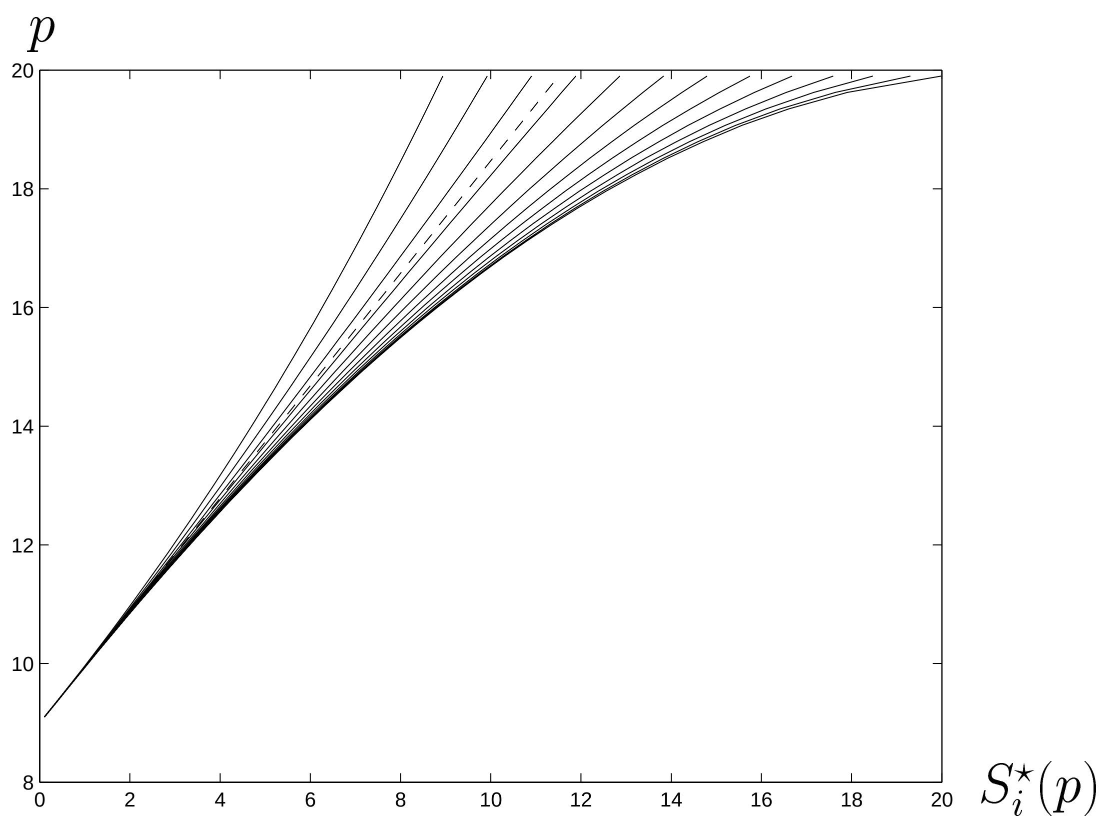
Outline of proof.



