Two-Dimensional Signals

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1 Introduction to the Course

- MECHANICS
  - Course descriptor
  - Other handouts: also available on the course Web page
  - Problem sets
    - Problems are useful only if you work them.
    - MATLAB and MATHEMATICA will be required sometimes.
  - Textbooks
    - Theory: Dan Dudgeon and Russell Mersereau, *Multidimensional Digital Signal Processing*, Prentice-Hall, 1984. Book is out of print.
    - Applications: Alan C. Bovik, ed., *Handbook on Image and Video Processing*, 2nd ed., Academic Press, ISBN 0-12-119792-1, 2006.
  - Project
    - 50% of the grade
    - Half of the project (grade) is a literature survey submitted in both written and oral form
    - Half of the project (grade) is a final project that includes an implementations and is submitted in both written and oral form
    - Get started as soon as possible
- GOALS
  - Develop insight to solve problems.
    - Filling in the details is easy once you know what the answer is.
  - Explore similarities and differences with 1-D
  - Learn specific techniques
    - These will be forgotten, but insight should remain.
  - Reinforce what has already been learned for 1-D.
  - Explore some applications.

- WHAT IS THE DIFFERENCE ?
  - Many things are the same.
    * Concepts generalized, details change.
  - Some mathematics does not generalize.
    * Polynomials do not factor.
  - Application assumptions different
    * Not causal
    * Inputs finite extent
  - Problems are bigger
    * 1-D example CD audio: 44.1 kHz $\times$ 2 channels $\times$ 2 bytes/sample = 176.4 kbytes/sec.
      M-D example high-definition TV (1080p): $1920 \times 1080 \times 60$ frames/sec $\Rightarrow$ 124.4 million color samples/sec $\Rightarrow$ 373.2 Mbytes/sec
    * Processor speed? Memory? Addressing capability?

2 2-D Signals

We represent a 2-D signal as $x(n_1, n_2)$ where x may be real-valued or complex-valued, and n_1 and n_2 are integer indices. We'll use n_1 as the horizontal index, and n_2 as the vertical index.
In image processing, the roles of n_1 and n_2 are interchanged in that n_1 is the row index and n_2 is the column index.

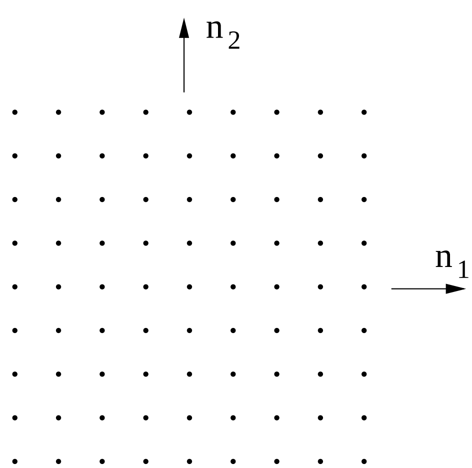

Figure 1: 2-D grid

- Examples

- Sampled B/W photograph: $x(n_1, n_2)$
- Sampled B/W video signal: $x(n_1, n_2, n_3)$ where n_1 and n_2 are the row and column indices of each frame and n_3 is the time index.
- Sampled color video signal:
  Three fundamental standards are:
  * RGB (red, green, blue): Monitors
  * CMYK (cyan, magenta, yellow, black): Printers
  * YUV (luminance, chrominance): Broadcast TV

  An RGB video signal can be represented as:

$$\left\{ \begin{array}{l} x_R(n_1, n_2, n_3) \\ x_G(n_1, n_2, n_3) \\ x_B(n_1, n_2, n_3) \end{array} \right\}$$

- Seismic array: $x(n_1, n_2)$

2.1 Human Visual System

A non-linear, non-uniformly sampled, spatially varying, non-separable system. We will find later in the course during the lecture on halftoning that we can model the human visual system as a linear, uniformly sampled, spatially varying, non-separable system for the purposes of assessing image quality.

2.2 Special 2-D Sequences

- Unit Impulse $\delta(n_1, n_2) = \begin{cases} 1 & n_1 = n_2 = 0 \\ 0 & \text{otherwise} \end{cases}$

  2-D unit impulse is separable since $\delta(n_1, n_2) = \delta(n_1)\delta(n_2)$.

- Line Impulses
  Vertical line impulse: $x(n_1, n_2) = \delta(n_1)$ is separable since $x(n_1, n_2) = \delta(n_1) \times 1$

  Horizontal line impulse: $x(n_1, n_2) = \delta(n_2)$ is separable since $x(n_1, n_2) = 1 \times \delta(n_2)$

  Diagonal line impulses are non-separable.

  A general diagonal line impulse can be written as: $\delta(Pn_1 + Qn_2)$ where P and Q are integers. The slope of the impulses is $-\frac{P}{Q}$.

- Unit step sequence:

$$u(n_1, n_2) = \begin{cases} 1 & n_1 \geq 0 \text{ and } n_2 \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

  The unit step function is separable since $u(n_1, n_2) = u(n_1)u(n_2)$.

