Deriving the Lens Transmittance Function

Thin lens transmission is given by a phase with unit magnitude.

$$t(x,y) = \exp\left[jk\Delta_o\right]\exp\left[jk(n-1)\,\Delta(x,y)\right]$$

Find the thickness function for left half of the lens first.

Use paraxial approximation for the square root

(R >> x or y). Add the two halves to get:

$$\Delta(x,y) = \Delta_o - \frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\left(x^2 + y^2\right)$$

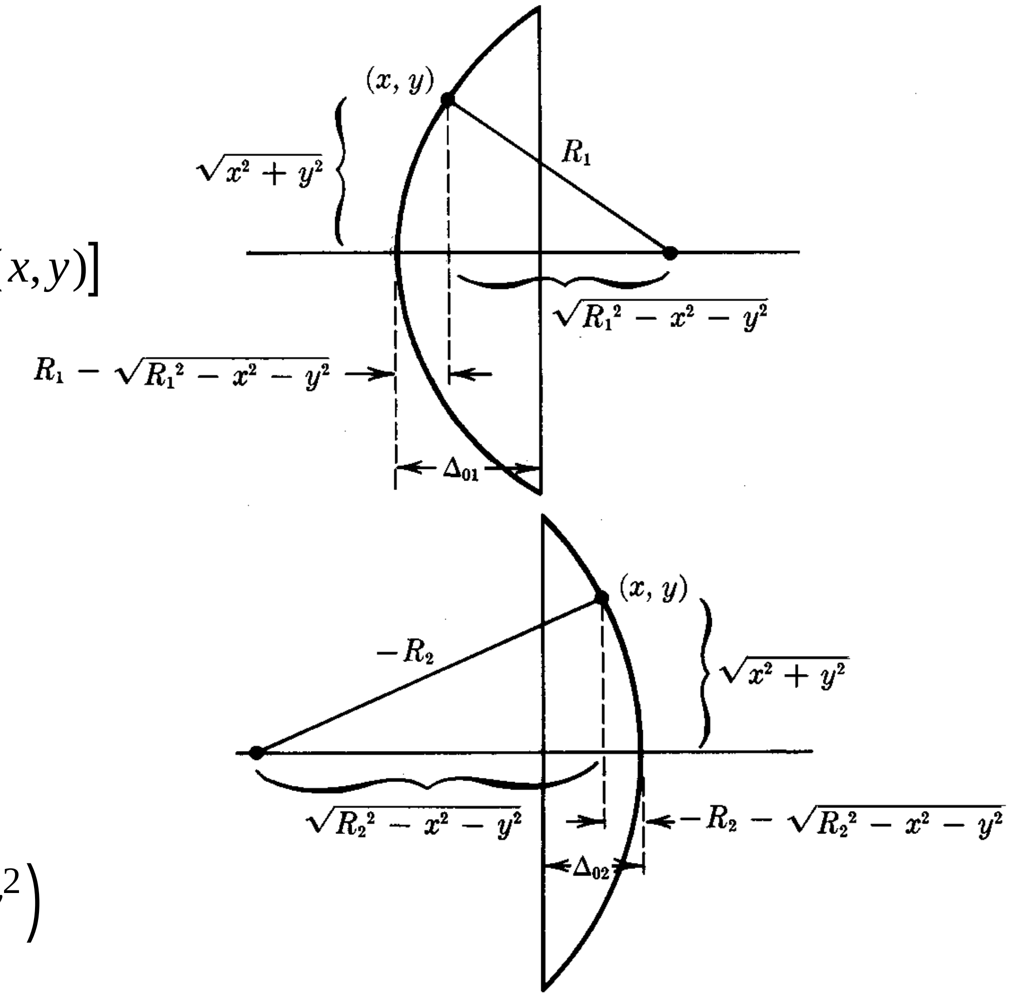
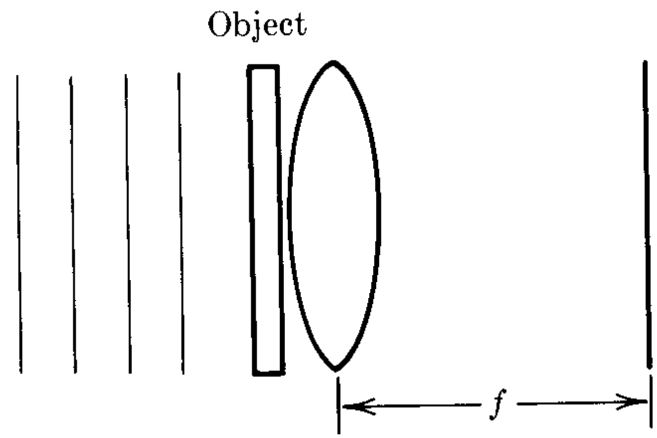


Figure 5-2 Calculation of the thickness function.

Fourier Transform Configurations

(a) Object in lens plane (front or back is the same).

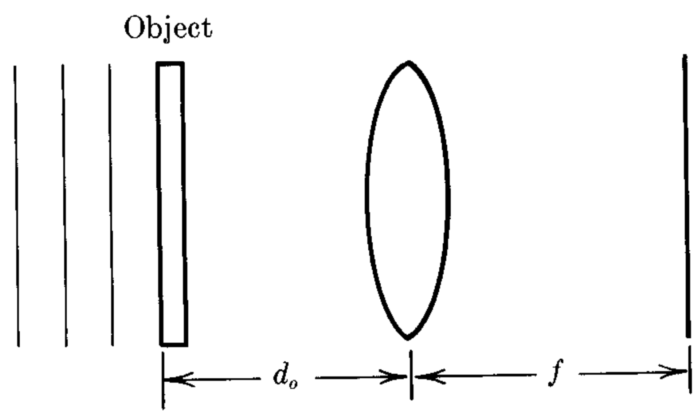
Gives Fourier transform with a phase factor.



