Converting Graphical DSP Programs into Memory-Constrained Software Prototypes

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Application Specific Software Environments

- Provide <u>syntax</u> that is natural for the application domain
- Incorporate appropriate <u>computational models</u>
  - may be streamlined to enable powerful optimization
- Optimize for appropriate <u>implementation constraints</u>

Embedded DSP systems

- **Computational characteristics**
  - **Infinitely iterated**
  - **Possibly multirate**
  - **Mostly deterministic control flow**
- **Implementation objectives**
  - **Target throughput**
  - **Memory**
  - **Latency**
  - **Power**

Computational Models for DSP Software

- **Synchronous dataflow**
  — Lee/Messerschmitt, 1987
- **Well behaved stream flow graphs**
  — Gao/Govindarajan/Panangaden, 1992
- **The token flow model**
  — Buck/Lee 1992
- **Multidimensional synchronous dataflow**
  — Lee 1992
- **Scalable synchronous dataflow**
  — Ritz/Pankert/Meyr, 1993
- **Cyclo-static dataflow**
  — Bilsen/Engels/Lauwereins/Peperstraete, 1994

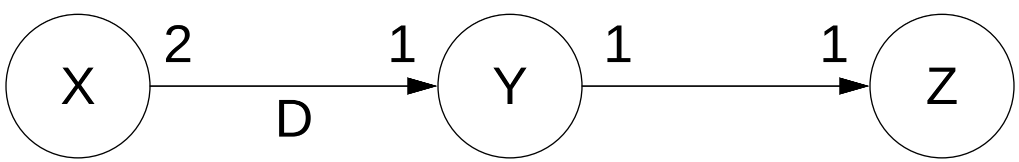
All are closely related to the synchronous dataflow model

Problem overview

- **Minimization of memory requirement (program *and* data) when synthesizing software from a synchronous dataflow program**
  - **Target throughput**
  - **Memory**
  - **Latency**
  - **Power**
- **May be critical to all other objectives**
  - **On-chip vs. off-chip memory**
  - **Limited on-chip memory on programmable DSPs**

Synchronous dataflow

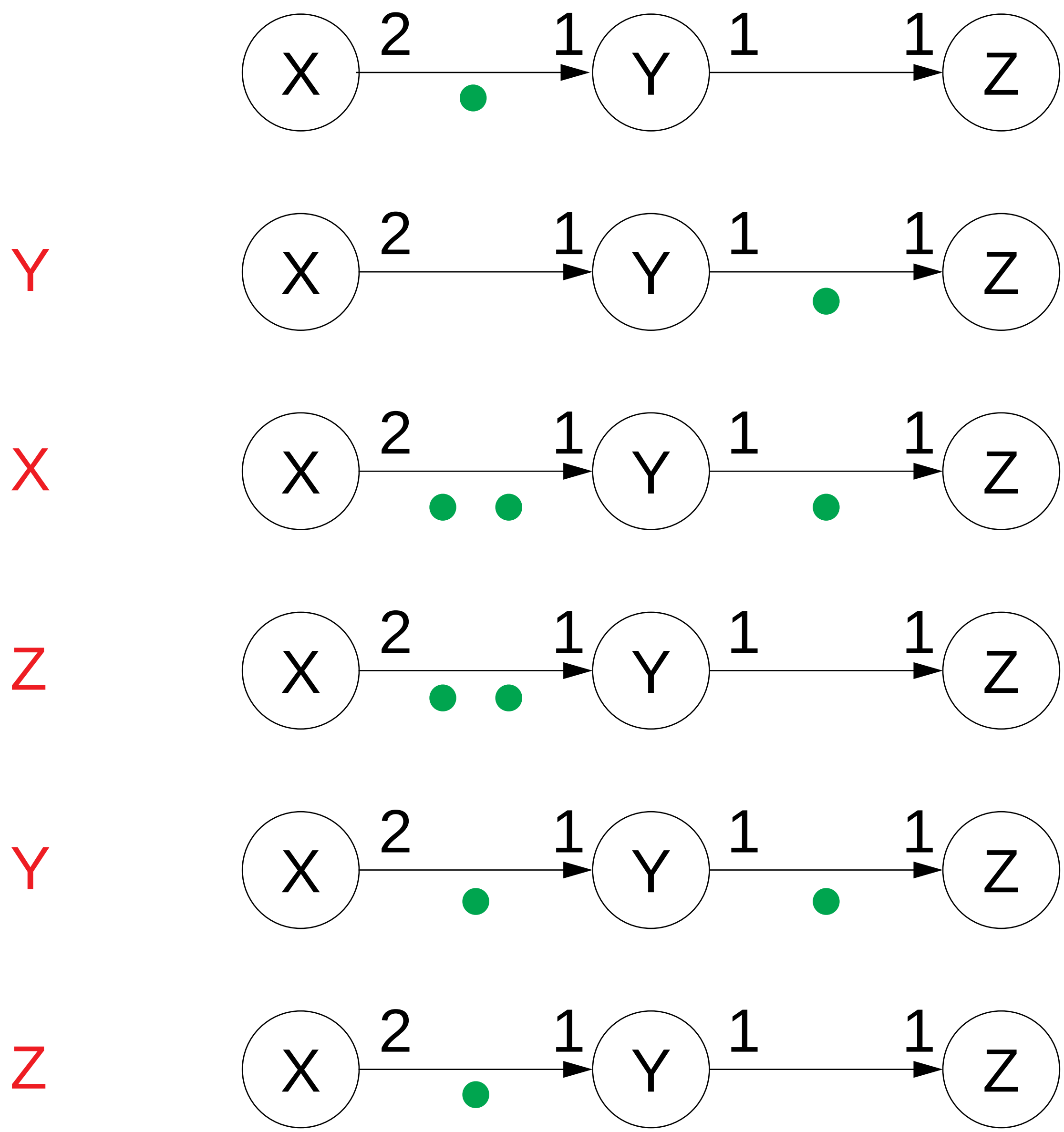
- **The number of tokens produced and consumed by each actor is fixed.**
- **Periodic schedules.**
- **Unique** *repetitions vector* **q.**



