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Unit Commitment With Probabilistic Reserve

Debabrata Chattopadhyay, and Ross Baldick, *Member, IEEE*[*]

***Abstract*-- This paper demonstrates how a probabilistic criterion based on the full capacity outage probability distribution (e.g., LOLP) could be integrated into the unit commitment (UC) optimization using simple statistical approximation. While this provides a direct and computationally cheaper means as compared to a recently published approach to locating the globally optimal SR allocation (and associated UC pattern), the usefulness of the results are contingent upon the accuracy of such approximation. A case study using the IEEE RTS 96 single area system discusses the implementation issues. It is shown that the approximation procedure provides a reasonably accurate and efficient means to integrate probabilistic reserve in UC, and such a criterion leads to a substantially improved risk profile as compared to the traditional deterministic criterion.**

Keywords: Unit commitment, Spinning Reserve, Risk, Mixed integer programming, Spot Pricing.

I. Introduction

A. Deterministic vs. Probabilistic Spinning Reserve

Spinning reserve (SR) is needed in the system to cover for unforeseen events such as sudden increase in demand, and/or loss of generators/lines. SR allocation (over time, and across the generating units) often has important bearing on the dispatch and unit commitment decision, because it comes at some cost, which ideally should be kept minimal. This can be achieved by adjusting the SR on various generating units to keep the total start-up/back-down and operating cost impacts at the minimum. This has long been recognized, and SR allocation forms part of the standard economic dispatch and unit commitment optimization procedures.

However, one important feature that has not received much attention is to recognize the probabilistic nature of reserves in the dispatch/commitment optimization. Although the probabilistic nature of reserve has been well understood for several decades, its *integration* into the dispatch/UC optimization has been addressed only recently [1]. Spinning reserve requirement is set in most traditional UC models using various deterministic criteria e.g., a fraction of demand, largest generator/line contingency, or maximum on-line generation during the dispatch period. Such criteria have widely been used, including applications in the market environment, mainly because they are easily understood and implemented. A probabilistic reserve criterion, on the other hand, is more complex, but represents the complete system outage probability *distribution* and enables dispatch of reserve to meet an acceptable *risk* level (e.g., maximum Loss of Load Probability - LOLP). Stochastic programming approaches have also been attempted in the past (e.g., [2]) but computational tractability of stochastic programming models have been a major impediment to their success.

B. State-of-the-art in Probabilistic Reserve Modeling

Gooi *et al* [1] is a recent approach to model probabilistic reserve in a comprehensive way. Their approach involves post-processing the UC schedule to check if the target risk level is satisfied for each hour, and adjust commitment to either cover deficit SR, or eliminate any excess SR. A summary of this procedure is presented because it helps to understand the concept of probabilistic reserve, and also is the motivation behind the present analysis:

1. Start with some estimate of spinning reserve requirement, or SRR;
2. Perform UC (using Lagrangian Relaxation (LR));
3. Develop capacity outage probability distribution/table (COPT) for each hour for the set of *online* units;
4. Check (using COPT) for each hour if the specified risk level is attained;
5. If not, re-specify SRR = (SRR obtained+1), which means an *additional* unit has to be committed;
6. Repeat steps 2-5 until risk level is attained for *all* hours.

Since the above procedure leaves the possibility of allocating excessive reserve, a problem that is compounded by the duality gap of the LR approach, [1] also performs a "de-commitment" procedure:

1. If reserve is higher than is needed, it checks for every hour if *excess* reserve is higher than the capacity of the *marginal unit*;
2. If "TRUE" for any hour, UC is re-performed *dropping* the marginal unit. *If it leads to cost reduction*, then unit is de-committed.

The approach in [1] has several strong merits, including it:

- Enables accurate calculation of risk (indices) based on the exact probability distribution;
- Retains the UC optimization procedure intact and hence does not add to the complexity of the basic model; yet
- Achieves the basic objective of introducing risk, and apparently helps to narrow the duality gap as well.

However, the approach has some drawbacks in that it:

- Could be computationally intensive given that several UC runs may be required as well as overheads needed to calculate the probability distribution for each hour of all UC runs; and
- Could yield sub-optimal SR allocation and UC

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schedule.

The present work is motivated by the latter observations, namely such post-processing, besides being computationally intensive, leaves the possibility of sub-optimal solutions. In theory, it should be possible to replace the post-processing by integrating the risk criterion as a constraint within the UC optimization. This is what we explore in this paper. We will focus specifically on the random generator outages, although the approach could easily be extended to take into account demand uncertainty.