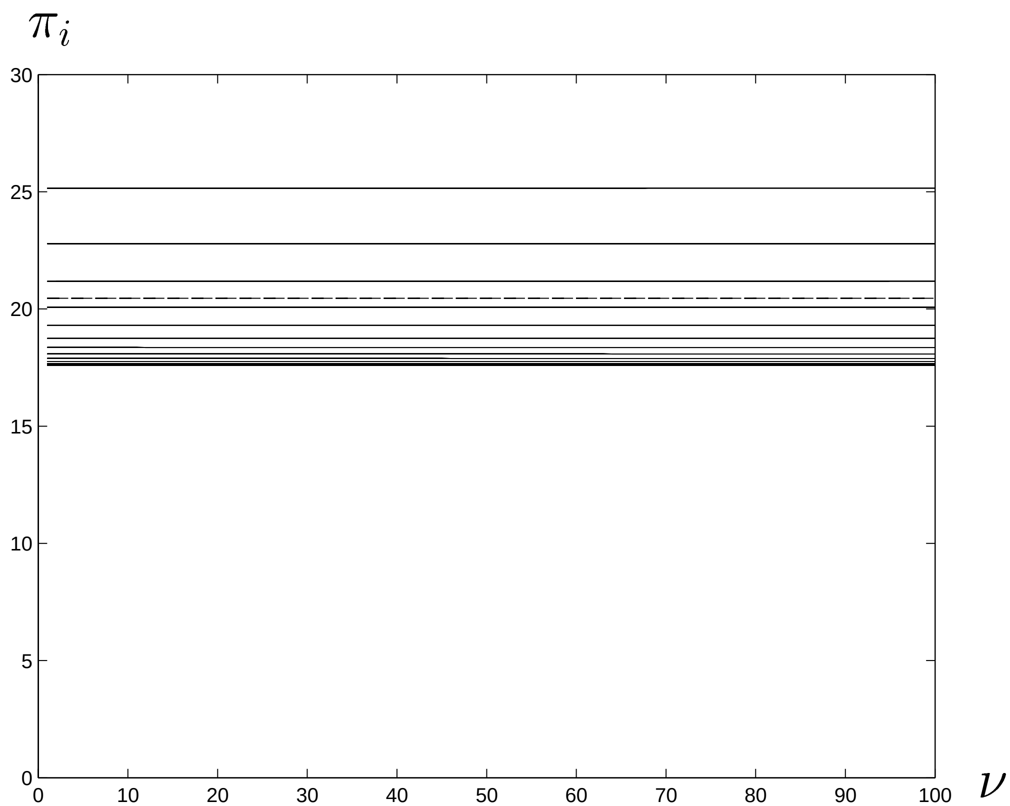
Numerical analysis with symmetric three firm system.

- **Cost functions based on Day and Bunn (2001),**
- **Cost functions are the same for each firm,**
- **No capacity constraints represented,**
- **Iterate in function space of allowed supply functions to seek equilibrium:**
  - **update supply function of firm i by seeking its optimal response to the supply functions of the other firms,**
- **Requires specification of a starting function.**

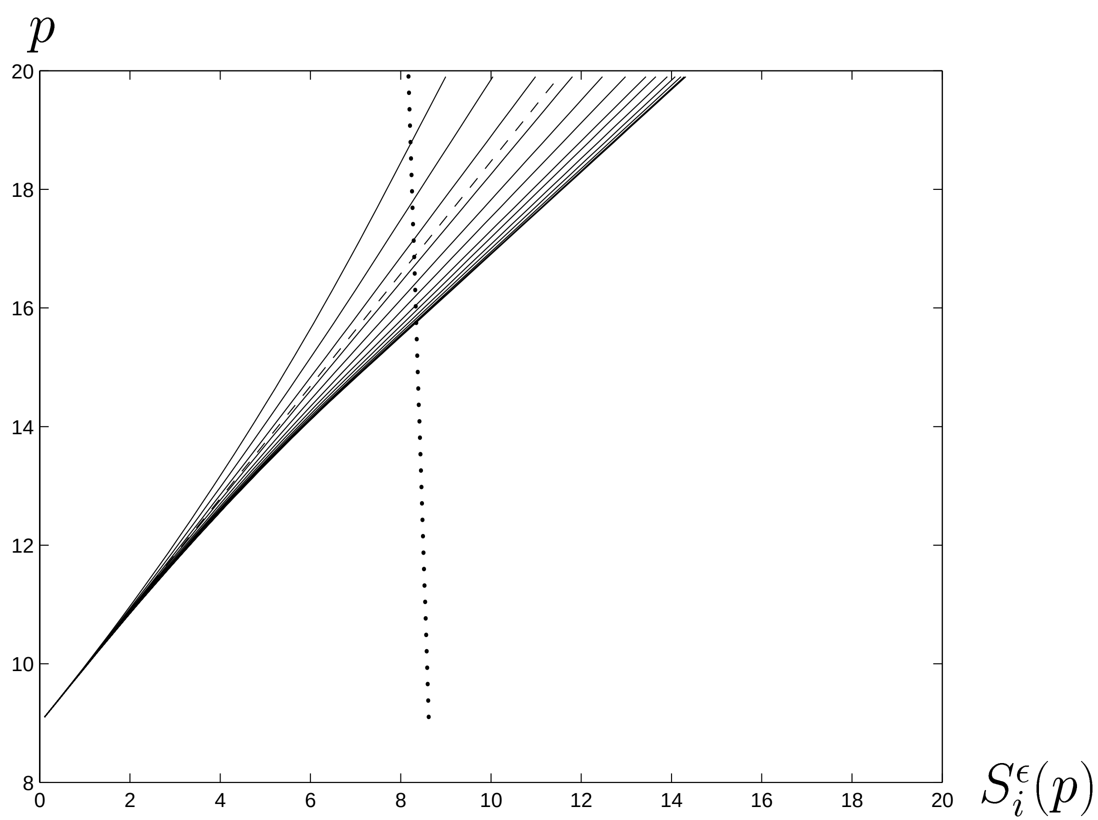
Numerical analysis with three firm system: Wide range of equilibria.