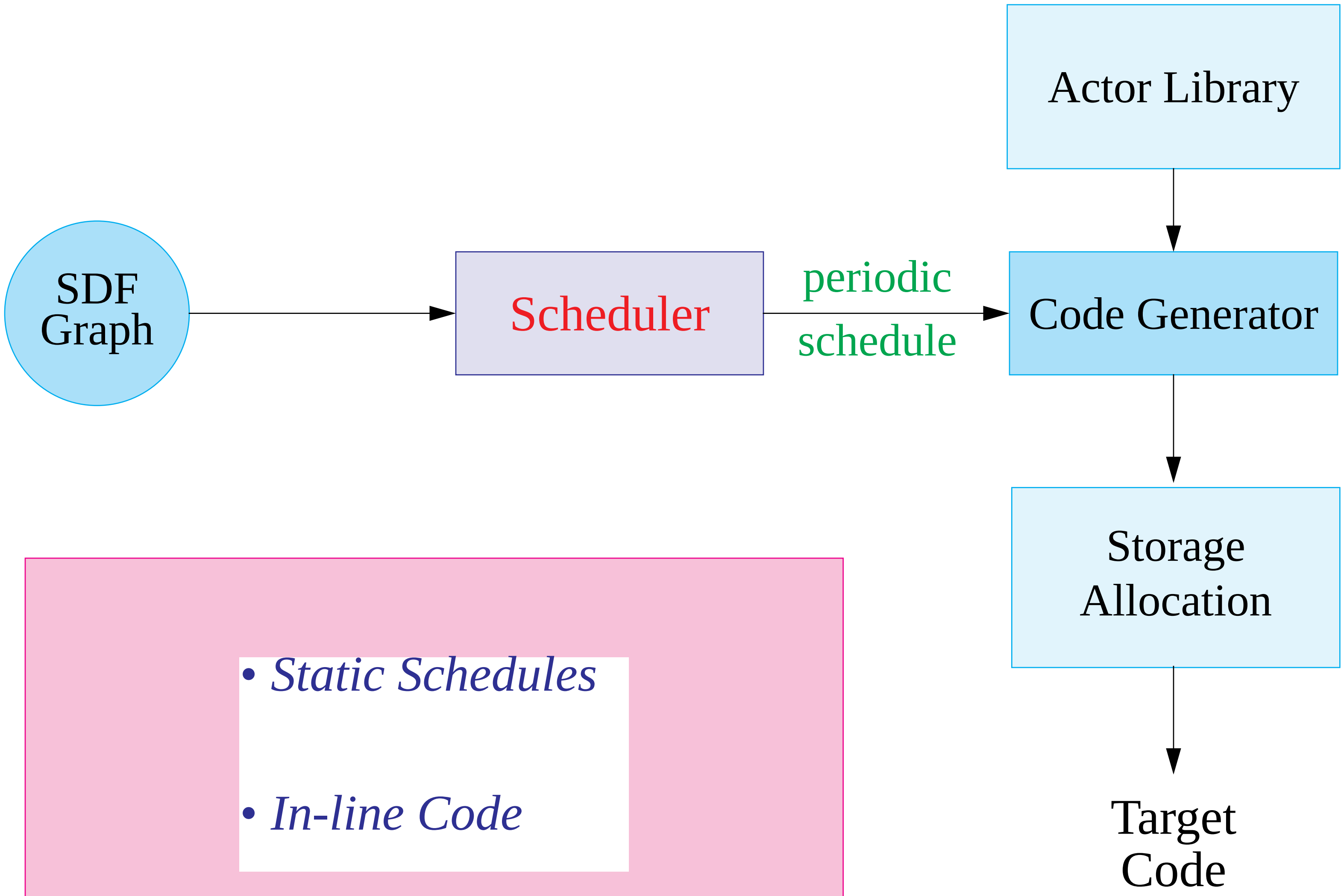
$$q_X = 1, \quad q_Y = 2, \quad q_Z = 2$$