II. RISK CONSTRAINED UNIT COMMITMENT

A. Risk Approximation: Deterministic and Probabilistic

The deterministic criteria can be adopted in a fairly straightforward way. For example, the Australian, Ontario and New Zealand linear programming based dispatch models use the following as one of the criteria to determine SR:

$$\forall\ i,\ t,\ SR_t \geq P_{i,t} \qquad (1)$$

where,
i indexes on-line units $1,\ldots,n$
t indexes the dispatch period
$P_{i,t}$ is the MW dispatch of unit i at time t.

It should be noted that the above criterion forces the SR to be at least as large as the maximum on-line generation, which itself is a decision variable in the dispatch optimization. The "maximum on-line generation" usually, but not necessarily, implies the generation of the largest *size* (and, often relatively cheap) generator. It also implies that in a "reserve scarce" situation, the cheaper, but larger, generators will be discouraged to generate in order to keep the SR requirement low. It may lead to strange spot pricing impacts that we will discuss later. Although none of the above electricity markets employ a formal UC model (i.e., incorporating integer decision variables) to determine the day-ahead pre-dispatch schedule, the risk criterion nonetheless can be adopted for the (integer) UC model. Other deterministic criteria are equally naïve e.g.,

$SR \geq$ *some fraction of demand,*
or the maximum import across a line (or, set of lines),
or some combination of the above two criteria.

The principal difficulty in directly representing probabilistic criteria in an optimization problem stems from the fact that there is no direct means of incorporating the discrete capacity outage probability distribution (or, the so-called COPT - capacity outage probability table) in the UC optimization procedure. As a result, risk indices such as LOLP are commonplace in probabilistic production costing models, but the indices have not been incorporated into dispatch/UC optimization.

This difficulty could, however, be overcome using an observation made by Garver [4], albeit in a different context of calculating the Load Carrying Capability, which is the incremental load a generator permits the system to carry. The COPT may be approximated using an exponential function:

$$\textit{Cumulative Probability of X MW or more on outage } Pr(X) = a_0 e^{(-X/M)} \qquad (2)$$

The parameters a_0 and M are specific to the system and can be statistically determined. It has been observed for a number of applications [4] that the above approximation works reasonably well for relatively small levels of (cumulative) probability (*Pr*) e.g., less than 0.05, that are typically of interest for real-life systems. Further, taking logarithms on both sides a linear function is obtained:

$$\ln\,[Pr(X)] = \ln(a_0) - X/M = A_0 - X/M \qquad (3)$$

In the UC problem, we are interested in determining the level of SR that attains a permissible risk level, for example, a maximum LOLP limit. The probabilistic risk limit can, thus, be expressed as a *linear* constraint as follows:

$$A_0 - SR_t/M \leq \ln\,(LOLP),\ \text{or}\ SR_t \geq M\,[\,A_0 - \ln(LOLP)] \quad (4)$$

The LOLP limit could either be based on some pre-determined criterion, e.g., 1% or, 5%, or implicitly determined using cost/benefit analysis [1].

This (continuous) linear risk approximation resolves one of the problems in adopting probabilistic risk criteria, namely, representing the discrete COPT in the optimization problem.

B. Commitment Adjusted Risk Approximation

Incorporating the linear risk constraint in the UC optimization would have been a trivial task but for one complication. The COPT and hence its approximation (i.e., A_0, M) would change as the UC schedule changes. It is practically impossible to compute the (A_0, M) values for all 2^N combinations of units even for a moderate system size, let alone model all these risk constraints in a practicable way. Simplifying assumptions are, therefore, inevitable to the extent that the accuracy of the approximation is not seriously compromised.

We propose the following:

1. Assume that A_0 is *constant for all commitment patterns*. This is based on the premise that the linear risk characteristics for all commitment patterns converge to the common point $Pr(0)=1$, and hence a "constant intercept" assumption is expected to be reasonably accurate. It should, however, be noted that the approximation for the low probability range (e.g., < 0.05) may yield an estimate of $Pr(0)$ which is significantly different from 1; and
2. Assume only *first order effects* for changes in M, and that such effects are *additive*. That is, we will calculate the incremental change in M due to a generator i (Δm_i) based on single generator (i) outage condition alone,

and further assume that such incremental changes for multiple outage conditions simply add up.