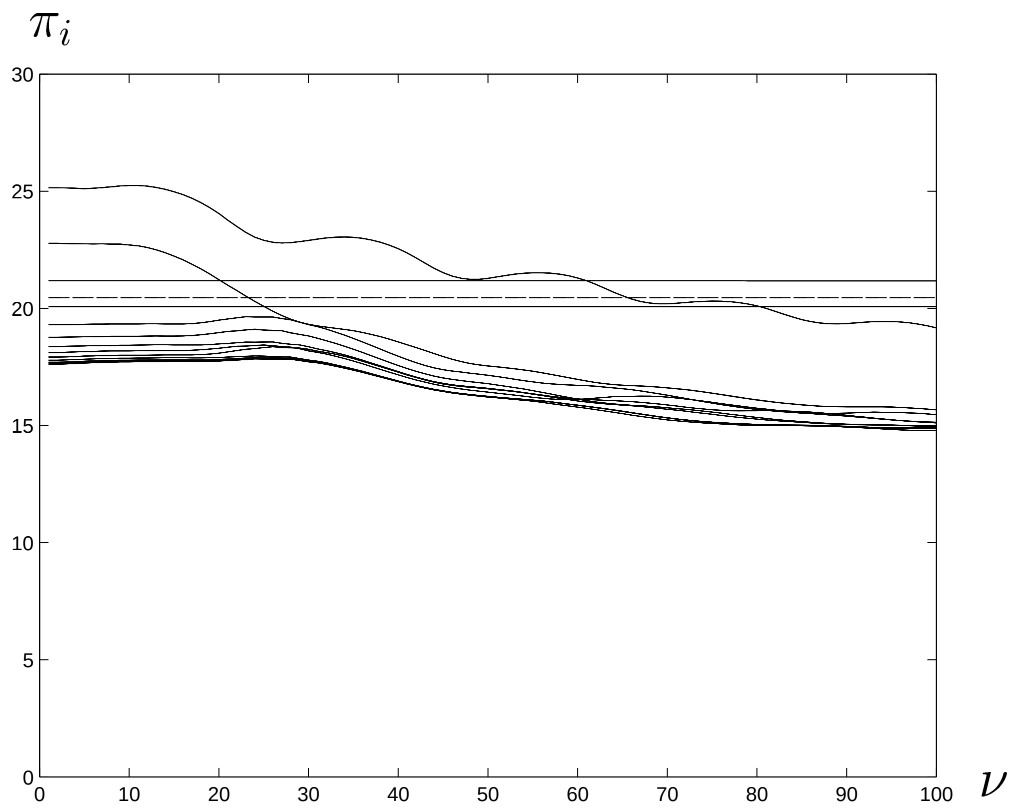
Iterating in function space of supply functions, Starting with equilibria.



