Outline

- 2-D Narrowband Signals
- 2-D Decimation Systems
- Interactive Design Method
- Example
- Conclusion

2-D Narrowband Signals

- **Image Processing**

  Digitized pictures are often oversampled

- **Video Processing**

  Quincunx decimation has little perceptual effect

- **Seismic Processing**

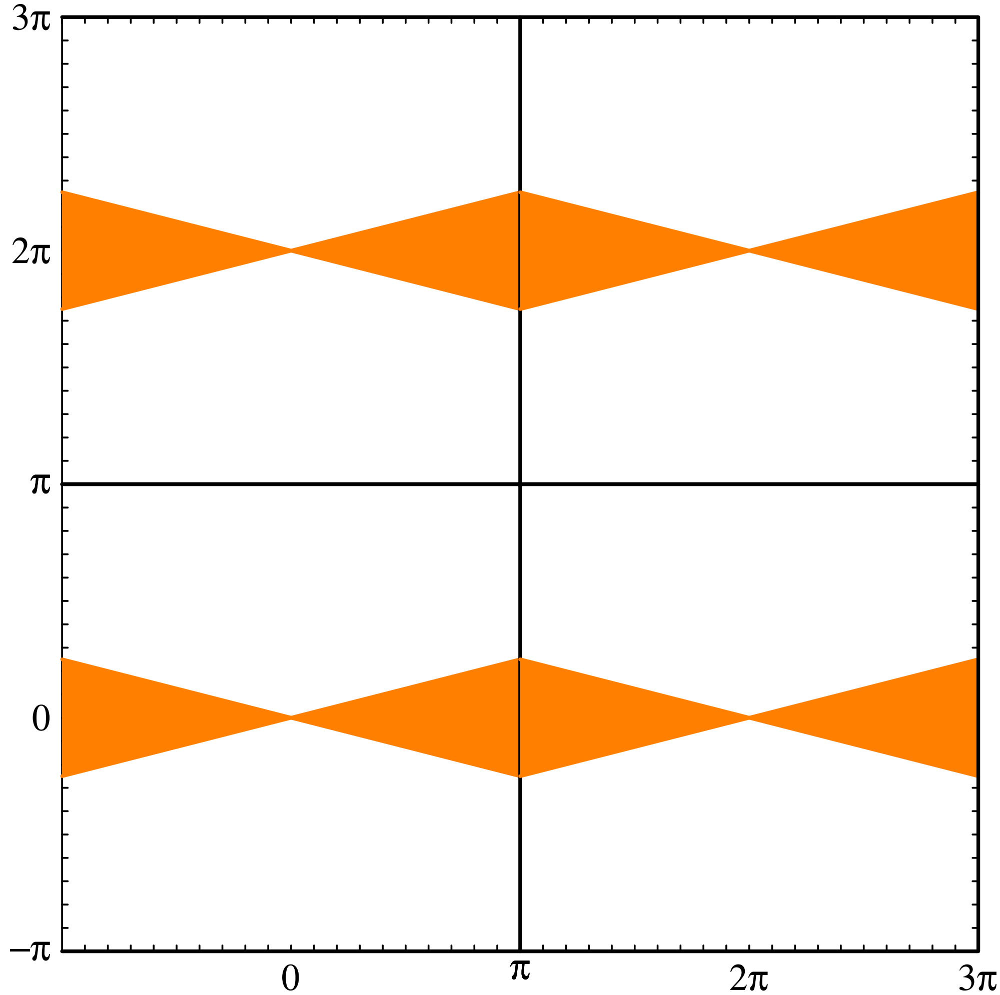
  Fan filtering

2-D Narrowband Signals