(a)

(b) $d_o = f$ gives exact Fourier transform.

Other values of d_o give a phase factor.

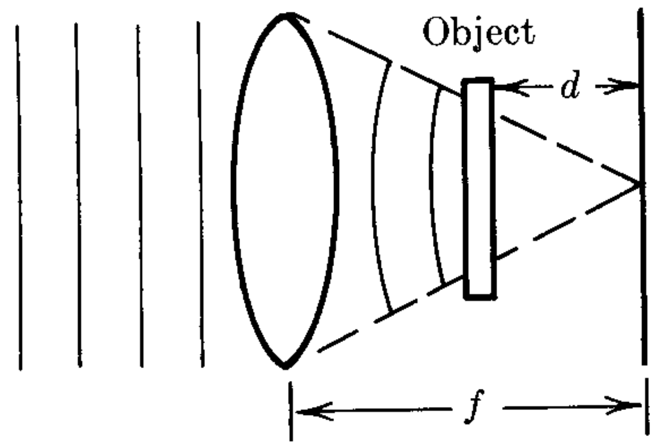


(b)

More Fourier Transform Configurations

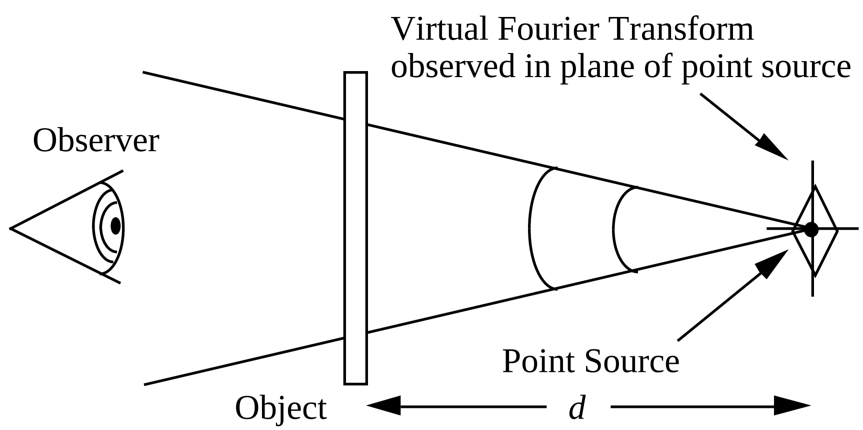
(c) Gives a scaled Fourier transform with a phase factor.

Omit the lens and you also get a Fourier transform in a converging spherical wave.



(c)

(d) Gives a (scaled) virtual Fourier transform in the plane of the point source of a diverging spherical wave.



(d)

Vignetting

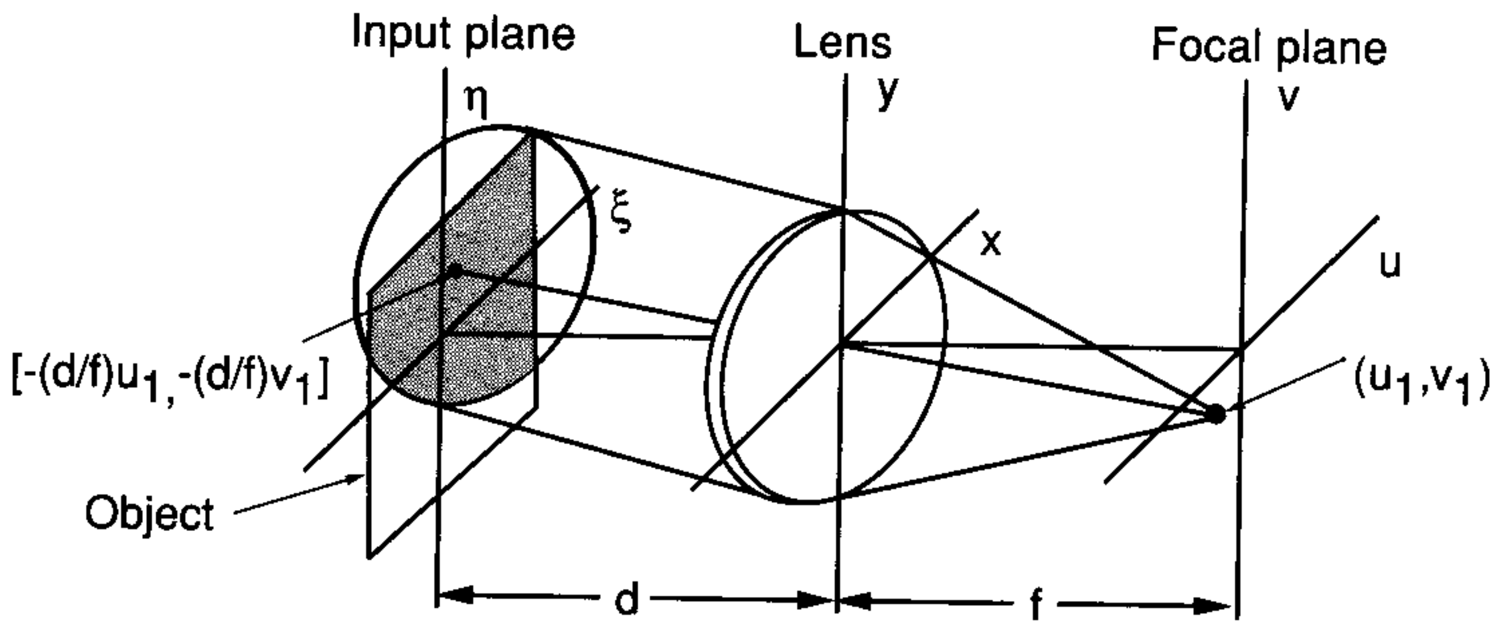


FIGURE 5.6
Vignetting of the input. The shaded area in the input plane represents the portion of the input transparency that contributes to the Fourier transform at (u_1, v_1).