Slight perturbations of equilibria.



Iterating in function space of supply functions, Starting with perturbed equilibria.



Numerical analysis with three firm system: Confirmation of stability analysis.

- Simulations confirm that there are multiple equilibria,
- For a starting function that is a slight perturbation from an equilibrium:
  - ∗ sequence of iterates typically drifts away from equilibrium,
  - ∗ results depend on simulation assumptions.
- Equilibria that are significantly different from the affine equilibrium are not stable from a numerical perspective.

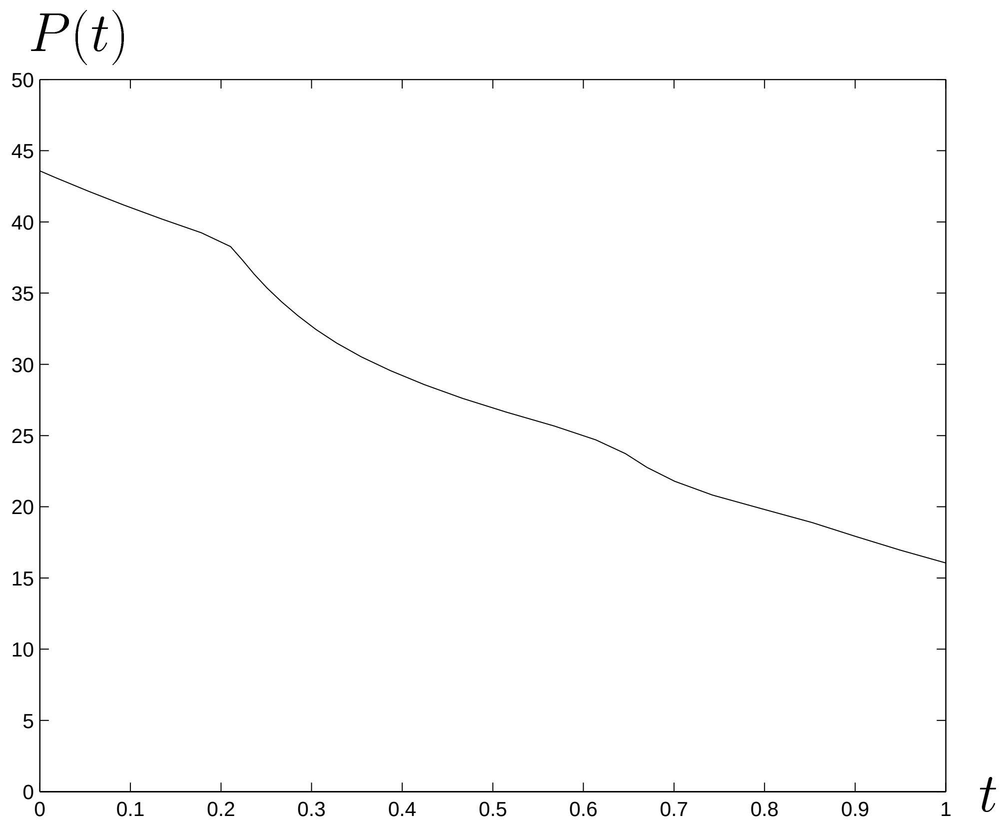
Numerical analysis with five firm system.

- Cost functions based on five non-nuclear firms in England and Wales circa 2000,
- Generators have capacity constraints,
- Load-duration characteristic chosen to require almost all generation on-peak,
- Economic dispatch at peak demand results in marginal costs of 27 pounds per MWh,
- Cournot prices at peak are 80 pounds per MWh.