YXZYZ, XYZYZ, X(2 YZ)

Periodic schedule example



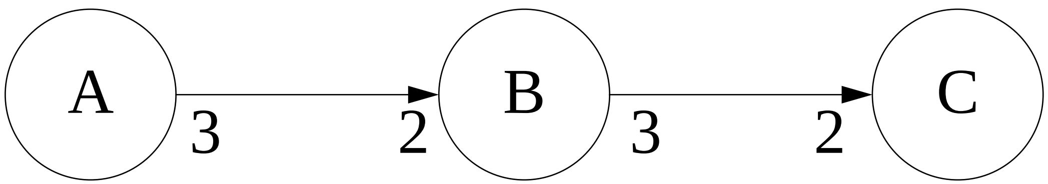
Code synthesis model



- *Static Schedules*
- *In-line Code*

Looped schedules

$$q_A = 4, \quad q_B = 6, \quad q_C = 9$$



Schedules

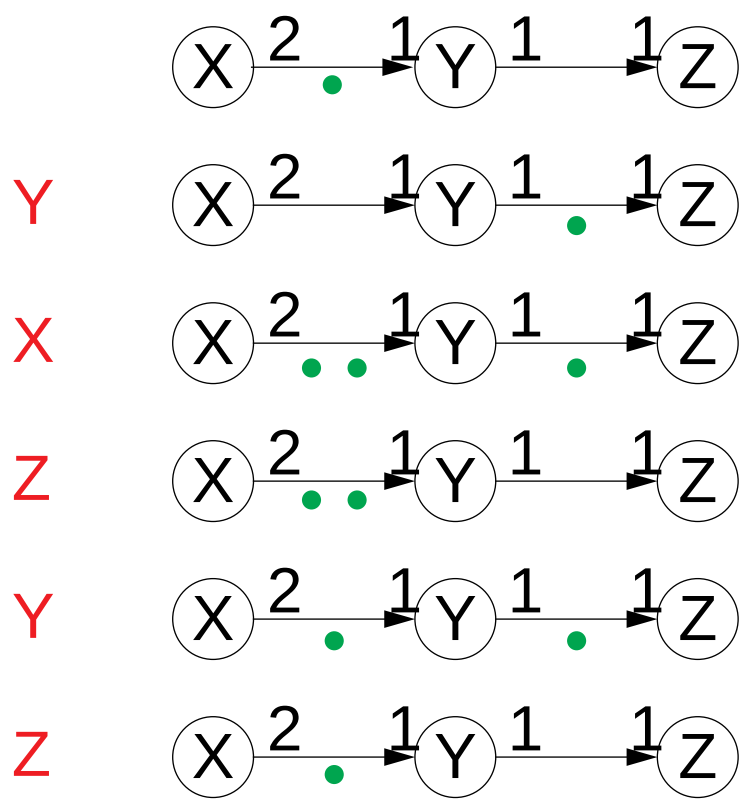
(2 ABC)CBCAB(2 C)A(2 BC)C

Single Appearance Schedules → **(4 A)(6 B)(9 C)**

→ **(4 A)(3 (2B)(3 C))**

Buffering model

- **Buffer on every arc in the graph.**
- **Size of a buffer is given by the maximum number of tokens queued on the arc in the schedule.**
- **Total buffering cost is sum of individual buffer sizes.**



