Image Restoration

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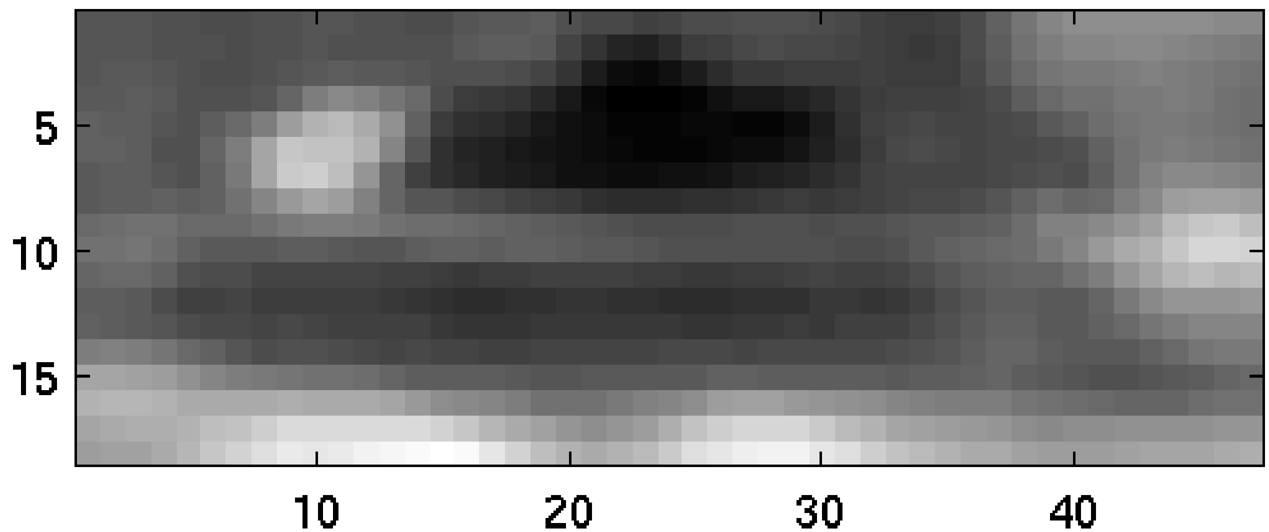
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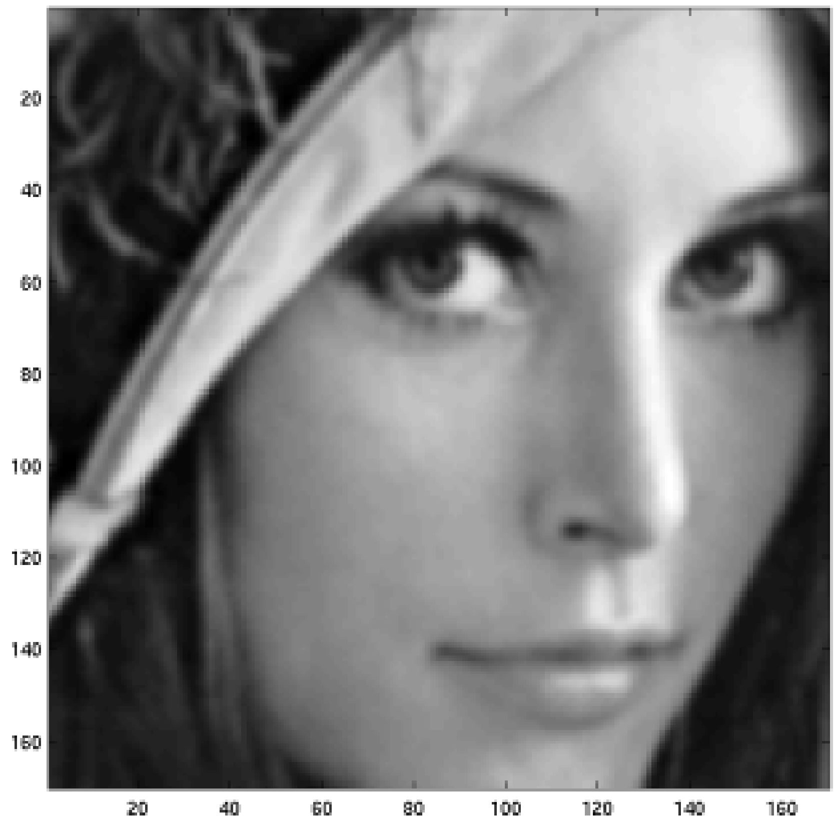
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Degradation Model