- **Pictorial Example: *Fan Filters***

  For example, velocity filters for position-time data

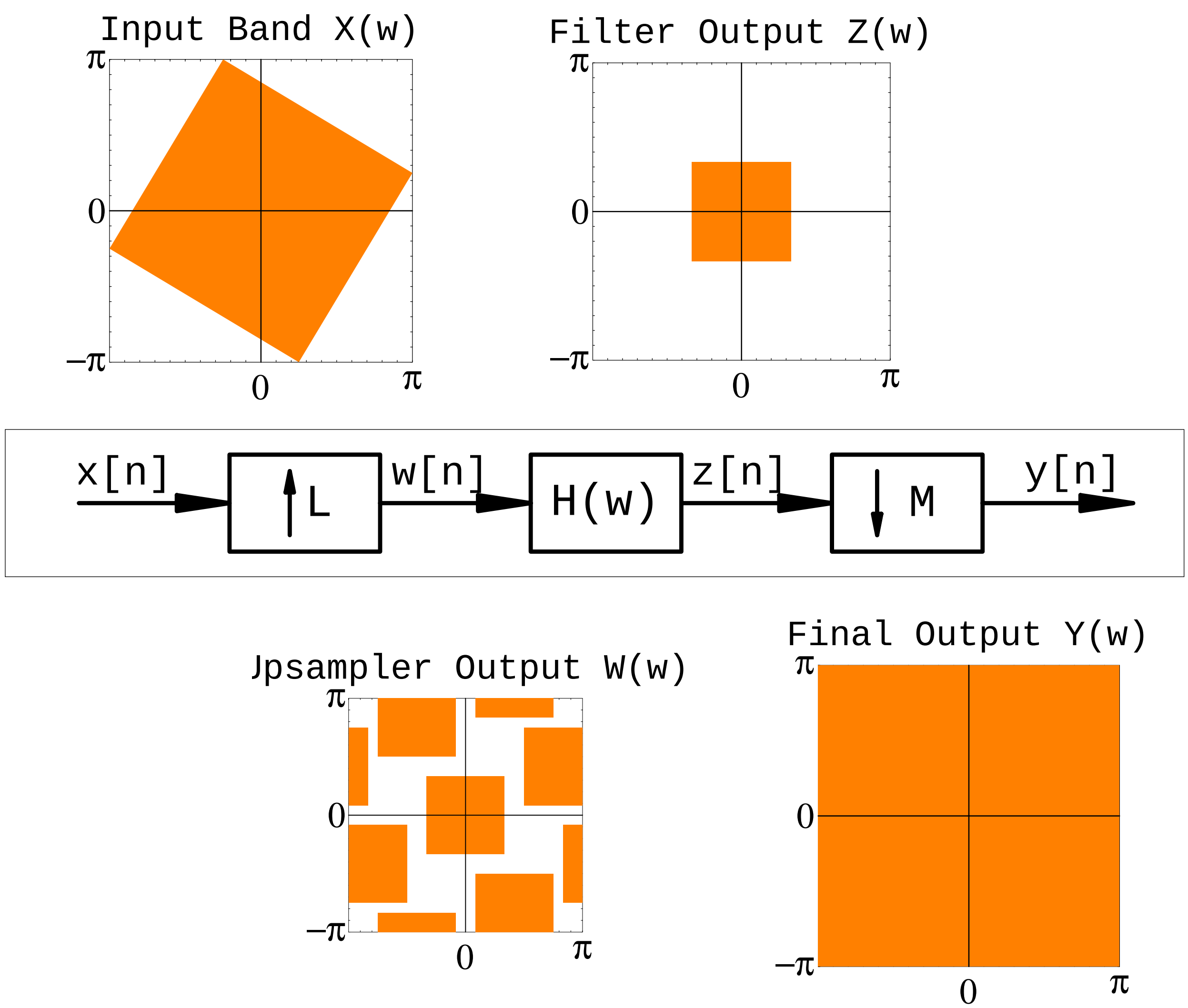
  If filtering specification includes ω_1 or ω_2 axis, periodic extension reveals parallelogram-shaped passbands



2-D Decimation Systems

Nyquist Resampling of Signals with Parallelogram-Shaped Passbands

Using only linear components [Chen & Vaidyanathan]