buffering cost $= 2 + 1 = 3$

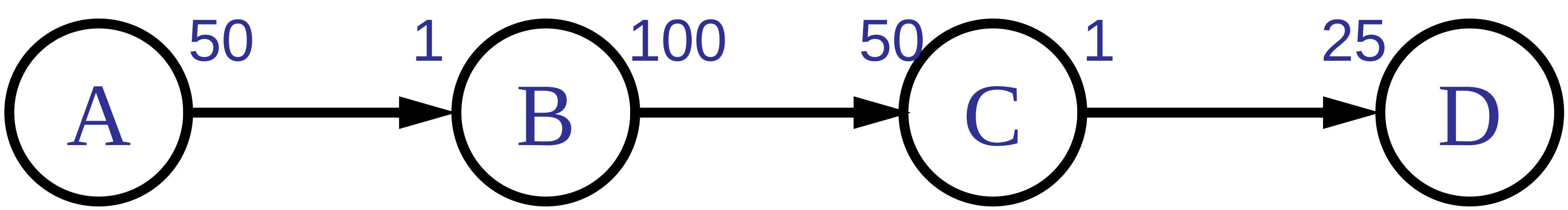
Advantages over alternative buffering models

Alternative #1: *Flat* single appearance schedules with shared buffers.

- Buffering requirement can be very bad for some graphs.
- Does not handle delays well.
- Latency is maximized.

Alternative #2: Use nested schedules with buffer sharing.

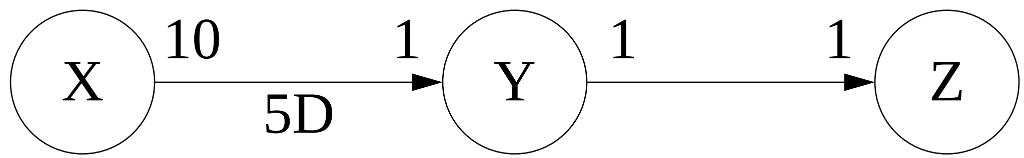
- More awkward to implement.
- Cost function is more complicated.