Imaging with Lenses (Diffraction Analysis)

1. Fresnel diffraction from U_o to U_l
2. Multiply by the lens transmittance (quadratic phase)
3. Fresnel diffraction from $U_{l'}$ to U_i

<u>Shortcut</u>: find the "impulse response" by making the input a point source (one pixel) in the object plane.

What part of the object really contributes to an image point?

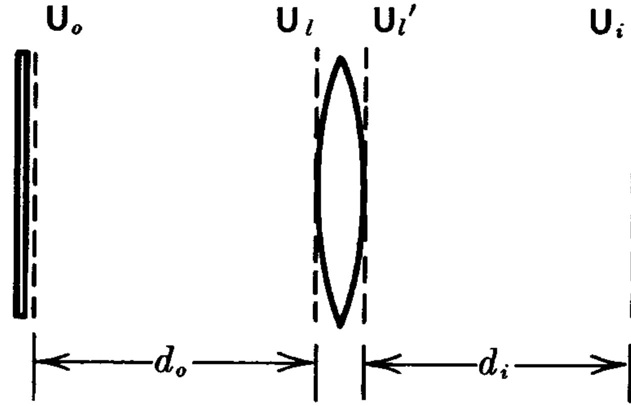


Figure 5-8 Geometry for image formation.

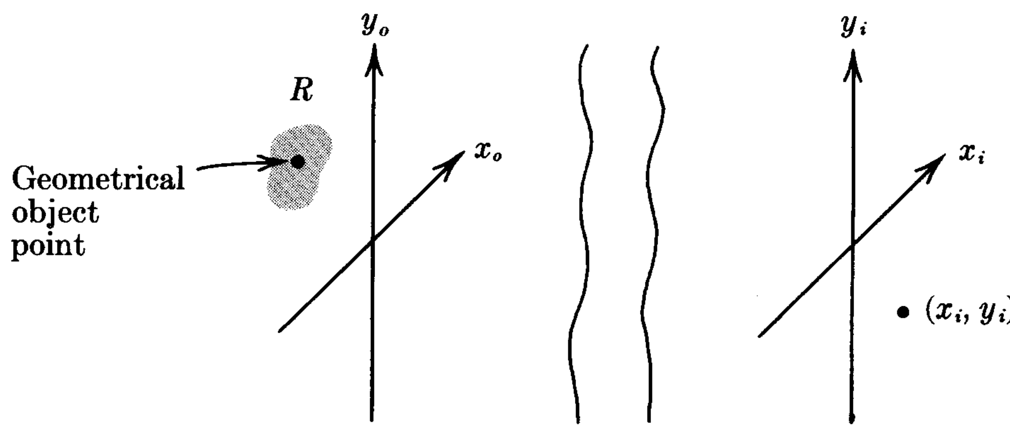


Figure 5-9 The region R where h has significant value for the particular coordinates (x_i,y_i) shown.

General Optical Imaging System with Diffraction

• **Diffraction determines spherical wave propagation from object to entrance pupil (or from exit pupil to image plane).**

• **Geometrical optics determines laght transfer from entrance to exit pupil. May contain aberrations.**

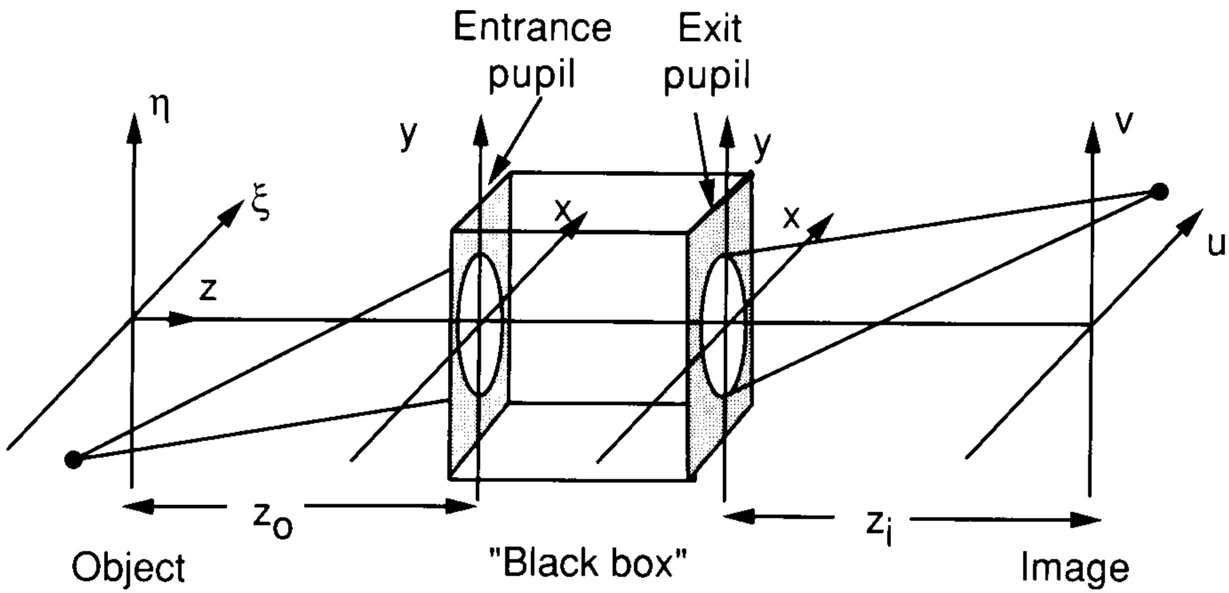


FIGURE 6.1
Generalized model of an imaging system.