- In noise-free cases, a blurred image can be modeled as

$$y = x * h$$

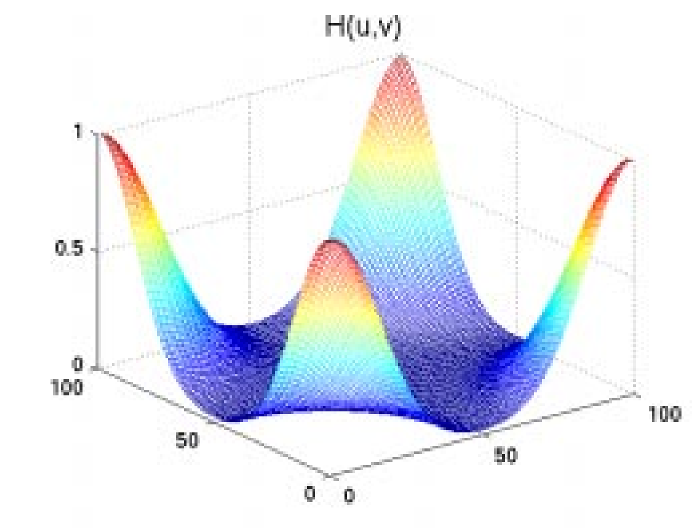
h : linear space-invariant blur function

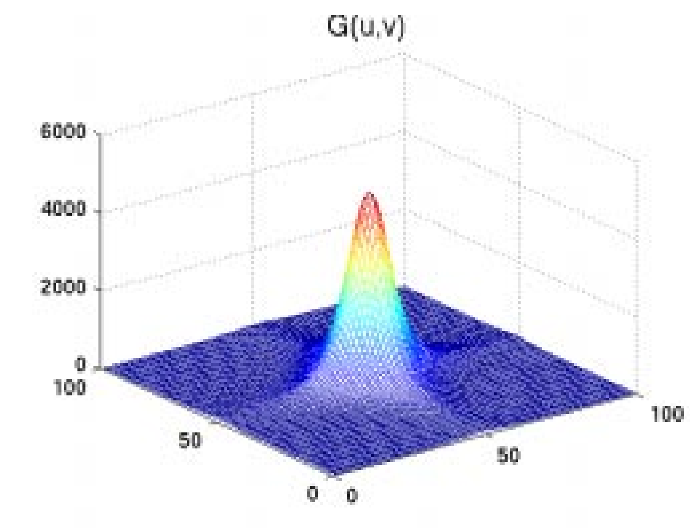
x : original image

- In the DFT domain, Y(u,v) = X(u,v) H(u,v)