A (50 B) (100 C) (4 D): Cost = 5000

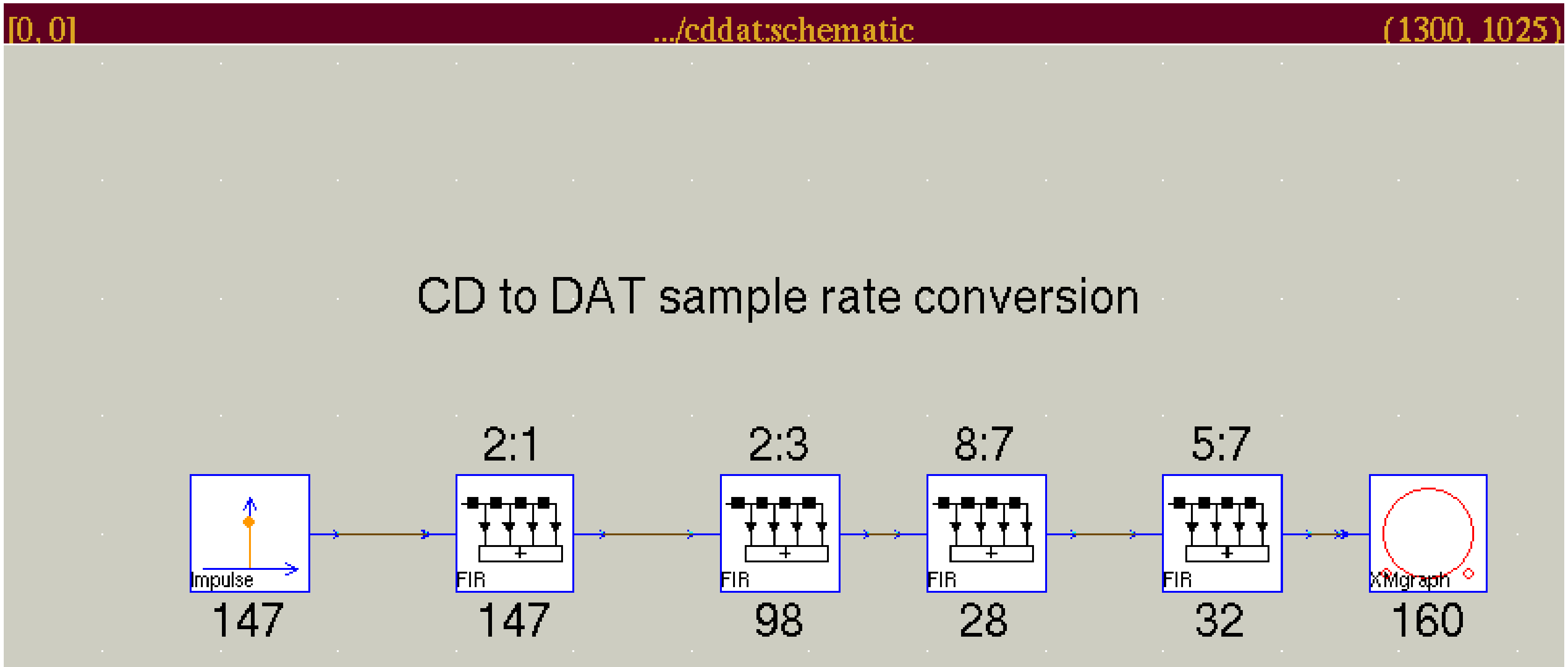
A (2 (25 (B (2 C))) (2 D)): Cost = 200

Code size vs. buffer memory trade-offs



single appearance schedules

(5 YZ) X (5 YZ)	X (10 Y) (10 Z)	X (10 Y Z)
buffering cost: $10 + 1 = 11$	buffering cost: $15 + 10 = 25$	buffering cost: $15 + 1 = 16$
code size: $S(X) + 2S(Y) + 2S(Z)$	code size: $S(X) + S(Y) + S(Z)$	code size: $S(X) + S(Y) + S(Z)$



	Code	Data
Minimum buffer schedule, no looping	**13735**	**32**
Minimum buffer schedule, with looping	**9400**	**32**
Worst minimum code size schedule	**170**	**1021**
Best minimum code size schedule	**170**	**264**

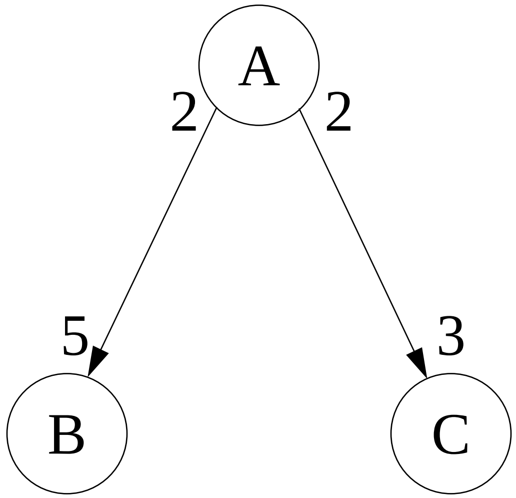
Minimum buffer single appearance schedules

Schedule	Buffering Cost
(9 A)(12 B)(12 C)(8 D)	36 + 12 + 24 = **72**
(3 (3 A)(4 B C))(8 D)	12 + 1 + 24 = **37**
(3 (3 A)(4 B))(4 (3 C)(2 D))	12 + 12 + 6 = **30**

- **Finding buffer-minimal single appearance schedules is NP-complete, even for acyclic, homogenous SDF graphs [Murthy, PhD thesis 1996].**
- **Thus, need to use heuristics.**

Minimum buffer single appearance schedules

single appearance schedule $\Leftrightarrow$ a parenthesization of a lexical ordering



q = (15, 6, 10)

- (3 (5 A) (2 B)) (10 C) buff. cost = 40
- (15 A) (2 (3 B) (5 C)) buff. cost = 60
- (15 A) (6 B) (10 C) buff. cost = 60
- (5 (3 A) (2 C)) (6 B) buff. cost = 36
- (15 A) (2 (5 C) (3 B)) buff. cost = 60

Dynamic programming post optimization (DPPO)

- **Given a lexical ordering, computes an optimal parenthesization.**
- **Time complexity $O(n^3)$.**