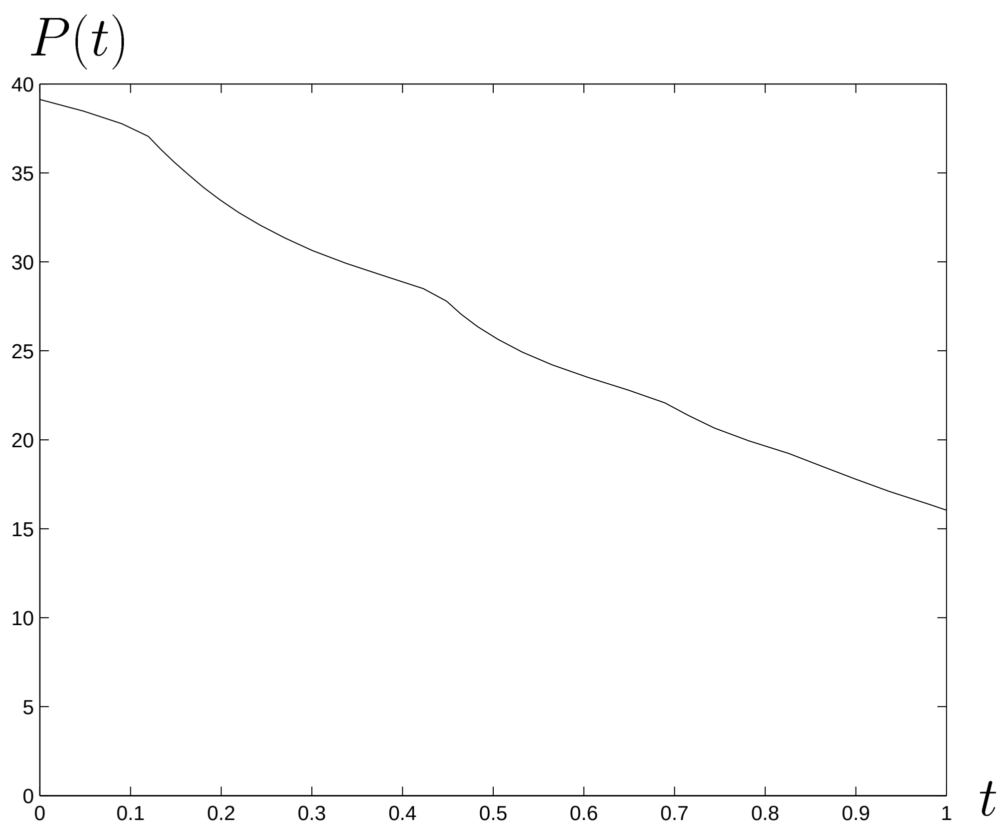
Numerical analysis with five firm system: Confirmation of stability analysis.

- Again iterate in function space of allowed supply functions to seek equilibrium.
- Obtain a relatively small range of observed equilibria,
- All equilibria have peak prices well below Cournot prices.