Abbé Concept of Image Formation

High spatial frequencies in the object do not pass through the lens aperture; low frequencies do. The frequencies that pass through the lens for a Fourier transform (with a phase factor) at the focal pland (source image plane) before passing on to the image plane.

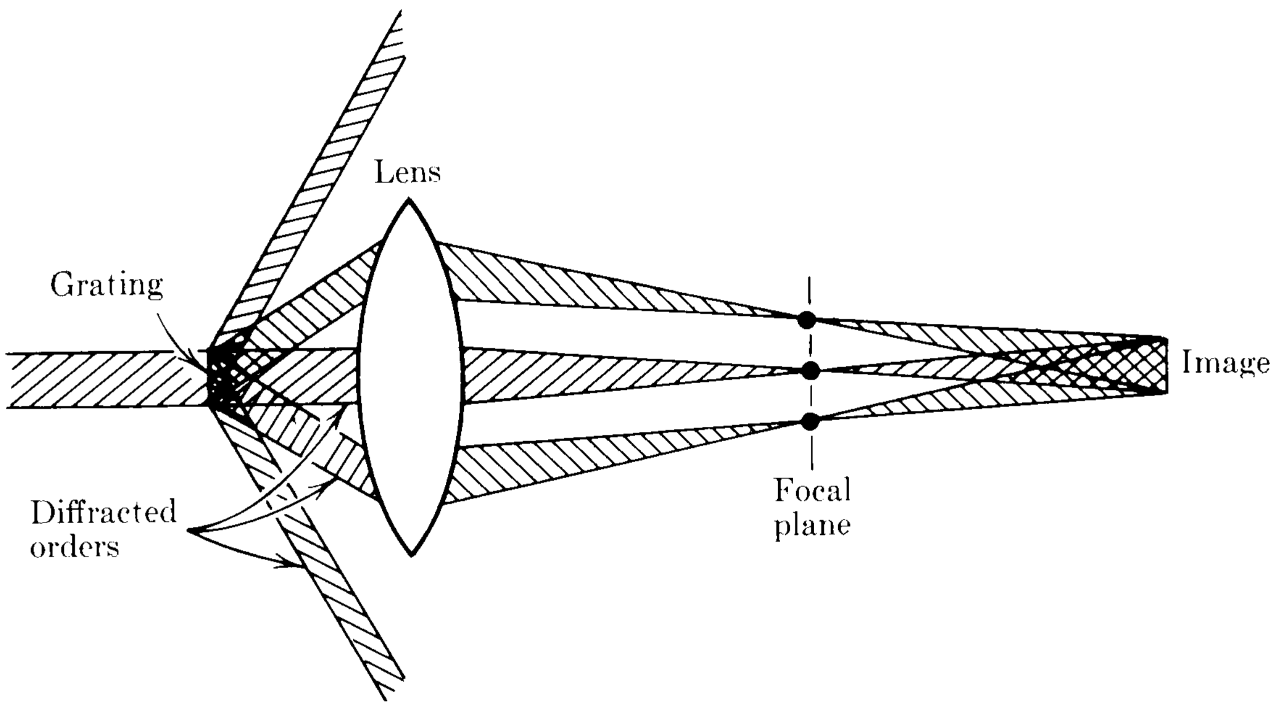


Figure 6-2 The Abbe theory of image formation.

OTF Calculation via Correlation Function

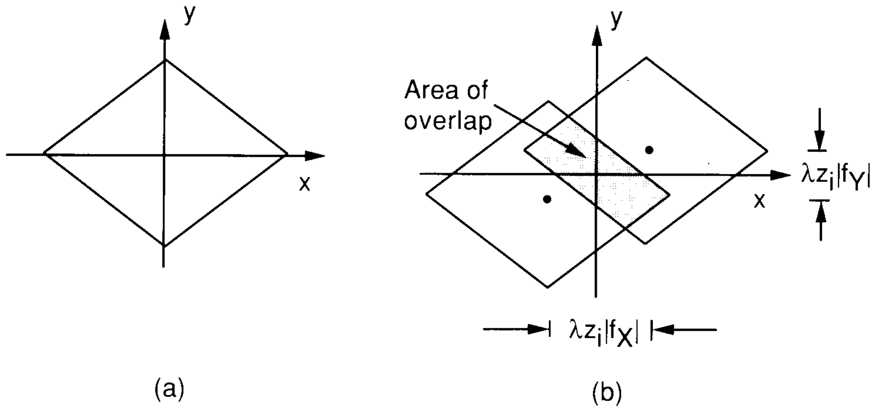


FIGURE 6.4
Geometrical interpretations of the OTF of a diffraction-limited system. (a) The pupil function—total area is the denominator of the OTF; (b) two displaced pupil functions—the shaded area is the numerator of the OTF.