$$\ldots((x_i x_{i+1} \ldots x_k)(x_{k+1} \ldots x_{j-1} x_j))\ldots$$

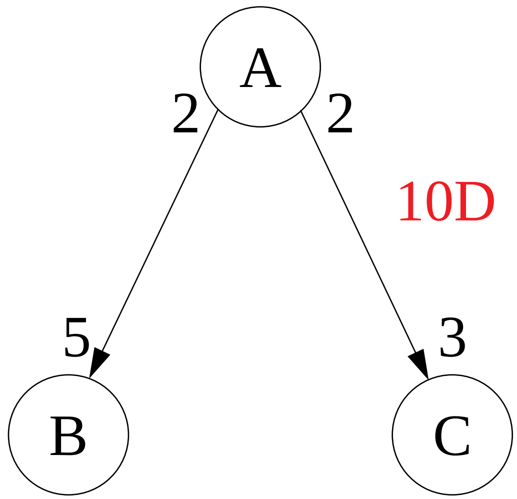
split cost

left subchain cost

right subchain cost

$$b[i, j] = MIN_{i \le k < j}\{b[i, k] + b[k+1, j] + c_{ij}[k]\}$$

Lexical orderings that are not topological sorts



- (5 (3 A) (2 C)) (6 B) buff. cost = 46
- (5 (2 C) (3A)) (6 B) buff. cost = 40

Two heuristics for constructing lexical orderings

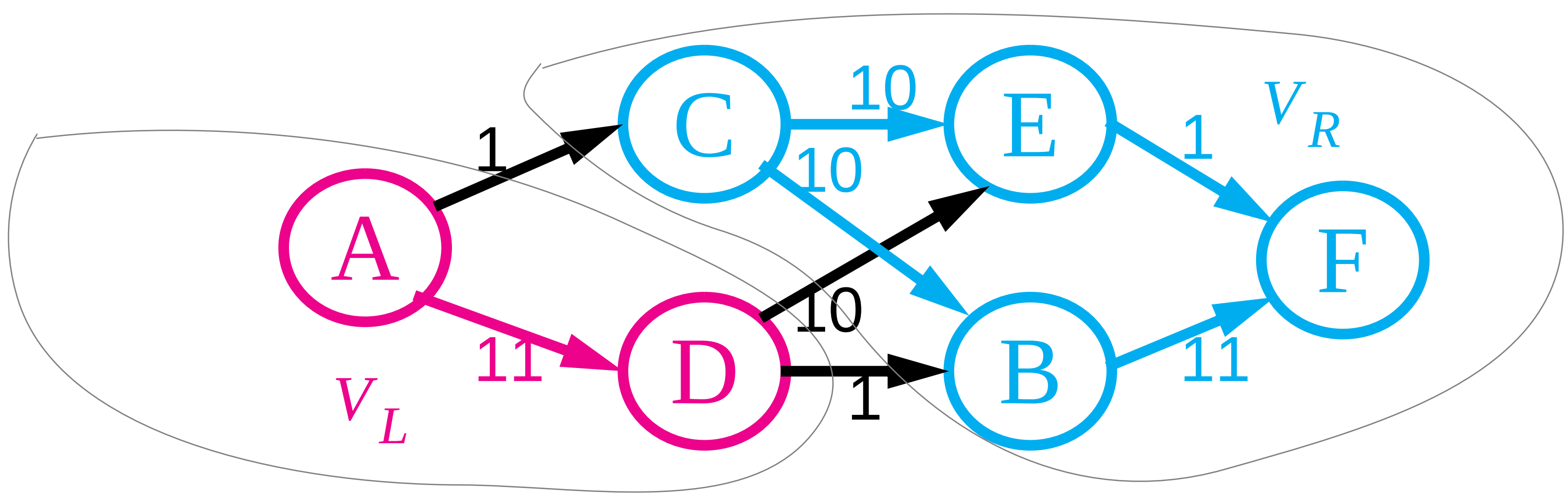
- **Pairwise grouping of adjacent nodes (PGAN)**
  - **Bottom-up algorithm**
  - **Effective for regular topologies**
  - **Optimal for a class of graphs**
- **Recursive partitioning by minimum cuts (RPMC)**
  - **Top-down algorithm**
  - **Effective for irregular topologies**

Complementary:
often, when one does poorly, the other does well

Recursive partitioning by minimum cuts

- **Idea: Find a *cut* of the graph such that**
  - **a) All arcs cross the cut in the forward direction.**
  - **b) The cut results in fairly even-sized sets.**
  - **c) Amount of data crossing the cut is minimized.**

Recursively schedule the nodes on the left side of the cut before nodes on the right side of the cut.



RPMC continued

- **Splitting the graph where the minimum amount of data is transferred is a *greedy* approach and is not optimal in general.**

- **Finding the minimum cut such that all of the conditions *a*, *b*, and *c* are satisfied is itself a difficult problem:**
  - **Methods based on max-flow-min-cut theorem do not work.**
  - **Graph partitioning when the size of the partition has to be bounded is NP-complete.**