The second assumption implies that the system M is dependent on the commitment decision $U_{i,t}$ (=1 if ON, 0 if OFF). The risk constraint can be restated accordingly as:

$$SR_t \geq \left[M - \Sigma_i (1-U_{i,t})\, \Delta m_i \right] \left[A_0 - \ln(LOLP) \right] \quad (5)$$

The term in the first parenthesis on the RHS reflects the “adjusted M” for different commitment pattern. If certain generators are not committed in hour t *(i.e.,* $U_{i,t} = 0$ *for such units)* , the risk characteristic is more steep (*i.e.*, lower M). The Δm_i values indicate the relative significance of the unit i to the overall system. The Δm_i values may easily be computed starting from the full-system COPT and applying the recursive unit removal formula [5]. The larger the unit and lower the forced outage rate, the higher is Δm_i. The summation is performed over all off-line units and reflect the assumption that the Δm_i are additive**.** Finally, if *all* units are on-line, the full system outage characteristic is restored.

C. Probabilistic Risk Constrained UC Optimization:

Augmenting the UC optimization procedure with the commitment adjusted probabilistic risk constraint is also easy, although the presence of commitment variables may entail additional computational burden, and/or complicate the analysis of dispatch/pricing outcome, as compared to the traditional UC model. Either of these issues would be highly specific to the system under study, though, and it is hard to generalize if computing time and dispatch/pricing outcomes would indeed be any worse. It could well be the case, for example, that a Branch and Bound procedure to solve the UC model is able to locate the integer optimal quicker in the presence of the risk constraint, because the underlying linear programming sub-problems are “tighter”. On the other hand, the dispatch and pricing impacts of deterministic criterion could well be “surprising” as we will discuss later.

The risk constrained UC model incorporating all the usual features of its traditional counterpart (*e.g.*, [6]) may be solved using any of the standard techniques traditionally employed *e.g.,* Lagrangian Relaxation, Dynamic or Mixed Integer Programming (MIP). The choice of any specific technique over another is largely irrelevant for the present purpose of demonstrating the risk approximation, as long as the model is solved to optimality without having to resort to any ad-hoc procedure that may blur the interpretation of results, in particular the dual/price information. In the present analysis, we have developed a MIP model using GAMS [7] for the *IEEE Reliability Test System (RTS 96)* [8] and solved it using XA [7], a commercial Branch-and-Bound based solver. The MIP model is used in the present analysis to avoid the duality gap problems associated with the LR based model, although the latter has substantial advantages for large system applications. The duality gap problem encountered in LR is especially relevant because such dual/pricing distortions will blur the analysis of spot prices.

The MIP based UC model may be stated as follows:

Minimize total generation and start-up costs, subject to,

- Meeting demand;
- Keeping enough spinning reserve to meet maximum LOLP level;
- Restrictions on $U_{i,t}$ e.g., minimum up/down time;
- Restrictions on $P_{i,t}$ e.g., max/min generation, ramp limits;
- Incurring costs for starting up from hot/cold state where start up decision $S_{i,t}$ $(1 \geq S \geq 0)$ defined as $S'_{i,t} = U_{i,t} - U_{i,t-1}$, and $S_{i,t} \geq S'_{i,t}$

It should be noted that $S_{i,t}$ and $S'_{i,t}$ are both defined as continuous variables, but they are *naturally integer*.

The overall procedure for probabilistic risk constrained UC optimization may be summarized as follows:

1. Develop capacity outage probability distribution for the full system (i.e., all generators in service), and also for all single generator outage conditions to calculate (A_0, M) for the system and Δm_i for each generator;
2. Approximate the distribution for the relevant range of probability (say, less than 0.05) using ordinary least square or its variants, as $Pr(X) = a_0 \cdot e^{(-X/M)}$
3. Incorporate the risk constraint in the UC optimization as,

$$SR_t \geq \left[M - \Sigma_i (1-U_{i,t})\, \Delta m_i \right] \left[A_0 - \ln(LOLP) \right]$$

The advantage of the proposed approach compared to that in [1] is that the optimization needs to be performed only once, and the MIP model gives the globally optimal solution within the limitations of the approximation to the outage probabilities. The disadvantage is that the results are dependent on the accuracy of the approximation. The approximation may be crude for smaller systems having a few large units, or systems having large share of units with high forced outage rate, *etc*. The other source of inaccuracy could be due to the assumptions made to extend the risk approximation for different unit commitment patterns. The idea of incremental M while simple and easy to implement, may not provide a good approximation of the exact distribution. Any improvement in accuracy under such circumstances will come at the cost of adding complexity to the risk constraint e.g., adding higher order terms, which in turn will add to the difficulty in solving the UC optimization problem.