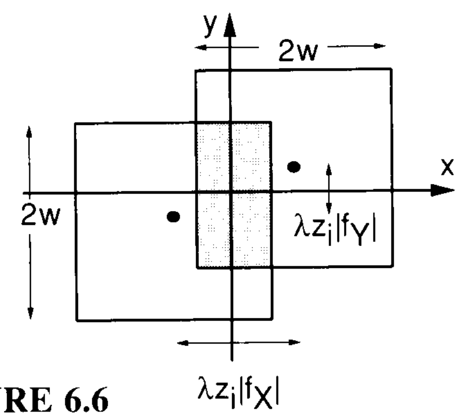


FIGURE 6.6
Calculation of the OTF for a square aperture.

<u>Note</u> that the OTF for a square aperture is linear along the axes and quadratic along the diagonals of the base, and that the base is twice the size of the square aperture or coherent transfer function.

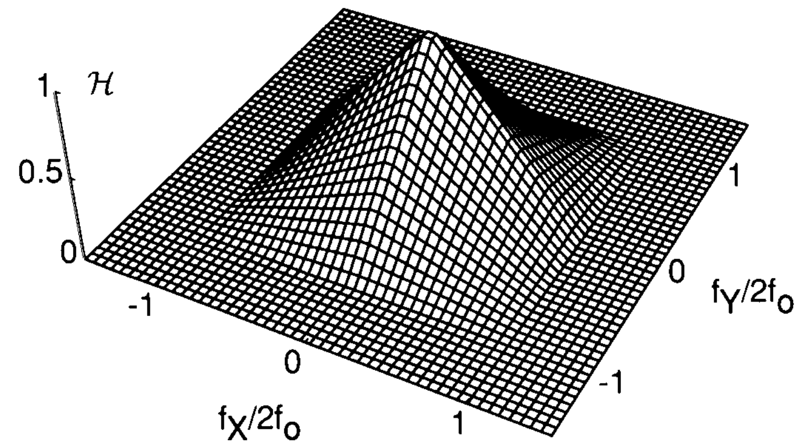


FIGURE 6.7
The optical transfer function of a diffraction-limited system with a square pupil.

OTF of a Circular Aperture

Again the base circle is twice the diameter of the aperture and the coherent transfer function.

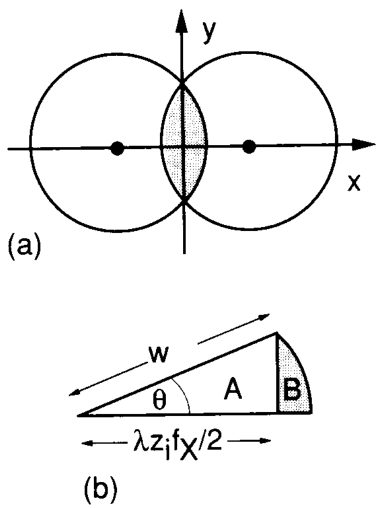


FIGURE 6.8
Calculation of the area of overlap of two displaced circles.
(a) Overlapping circles, (b) geometry of the calculation.

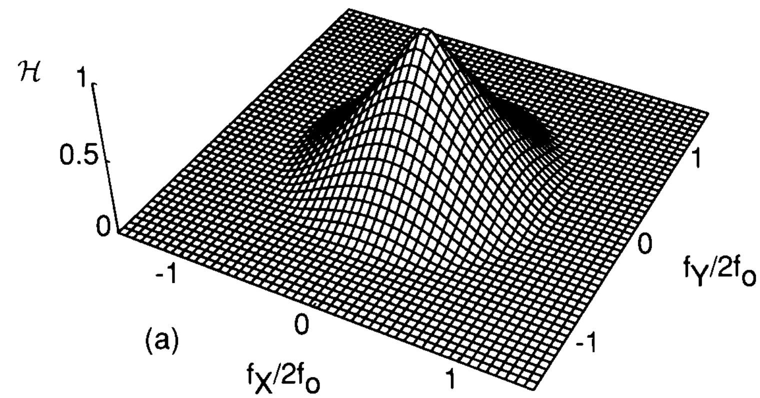
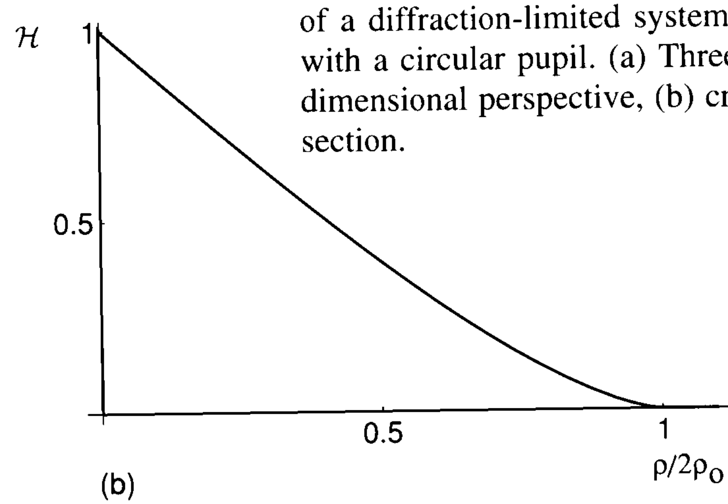


FIGURE 6.9
The optical transfer function of a diffraction-limited system with a circular pupil. (a) Three-dimensional perspective, (b) cross section.



Effect of Aberrations - Misfocus Example

General aberration analysis looks at deviation from spherical wavefronts.

Misfocus with a square aperture is a case that can be analyzed.