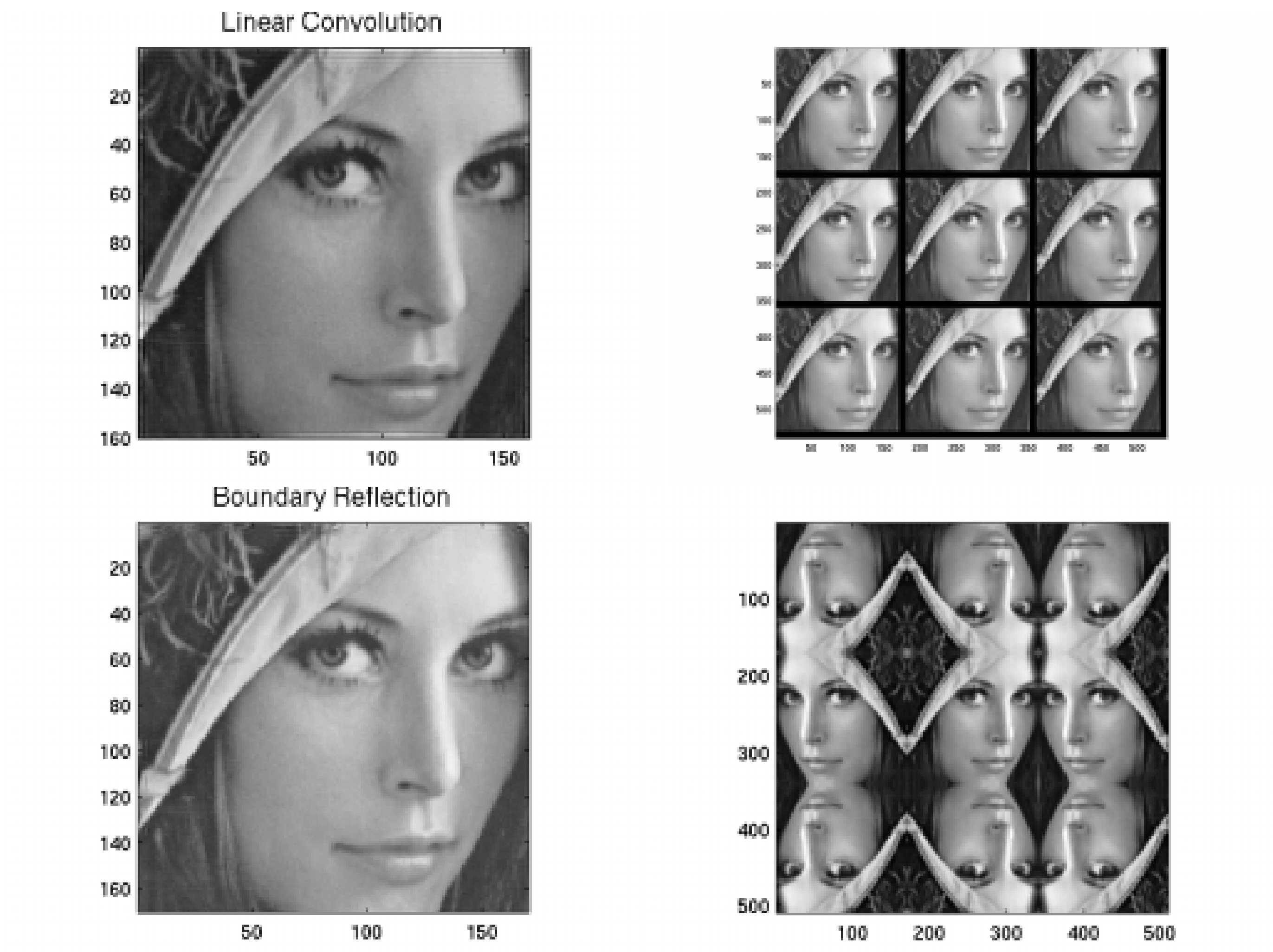
Inverse Filtering

- Assume h is known (low-pass filter)
- Inverse filter $G(u,v) = 1 / H(u,v)$
- $\tilde{X}(u, v) = Y(u, v)\, G(u, v)$