Summing up, if the present approach gives a reasonable approximation to the probability distribution, then it has advantage over [1], else [1] is a better option, unless the solution is grossly sub-optimal.

III. CASE STUDY BASED ON IEEE-RTS 96

The risk constrained MIP model has been implemented for the *RTS 96* [9] single area system for the “Winter weekday”. The system has two must-run nuclear units and 24 thermal units. We have assumed the following prices [6]: oil \$6/MBtu, coal \$4/MBtu and nuclear plant input \$2/MBtu. Piecewise linear cost characteristics for the generators are constructed based on the incremental heat rate data (Table 9, [8]). The hourly load distribution is same as in Table 4 [8], but the total system (peak) load is assumed to be lower at 2,400 MW to enable imposing realistic hourly LOLP targets (5%). We have not considered minimum up/down time and ramp limits because these are not particularly relevant for our purpose and omitting them helps to make the computation faster as well as interpretation of results simpler. These are not requirements though – in fact, the MIP model sometimes requires less branching if there are more constraints.

Firstly, the COPT for the 26 generators system has been developed and approximated using OLS regression. The R^2 or "goodness of fit" is fairly high at 0.985 and so are the *t*-statistics for the regression parameters (A_0, M). Fig.1 shows the original distribution and the fitted curve. The approximation is generally quite accurate especially for LOLP values < 0.02.

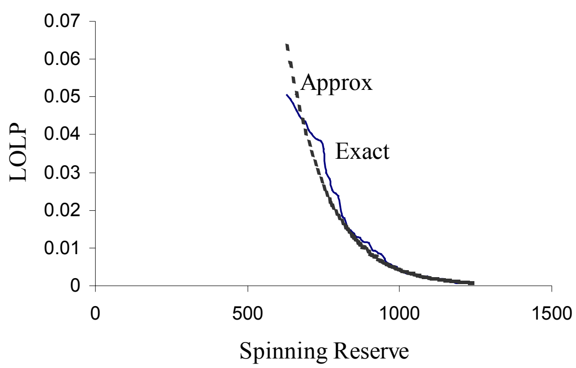

Fig.1: Approximation of capacity outage probability distribution

The slightly high level of inaccuracy at LOLP values > 0.02 shown in Fig.1 is attributable to the relatively small number of generators, and also the high (largest unit/system capacity) ratio. This is evident from the improved accuracy of approximation (R^2 = 0.995) for the 104 generator RTS four region system which is obtained by replicating the single area system. This is shown in Fig.2.

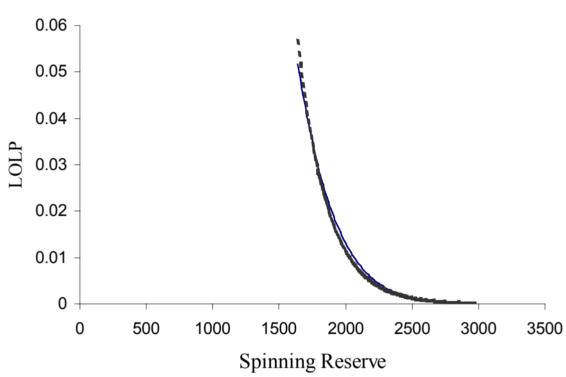

Fig. 2: Approximation for 104 generator RTS four region system

The exponential variation of LOLP with SR is noteworthy because it explains why a constant spinning reserve margin for all time periods may not be a good idea. It further becomes clear when we look at the Δm_i values for different size generators (Fig.3).

The smaller generators (≤ 100 MW) barely have any perceptible impact on the reliability characteristics, while the largest generator (400 MW) has nearly 29 times as much impact as a 100 MW generator. This also explains why a system having a few large generating units is much more vulnerable than one having same capacity distributed over a large number of smaller units (but with similar failure rates).

Fig.3: Δm_i values for different generators

While the M values (slope of the risk characteristic line) vary considerably across various generator outage configurations from 136.7 for full system to 98.8 with the 400 MW unit out of service, the A_0 values are relatively constant.