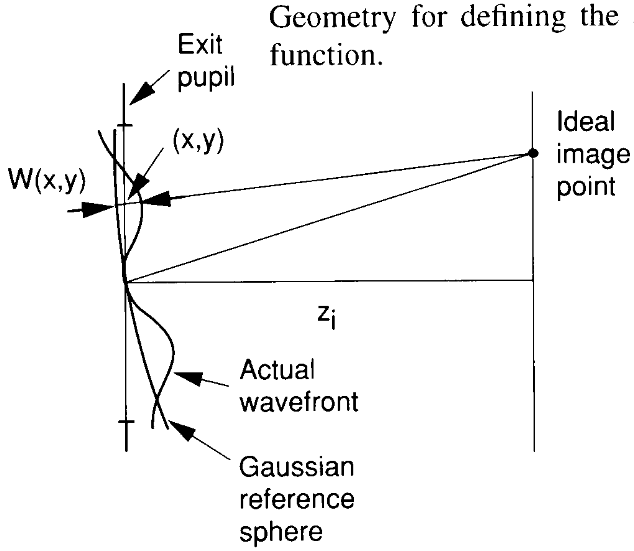


FIGURE 6.10
Geometry for defining the aberration function.

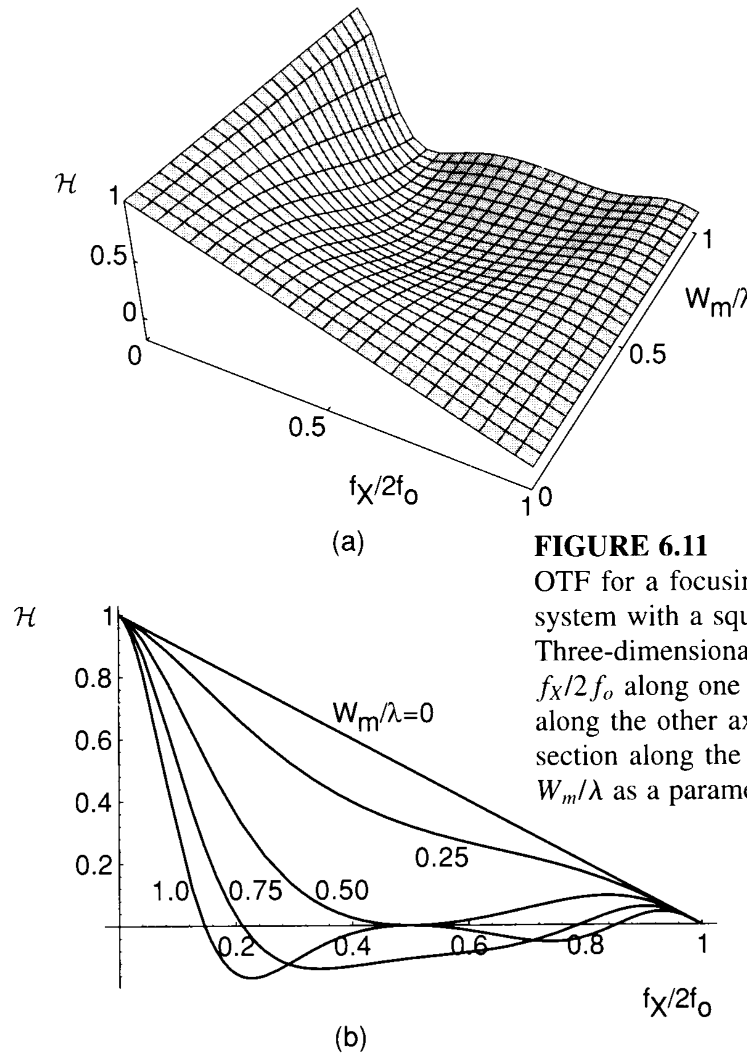


FIGURE 6.11
OTF for a focusing error in a system with a square pupil. (a) Three-dimensional plot with $f_X/2f_o$ along one axis and W_m/λ along the other axis. (b) Cross section along the f_X axis with W_m/λ as a parameter.

Aperture Shape Effects

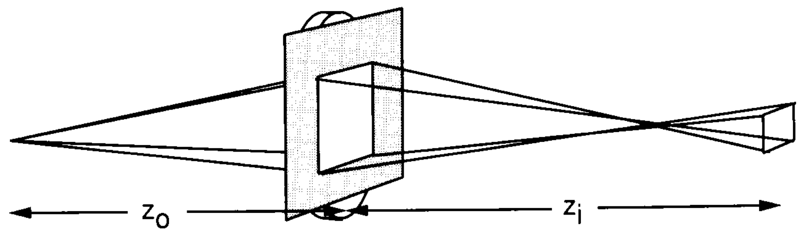


FIGURE 6.13
Geometrical optics prediction of the point-spread function of a system having a square pupil function and a severe focusing error.

Severe misfocus error goes to geometrical optics limit. Interesting example is pinhole camera homework problem. One limiting case is simply shadow casting.

- **One way to improve the point spread function is to apodize the aperture. What is gained and what is lost?**