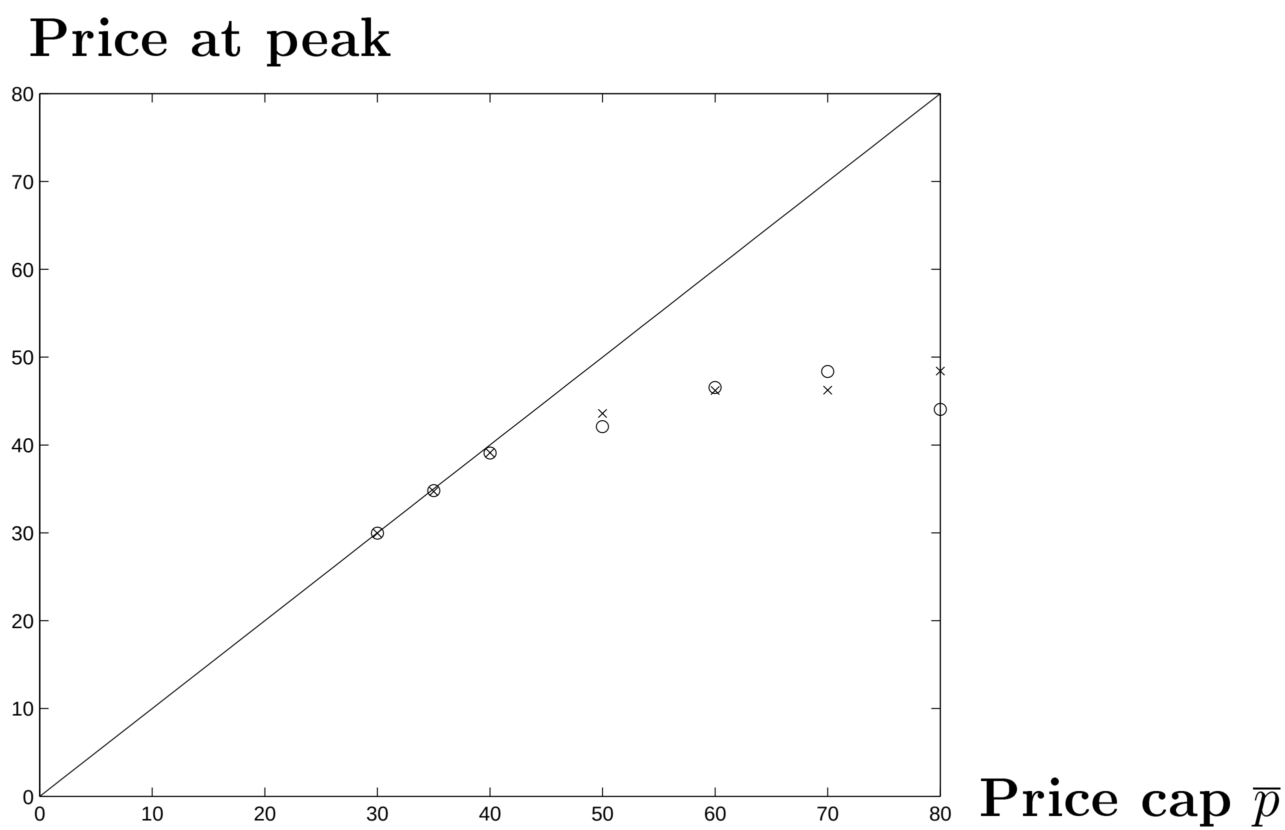
Numerical analysis with five firm system: Price-duration curve, No price cap.



Numerical analysis with five firm system: Price-duration curve, Price cap of 40 pounds per MWh.



Numerical analysis with five firm system: Price at peak versus price cap.