The standard deviation of A_0 across the 26 single outage scenarios is only 0.019 as compared to the mean value of 1.85. This corroborates our first assumption that A_0 is constant. We also verified the second assumption regarding the additivity of Δm_i values for multiple uncommitted unit conditions. In particular, we have considered the worst case when both 400 MW units are not committed, i.e., 800 MW out of 3,105 MW system capacity is not committed. In fact, this is never going to happen because both the units are designated as must-run units. We calculated the system M value (using OLS) as 63.7 which is reasonably close to [136.7– 2 *(136.7 – 98.8)=] 60.9. The latter calculation is based on the additivity assumption i.e.,

$$M \text{ (with Generator A and B out)} = M \text{ (full system)} - \Delta m_A - \Delta m_B$$

The impact of this inaccuracy on LOLP is relatively minor, and almost negligible for smaller uncommitted units. In fact, the risk constraint is likely to be a critical consideration during the peak hours when the majority, if not all, of the large generators are likely to be committed. If we combine this with the observation that the Δm_i values for relatively smaller generators are very low, the additivity assumption appears to be reasonable for a practical system.

Having obtained all the relevant parameters (A_0,M, Δm_i), the risk constraint is formulated for different (hourly) LOLP targets as:

$$SR_t \geq 6.45 \left[136.7 - \Sigma_i (1-U_{i,t}) \Delta m_i \right] \text{ for LOLP} = 1\%$$
$$SR_t \geq 4.85 \left[136.7 - \Sigma_i (1-U_{i,t}) \Delta m_i \right] \text{ for LOLP} = 5\%$$
$$SR_t \geq 4.15 \left[136.7 - \Sigma_i (1-U_{i,t}) \Delta m_i \right] \text{ for LOLP} = 10\%$$

We have systematically investigated how the risk constrained UC adjusts the commitment pattern as we progressively enforce tighter risk criterion, and further how these compare with the deterministic (largest generation) solution. Table-1 compares some of the broad features to start with.

TABLE-1: COMPARISON OF RESULTS UNDER DIFFERENT RISK CRITERIA

	No risk	10% LOLP	5% LOLP	1% LOLP *	Deter-ministic **
Total cost	1458746	1459394	1461302	4510343	1459117
Min SR	82	511	599	705	398
Max LOLP	70%	9.7%	4.8%	3.7% *	28.6%
Max spot price	53.2	52.9	52.9	52.9	79.5
Deficit SR	0	0	0	Hr 9-20 Max 176	0

* Note: 1% LOLP could not be achieved as evident from deficit SR. The high cost is an outcome of the penalty $3/kWh applied on violation of SR constraint.

** We have used $SR_t \geq P_{i,t}$ as the deterministic criterion

Some remarks based on Table-1 results are in order:

- The "no risk" case has the lowest cost, but highly unacceptable risk level;
- The risk constrained UC model does remarkably well to accommodate LOLP targets to the extent possible at little cost by adjusting commitment and dispatch. For example, 10% LOLP target is achievable at an additional cost over the no risk case of $648 (0.04% of the system cost without any risk constraint)) and 5% at an additional $2,556 (0.17% of the system cost without any risk constraint);
- The LOLP level of 1% is unachievable in the present case because of the high spinning reserve requirement that it entails. The system, being small with relatively low margin for about 50% of the day, is unable to comply with the 1% LOLP criterion. What is interesting to note though is that the UC optimization is capable of trading the benefit from additional SR (assumed to be valued at $3/kWh by the customers in this example), against the cost of providing it, to determine the *optimal* level of LOLP;
- The deterministic criterion $SR_t \geq P_{i,t}$ has slightly lower cost impact $371 (0.025%), but unacceptably high LOLP implications for the peak hours (hours 18,19). Probabilistic risk constraints, in comparison, achieve much better spread of risk at a marginally higher cost. In this case, the high LOLP of 28.6% (hour 18, 19) is entirely due to the commitment of Unit 1(20 MW) instead of Unit 13 (197 MW) [in the case of 10% LOLP]; a difference of 177 MW which brings down the LOLP to 9.7%! Though the deterministic criterion could be important from some other respect e.g., security, it fails to achieve requisite system adequacy performance primarily because the non-linear overall outage probability distribution is largely ignored. As we have seen, this is approximated well in the probabilistic risk constraint;
- All the hourly spot prices for no risk as well as probabilistic risk cases (Fig.4) are equal to the marginal generation offer. The fact that the max spot prices (hour 18, 19) have slightly decreased from the "No risk" level (although, total cost has increased), indicates that the underlying commitment schedules are different. These price differentials also reflect the judicious selection of units to build up enough capacity early on to meet the LOLP target;