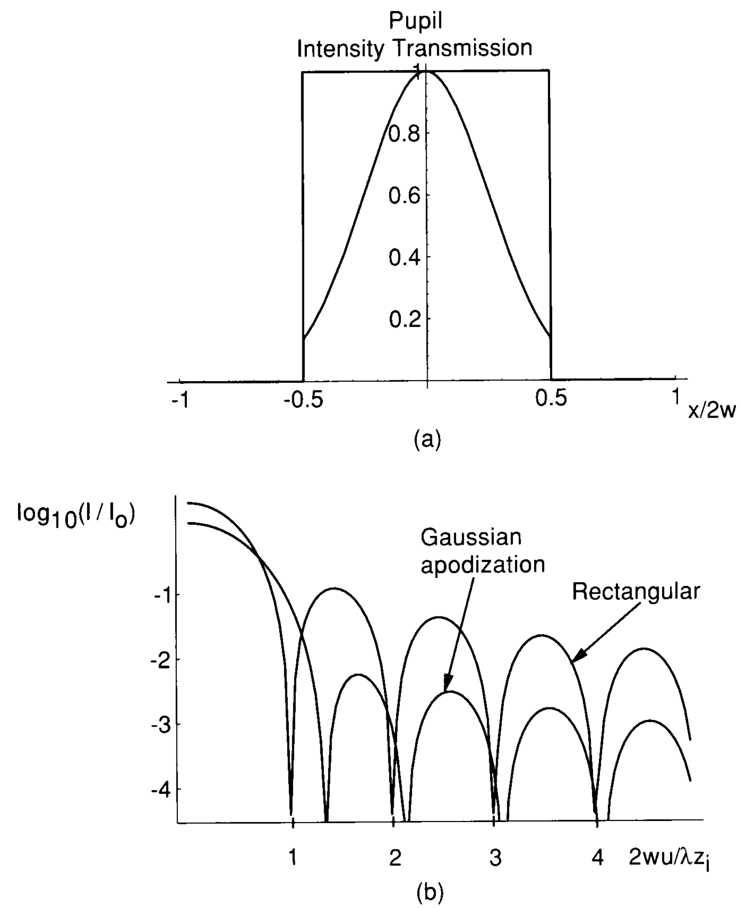


FIGURE 6.14
Apodization of a rectangular aperture by a Gaussian function.
(a) Intensity transmissions with and without apodization.
(b) Point-spread functions with and without apodization.

Apodization Continued

Gains and losses appear in the frequency domain resulting from apodization. Don't confuse this with the inequality that applied to (phase) aberrations.

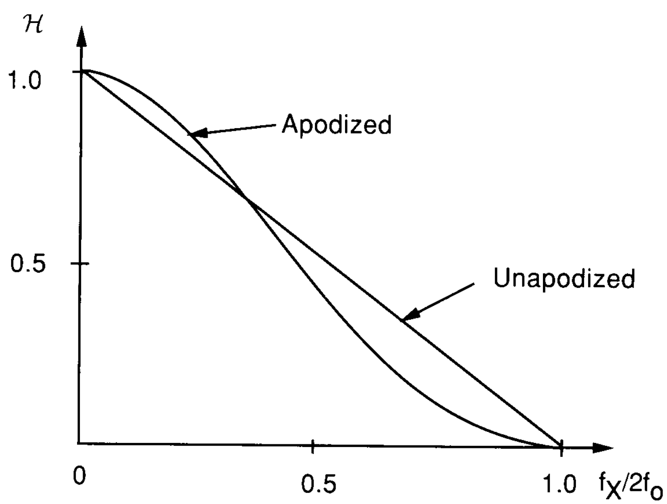


FIGURE 6.15
Optical transfer functions with and without a Gaussian apodization.

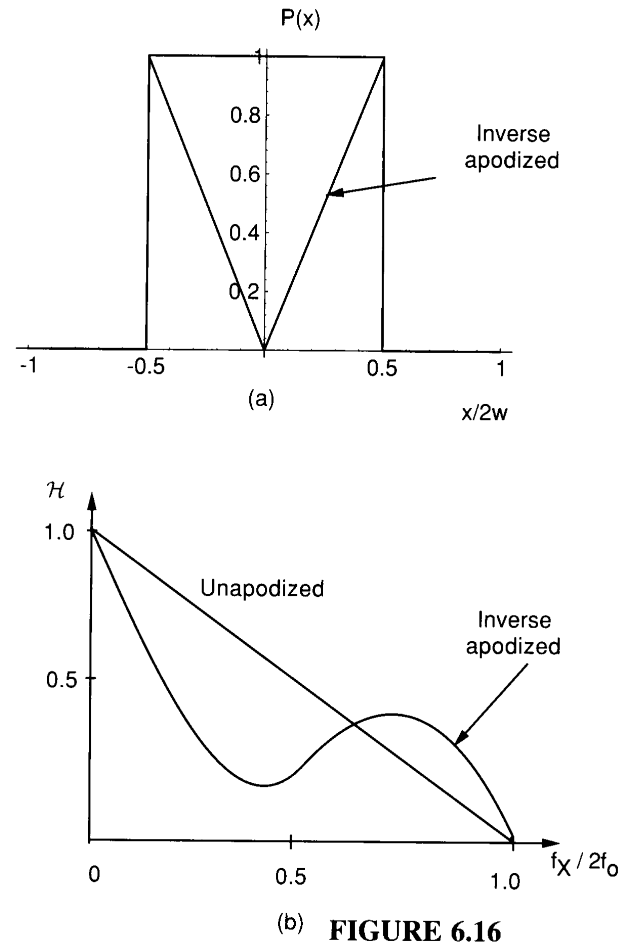


FIGURE 6.16
Pupil amplitude transmittance and the corresponding OTF with and without a particular "inverse" apodization.

Comparison Example - Coherent vs. Incoherent

1. Amplitude spectrum of cosine (left) and intensity spectrum of the same function (right)

[Object or input function]

2. Coherent (amplitude transfer function (left) and

OTF (right) [square aperture]

for imaging systems.