2-D Decimation Systems

Underlying Theory

- **Reported by Chen & Vaidyanathan at ICASSP '93**

- **Pick vertices of parallelogram to be rational multiples of π**

- **Compute rational matrix H from the vertices**

  (a) **H** maps parallelogram *onto* square fundamental frequency tile: $-\pi < \omega_1 < \pi$ and $-\pi < \omega_2 < \pi$

  (b) $\mathbf{H} = \mathbf{L}^{-1}\,\mathbf{M}$ where **L** and **M** integer matrices

2-D Decimation Systems

- **Converting Theory to an Algorithm**

  [Evans & Sakarya]

- **Factor $H = L^{-1} M$ using Smith-McMillan form of H**

$$H = U \Lambda V = U \Lambda_L^{-1} \Lambda_M V$$
$$= (\Lambda_L U^{-1})^{-1} (\Lambda_M V) = L^{-1} M$$

- **Add a modulator to the decimator structure at input**

  To make regions symmetric by shift centers to origin

- **Circumscribe arbitrary region with parallelogram**

  Sub-optimal method based on rotated rectangles $O(N^2)$

2-D Decimation Systems

Improved Algorithm

- **Find convex hull of arbitrary region first**

  Using Graham Scan to produce a convex polygon

- **Minimal enclosing parallelogram**

  Two adjacent sides overlap with two polygon edges

  Other two adjacent sides intersect polygon vertices

- **Circumscribe arbitrary region with parallelogram**

  Optimal method based on exhaustive search $O(N^2)$

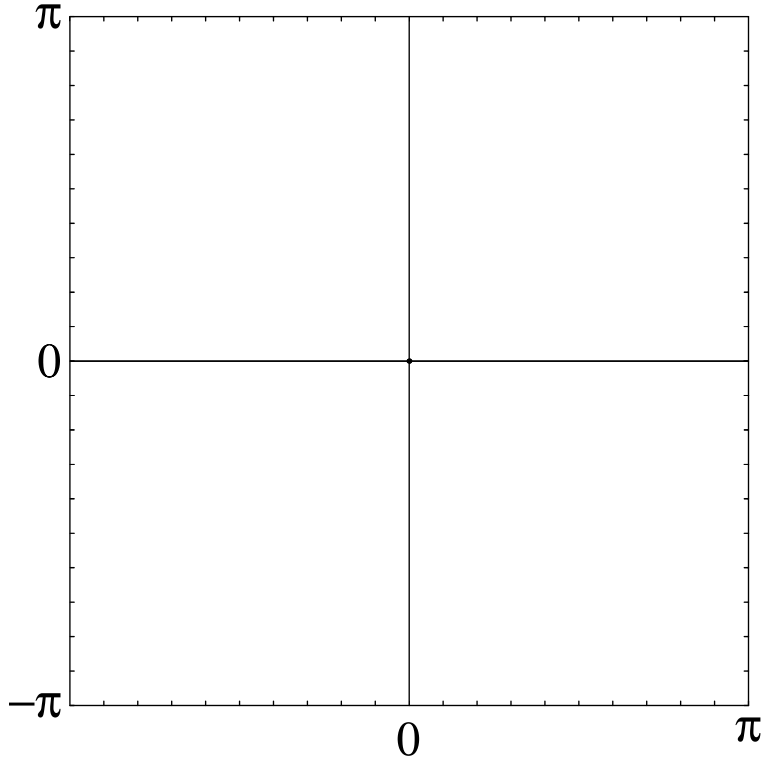
  Optimal method based on anti-podal pairs $O(N)$