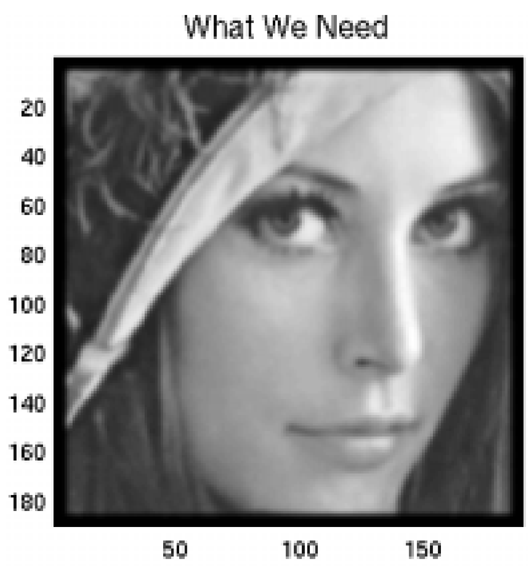


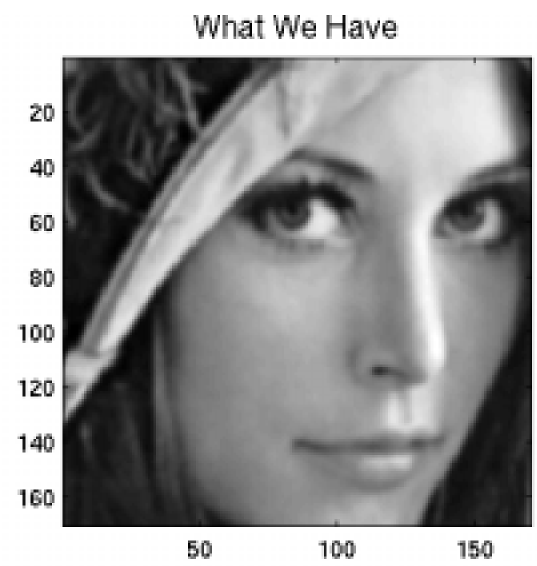


Implementing Inverse Filtering



Lost Information



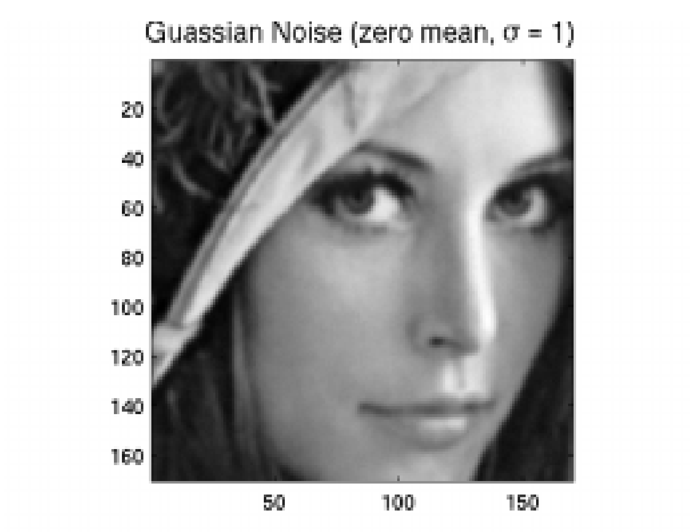


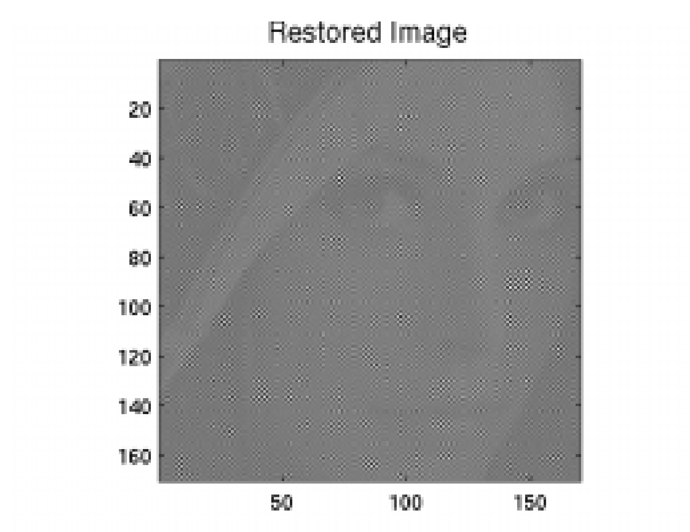
Problems with Inverse Filtering

- H(u,v) = 0, for some u, v
- In noisy case,

$$y = x * h + n$$

$$n : \text{additive noise}$$





Wiener Filter Formulation

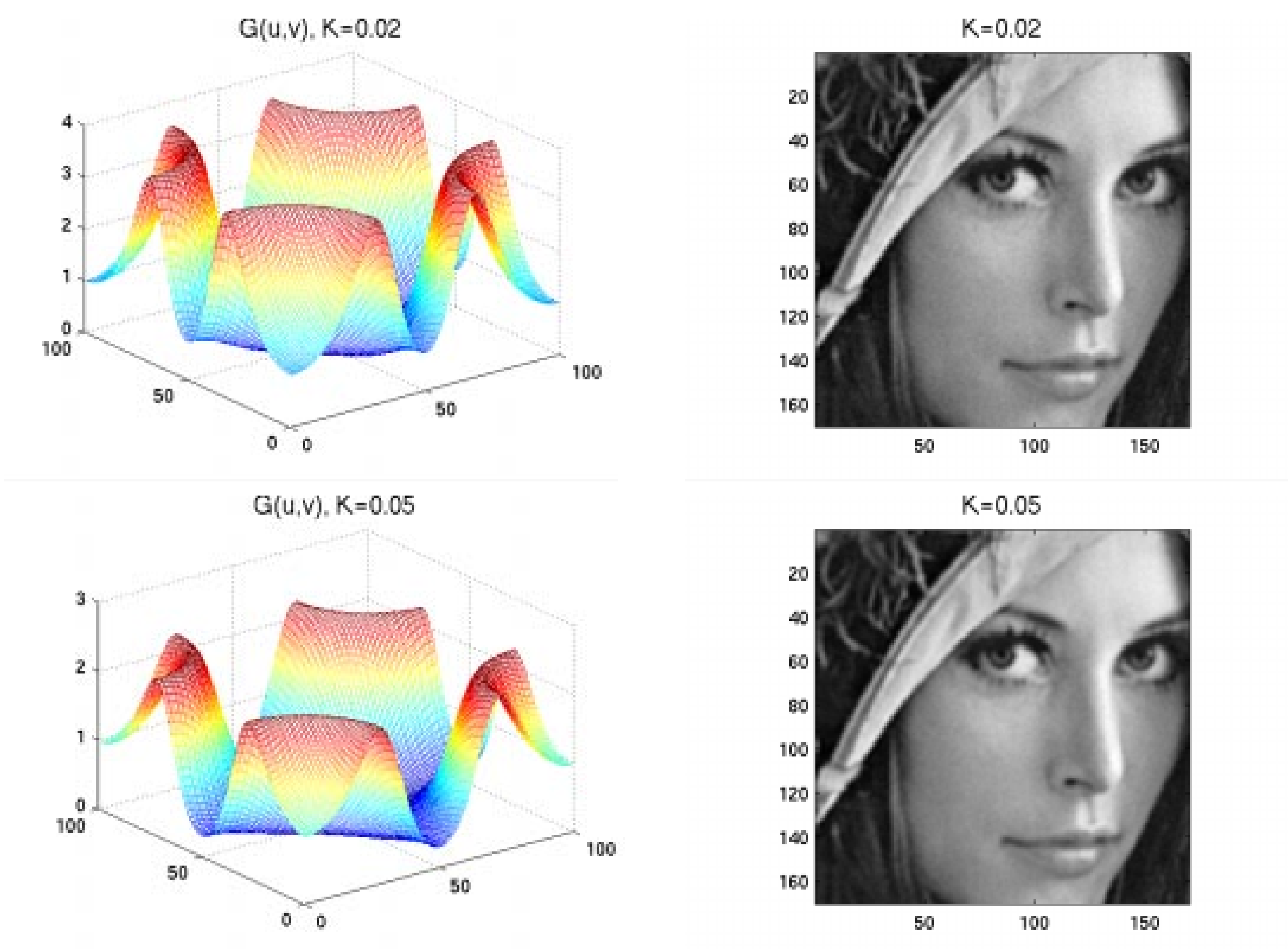
- Least Mean Square Filter

$$G(u,v) = \frac{H^{*}(u,v)}{\left|H(u,v)\right|^{2} + \left[S_{n}(u,v)/S_{x}(u,v)\right]}$$

- In practice

$$G(u,v) = \frac{H^{*}(u,v)}{\left|H(u,v)\right|^{2} + K}$$

Wiener Filter Results



Maximum-Likelihood (ML) Estimation

- h is unknown
- Assume parametric models for the blur function, original image, and/or noise
- Parametric set θ is estimated by

$$\theta_{ml} = \arg\{\max_{\theta} p(y \mid \theta)\}$$

- Solution is difficult

Expectation-Maximization (EM) Algorithm

- Find *complete set* Z: for z $\in$ Z, *f*(z)=y
- Choose an initial guess of θ
- Expectation-step