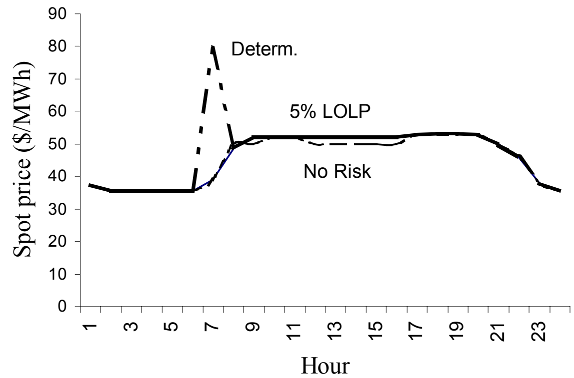

Fig.4: Comparison of hourly spot prices

- This is, however, not true about the deterministic criterion. Whenever there is a cost implication due to SR, the shadow prices of all binding spinning reserve constraints will get added to the marginal generation offer, and thereby lead to surprising price impacts e.g., the strong peak at hour 7. The peak price may occur during off-peak hours due to the largest (but relatively cheap) generator being *constrained down* by the SR constraint. In the present case, the SR constraint is binding for the 400 MW nuclear units during hour 7 (74% of peak demand). Even though the marginal generation offer at hour 7 is only $39, the shadow prices of spinning reserve constraints ($20.25 for both nuclear units) get added to it to produce the high price of $79.50. The shadow price indicates the opportunity cost of backing off these cheapest (but largest) units and equals the difference between the nuclear generation cost and the marginal generation cost.

Comparison with [1]: We have also compared the solution of the MIP model for the 5% LOLP case with that of the algorithm proposed in [1]. The summary of the procedure is included in section 1.2. We started with the "No risk" case and gradually tightened the SR requirement as the risk level for some hours fall short of 5%. However, incrementing the SR requirement by 1 MW leads to very slow convergence, and no feasible solution is encountered after 100 UC runs. The increment is raised to 10 MW and it still required 27 runs. The relatively high number of UC runs in this case is due to the fact that satisfying the risk limit for one hour often led to violating it in another hour. Fig. 5 shows how the LOLP declines as SR requirement goes up. It was verified that the surplus SR for all hours is less than the capacity of the marginal generator. No de-commitment is, therefore, necessary.

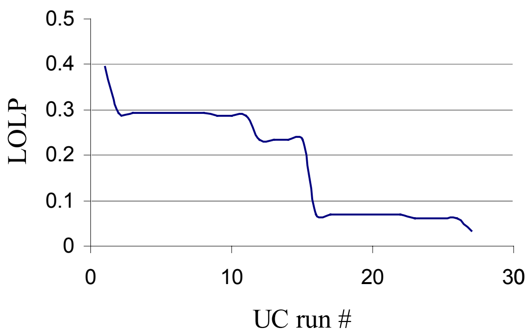

Fig.5: LOLP for hour 1 over the UC runs

The total system cost for the 27th UC run (when all hourly LOLPs are below 5%) is $1,539,725 which is approximately 5.3% higher than that associated with the optimal schedule (i.e., 5% LOLP case in Table-1). The additional cost over and above the optimal schedule is mostly due to unnecessary commitment of some units.

The integrated risk constrained UC model in this case proves to be superior both in terms of computational performance (requires only one UC run as compared to 27 runs), as well as locating a significantly (5.3%) cheaper solution, albeit at the expense of slight inaccuracy in calculating the LOLP values.

IV. CONCLUDING REMARKS

This paper proposes a simple statistical approximation to integrate the capacity outage probability distribution in unit commitment optimization. It has been demonstrated using the *IEEE Reliability Test System 1996* that the approximation is reasonably accurate for realistic LOLP values. The advantage of this approach over the state-of-the-art risk constrained unit commitment procedure [1] is that it is computationally less intensive and provides the globally optimal solution. We have extensively analyzed the risk implications of the deterministic, probabilistic and composite criteria. The test results indicate that the deterministic criterion may lead to unacceptably high risk and also may have strange pricing implications. The probabilistic risk constrained model, on the other hand, ensures a much better risk profile, but may fall short of covering the deterministic criterion, e.g., largest on-line generation source. We have also compared the outcome for 5% LOLP case with the algorithm proposed in [1]. The algorithm in [1], while enabling accurate calculation of LOLP, requires a very high number of UC runs and locates a sub-optimal UC schedule. The slight inaccuracy of the present method in calculating LOLP may be worthwhile given the improved quality of the solution and the requirement of solving the optimization problem only once.

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