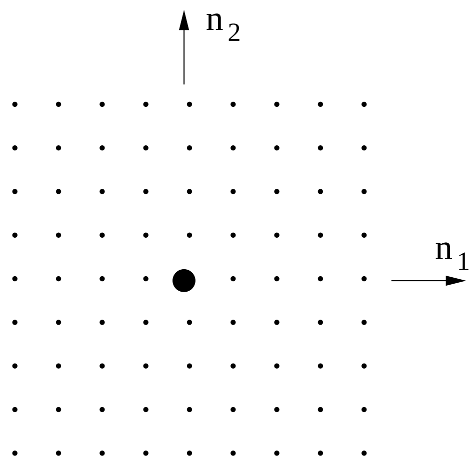

Figure 2: 2-D Unit impulse

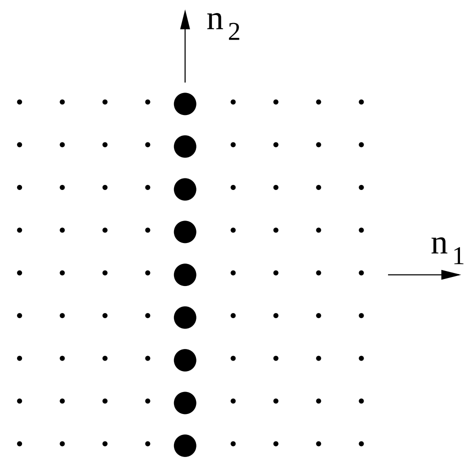

Figure 3: 2-D vertical line impulse

2.3 Separable Signals

- A 2-D signal x is separable if

$$x(n_1, n_2) = f(n_1)\, g(n_2)$$

- An M-dimensional signal x is separable if

$$x(n_1, n_2, ..., n_M) = \prod_{i=1}^{M} f_i(n_i)$$

2.4 Finite Extent Sequence

A finite-extent sequence is one with a finite number of non-zero samples.

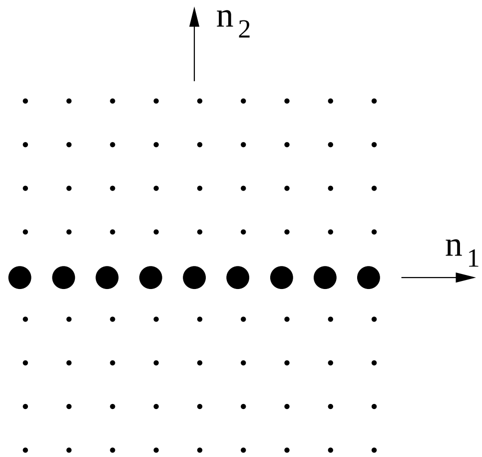

Figure 4: 2-D horizontal line impulse

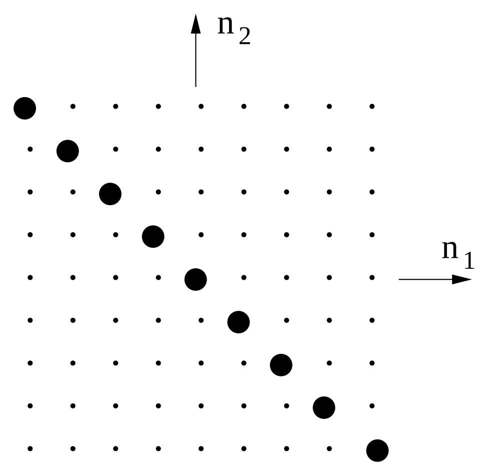

Figure 5: Diagonal line impulse: $\delta(n_1 + n_2)$

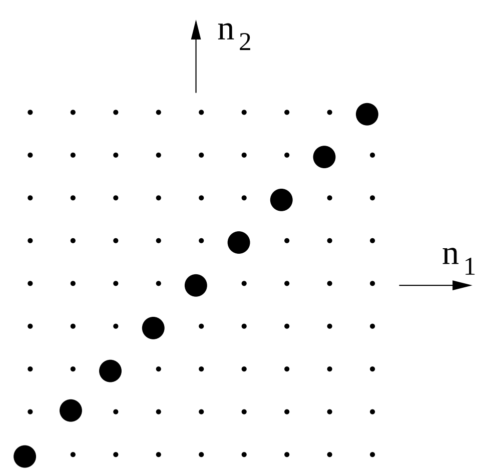

Figure 6: diagonal line impulse: $\delta(n_1 - n_2)$

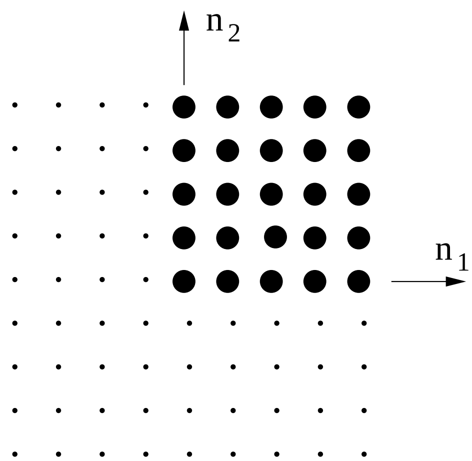

Figure 7: 2-D unit step

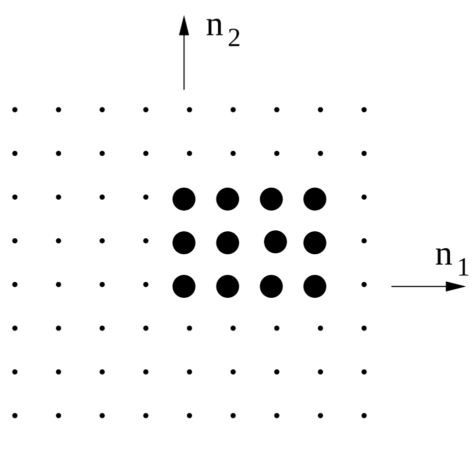

Figure 8: A finite extent sequence