$$g(\theta \mid \theta^{k}) = E[p(z \mid \theta) \mid y, \theta^{k}]$$

- Maximization-step

$$\theta^{\mathbf{k}+1} = \arg\max_{\theta} g(\theta \mid \theta^{k})$$

Subspace Methods

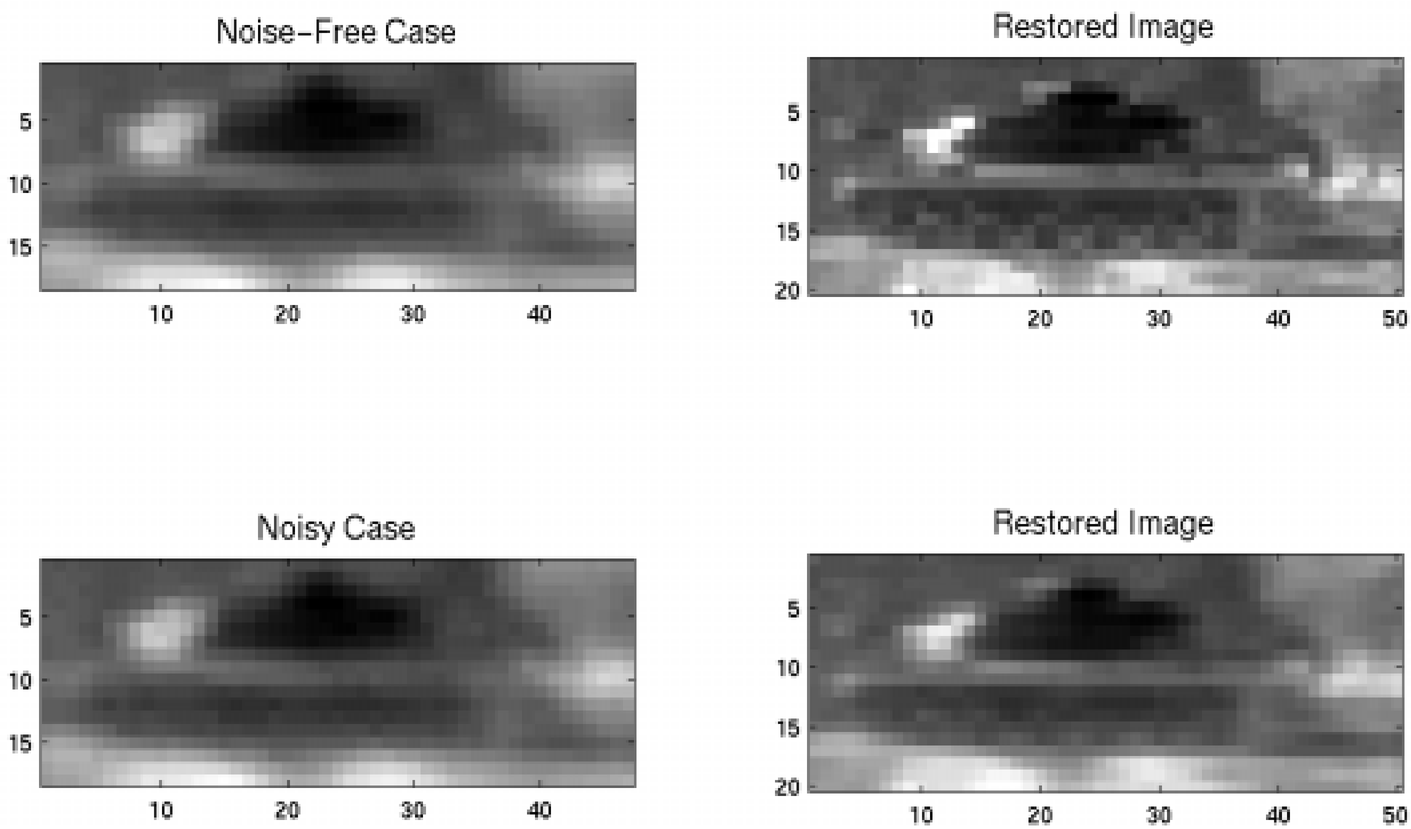
- Observe

$$\begin{bmatrix} a_0 & a_1 & a_2 \\ a_1 & a_2 & a_3 \\ a_2 & a_3 & a_4 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} b_0 & b_1 & b_2 & 0 & 0 \\ 0 & b_0 & b_1 & b_2 & 0 \\ 0 & 0 & b_0 & b_1 & b_2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

Subspace Methods

- Several blurred versions of original image are available
- Construct a block Hankel matrix X of blurred images
- $X = H\Sigma$, where H is a block Toeplitz matrix of the blur functions and Σ is a block Hankel matrix of the original image

Subspace Methods Results



Conclusions

- Noise-free case: inverse filtering
- Noisy case: Weiner filter
- Blind case: Maximum-Likelihood approach using the Expectation-Maximization algorithm
- Multichannel blind case: subspace methods

Further Reading

- M. R. Banham and A. K. Katsaggelos "Digital Image Restoration, " *IEEE Signal Processing Magazine,* vol. 14, no. 2, Mar. 1997, pp. 24-41.
- D. Kundur and D. Hatzinakos, "Blind Image Deconvolution," *IEEE Signal Processing Magazine,* vol. 13, no. 3, May 1996, pp. 43-64.