- **Therefore, a heuristic solution is needed.**

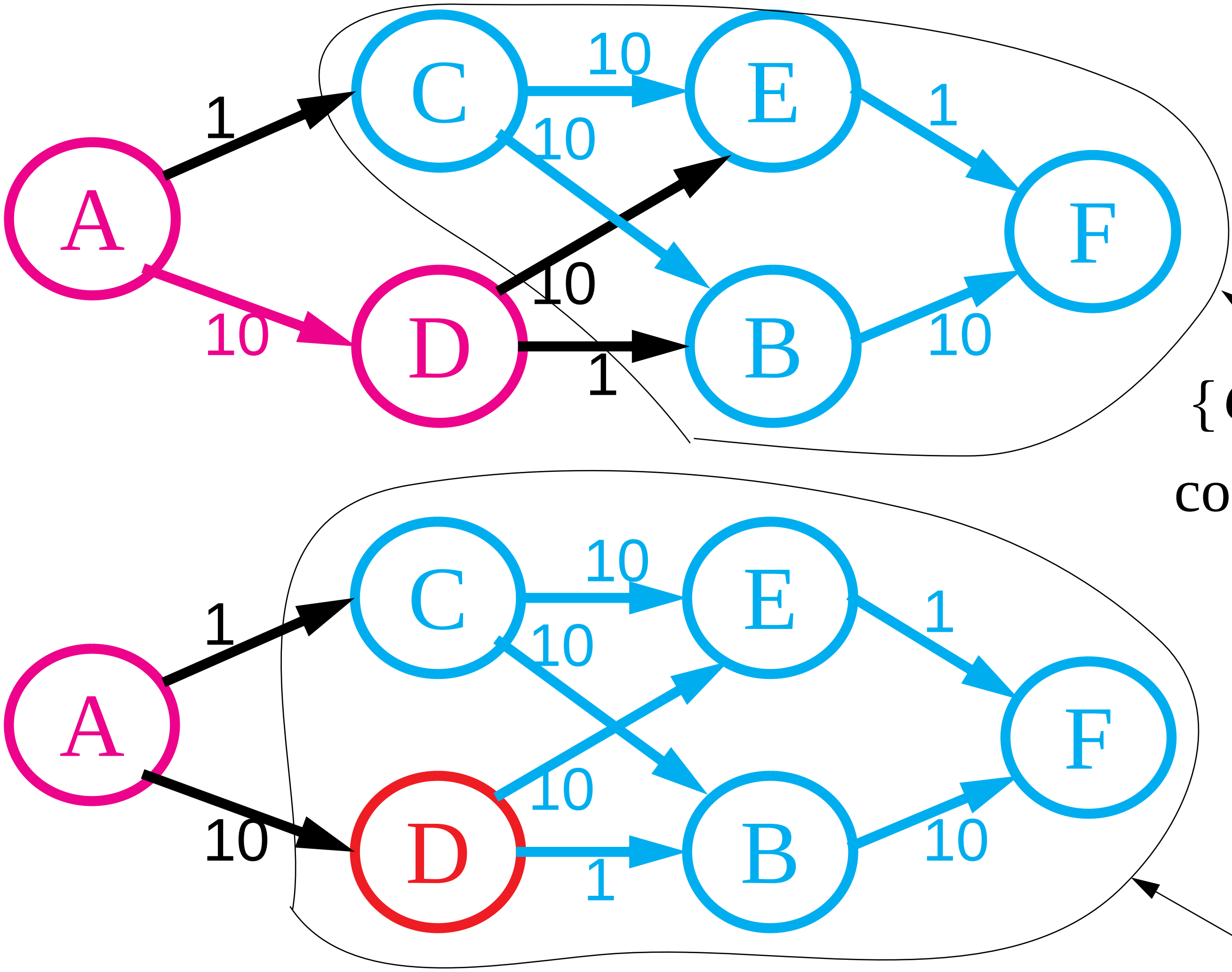
A heuristic for legal minimum cuts

- **Let $V_R(u)$ be the set of nodes consisting of u and its descendents. Let $V_L = V \setminus V_R(u)$.**
- **This forms a cut satisfying condition (*a*).**
- **Perform a local optimization by moving those nodes from V_L that reduce the cost into $V_R(u)$.**
- **Do this for all nodes u in the graph.**

- **Repeat above steps to generate cuts obtained by letting $V_L(u)$ be the set of nodes consisting of u and it ancestors, and letting $V_R = V \setminus V_L(u)$.**
- **Keep the cut with the lowest cost.**

- **Runs in time $O(|V||E| + |V|^2 \bullet \log(|V|))$.**

RPMC example



$\{C\} \cup desc(C)$,

$cost = 12$

$\{C\} \cup desc(C) \cup \{D\}$,

$cost = 11$

Acyclic pairwise grouping of adjacent nodes

Idea: Develop a loop hierarchy by clustering two adjacent nodes at each step.

Definition: *Clustering* means combining two or more nodes into one hierarchical node.

- The graph with the hierarchical node instead of the nodes that were clustered is called the *clustered graph*.

Definition: A *clusterizable* pair of nodes is a pair of nodes that, when clustered, does not cause *deadlock*.