Interactive Design Method

- **Initialization: Sketch Passband**

  Given empty graph of one period of frequency domain

  (a) outline the region of interest using the mouse



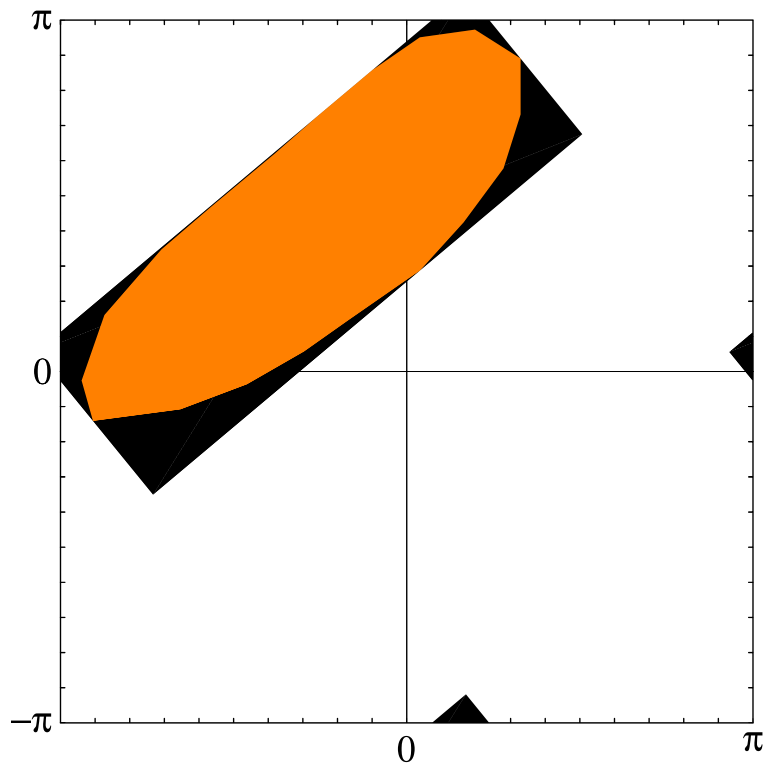
  (b) copy and paste the selected points to define the vertices of a polygon that represents the region

Interactive Design Method

- **Step 1: Circumscribe Passband with Rectangle**

  Input: polygon (*in* **orange**) passband whose vertices are defined by floating-point coordinates

  Output: rectangle (*in* **black**) whose vertices are defined by floating-point coordinates



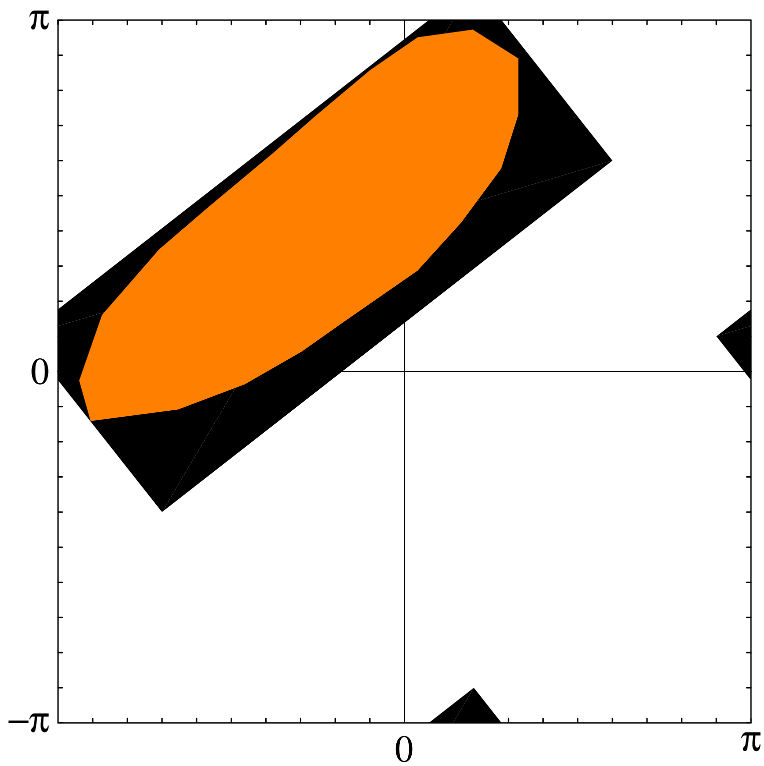
Improved Algorithm: computes convex hull