3. Output intensity spectra for coherent imaging system (left) and Incoherent system (right)

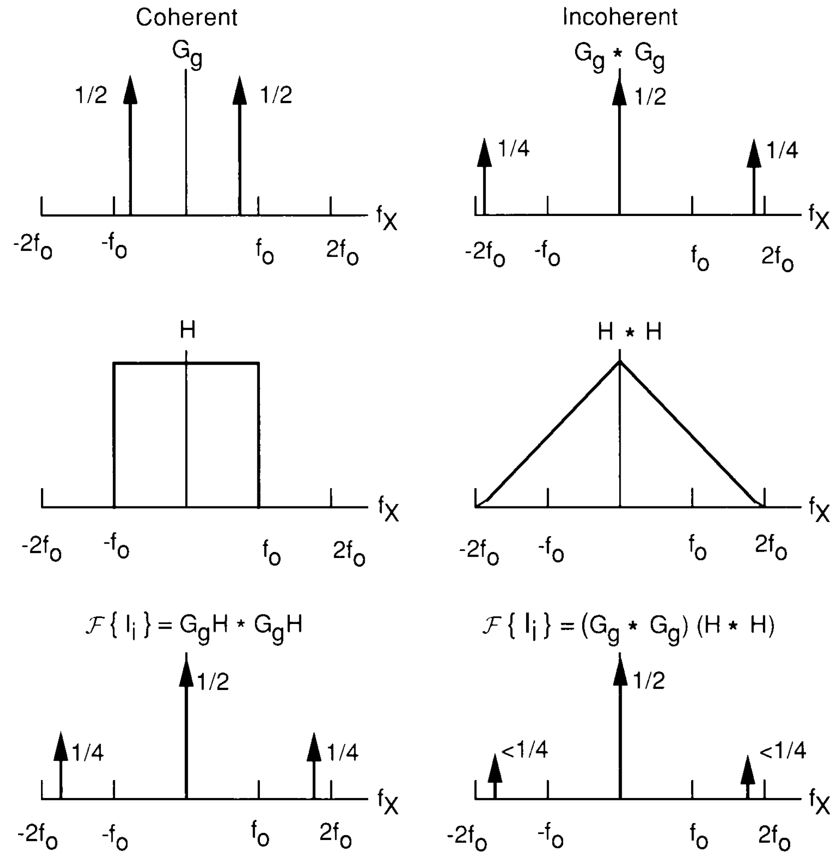


FIGURE 6.17
Calculation of the spectrum of the image intensity for object *A*.

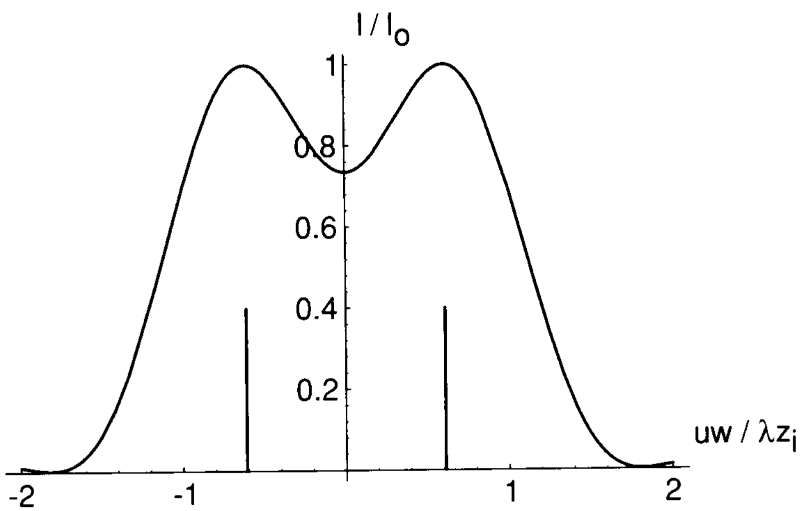
$$A: \quad t_A(\xi, \eta) = \cos 2\pi \tilde{f} \xi$$

$$B: \quad t_A(\xi, \eta) = |\cos 2\pi \tilde{f} \xi|.$$

Resolution Criteria

Rayleigh criterion for two incoherent point sources yields a single result (left). But, for the coherent case, the result depends on the relative phase of the two coherent point sources

FIGURE 6.18
Image intensity for two equally bright incoherent point sources separated by the Rayleigh resolution distance. The vertical lines show the locations of the two sources.



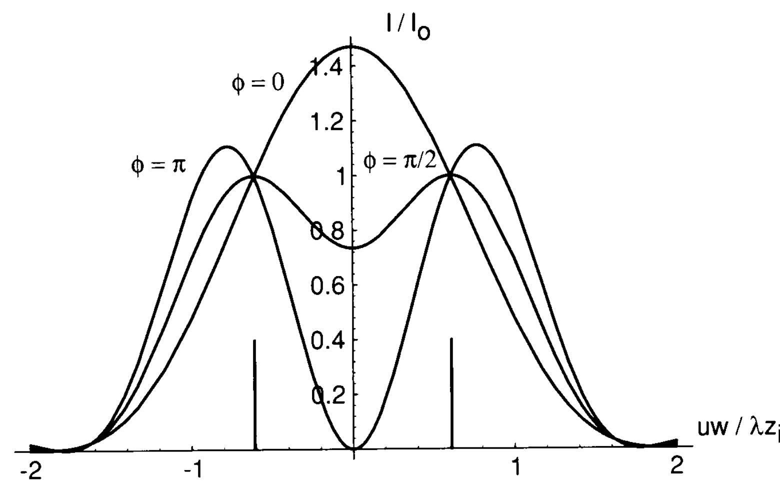


FIGURE 6.19
Image intensities for two equally bright coherent point sources separated by the Rayleigh resolution distance, with the phase difference between the two sources as a parameter. The vertical lines show the locations of the two point sources.