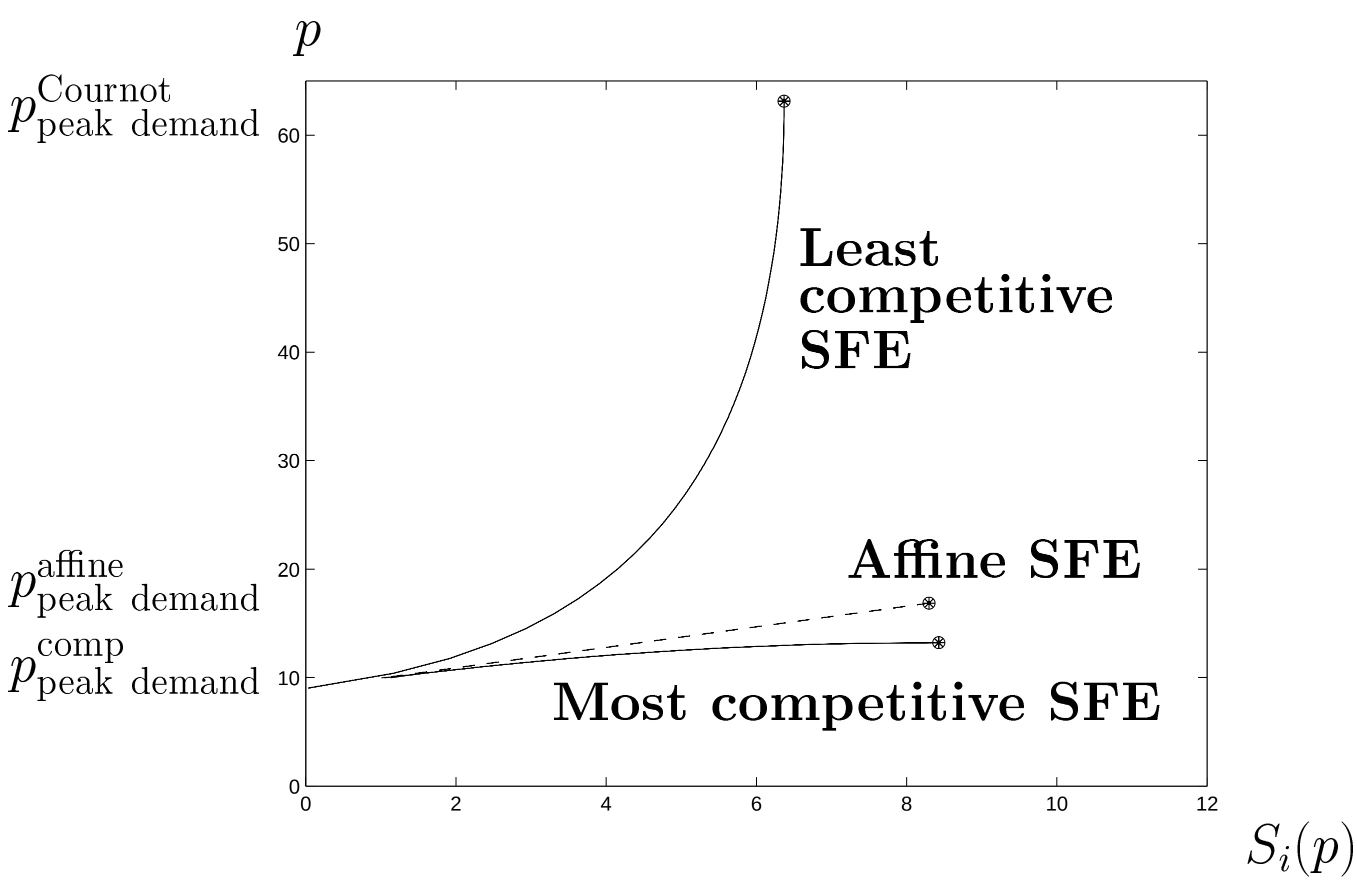
Significance of stability theorem.

- **If there is no requirement to bid consistently over the time horizon:**
  - **California Power Exchange,**
  - **any market without a mandatory pool,**
- **Cournot prices are a possible outcome in each pricing period:**
  - **much less competitive than even the least competitive supply function equilibrium.**

Obligation to bid consistently over a time horizon having many pricing periods.

- Firm must trade off profits from high prices at peak against sales at off-peak,
- Reduces mark-up at off-peak to below that of Cournot, (Green and Newbery, 1992),
- Reduces mark-up at all times to below that of Cournot, (Baldick and Hogan, 2001),
- Small range of stable SFEs, (Baldick and Hogan, 2001).

Obligation to bid consistently over a time horizon having many pricing periods.



Policy implications.

- Requirement to bid consistently over a time horizon with multiple pricing periods can help limit the exercise of market power,
- Compatible with and additional to other proposals for mitigation of market power (Borenstein, 2001):
  - ∗ long-term contracting,
  - ∗ real-time pricing,
  - ∗ price caps.

Conclusion

- **Analyzed stability of equilibria in a bid-based pool market,**
- **Found that requirement to bid consistently across a time horizon can limit the exercise of market power,**
- **Small range of stable SFEs,**
- **Seemingly arcane differences in specification of market rules can have large effects on outcomes.**