Interactive Design Method

- **Step 2: Circumscribe Rectangle with Parallelogram**

  Input: rectangle (*not shown*) whose vertices are defined by floating-point coordinates

  Output: parallelogram (*in* **black**) whose vertices are coordinates that are rational multiples of π



Improved Algorithm: find min. enclosing parallelogram

Interactive Design Method

- **Step 3: Shift Parallelogram to Origin**

  Shift the center of the parallelogram to the origin

  Shift vector is the modulation vector $\mathbf{n_0}$

- **Step 4: Resample Parallelogram**

  Map symmetric parallelogram to square by **H**

  (a) square contains one period of the frequency domain

  (b) **H** is a rational matrix

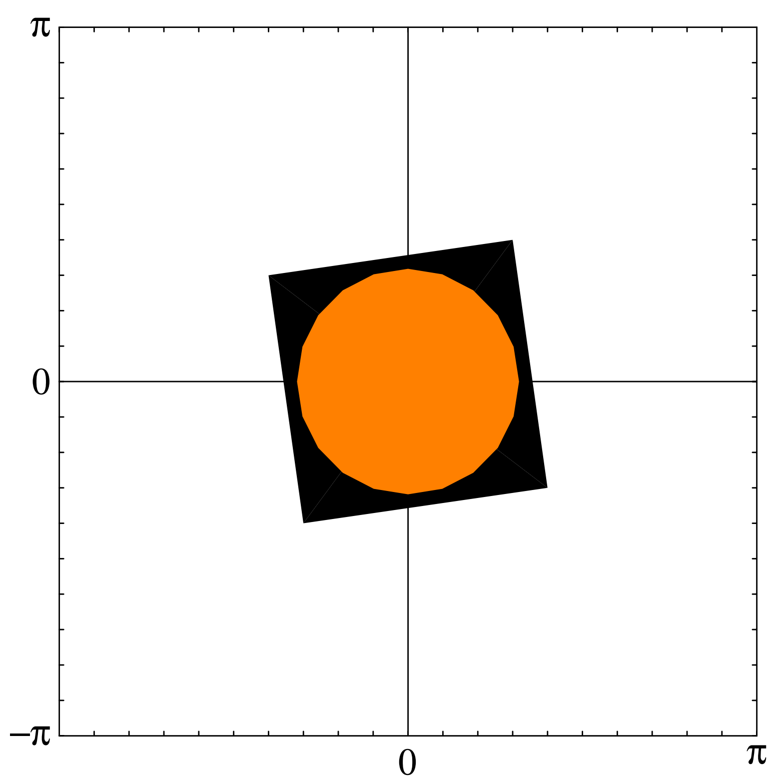
- **Step 5: Factor Matrix H into L and M**

Interactive Design Method

Example

```
{ shiftVector, upMatrix, downMatrix } =
  DesignDecimationSystem[
    poly, Dialogue -> All, Mod -> 10 ]
```

Efficiency of parallelogram: 62.6%



The compression ratio is 8-to-1.

n = (0, 0) $L = \begin{matrix} 7 & 1 \\ 2 & 14 \end{matrix}$ $M = \begin{matrix} 2 & 14 \\ 0 & 20 \end{matrix}$

Conclusion

Quick Prototyping of Interactive Design Procedure

- **Chen & Vaidyanathan report theory at ICASSP '93**

- **Convert theory to an algorithm**

  Add method to compute bounding parallelogram

  Add method to factor rational matrix $\mathbf{H} = \mathbf{L}^{-1}\,\mathbf{M}$

- **Choose environment for rapid implementation**

  Interpreted programming language

  Graphical user interface to sketch passbands

Future Research

2-D Decimator Design

- **Design of 2-D filter**
- **Based on hexagon spectrum**

  Hexagons are better for circular passbands

2-D Filter Bank Design

- **Polygonal tiling of the frequency plane**