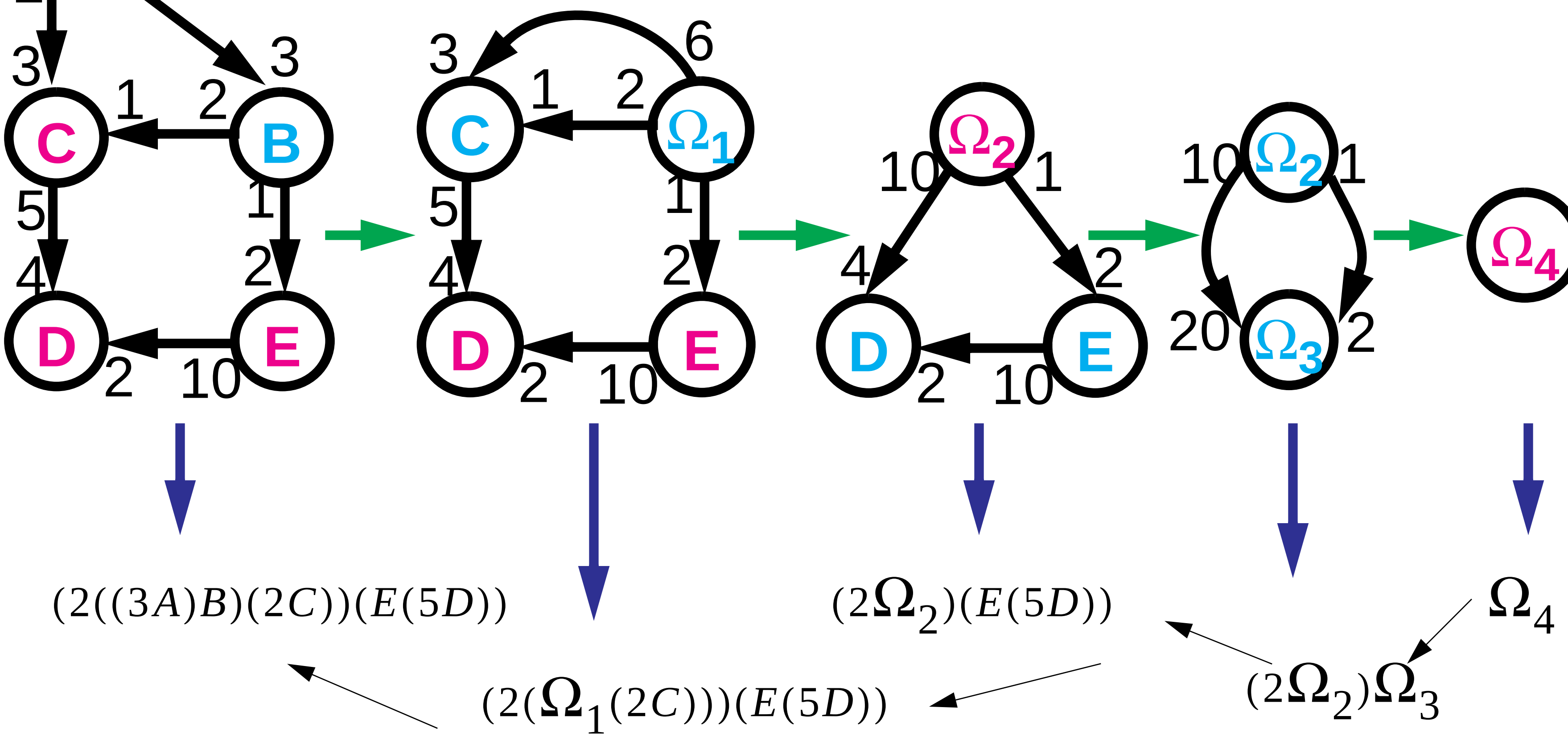
- A sufficient condition for clusterizability: Two nodes are clusterizable if clustering them does not introduce a cycle in the clustered graph.

APGAN algorithm

- **Cluster two nodes that maximize** $gcd\{r(A), r(B)\}$ **over all clusterizable pairs** $\{A, B\}$ **.**
- **Continue until only one node is left in the clustered graph**
- **This is similar to the Huffman coding algorithm.**
- **After constructing cluster hierarchy, retrace steps to determine the nested schedule.**
- **Post-process the schedule using dynamic programming to generate an optimal nesting for the lexical ordering generated by APGAN.**
- **Runs in time** $O(|V|^3)$ **for sparse graphs.**

APGAN example

Cyan nodes are clustered at each step.



Optimality of APGAN

Definition: The *buffer memory lower bound* for a (delayless) arc (e) is given by

$$BMLB(u, v) = \frac{cons(e)prod(e)}{gcd\{cons(e), prod(e)\}}$$

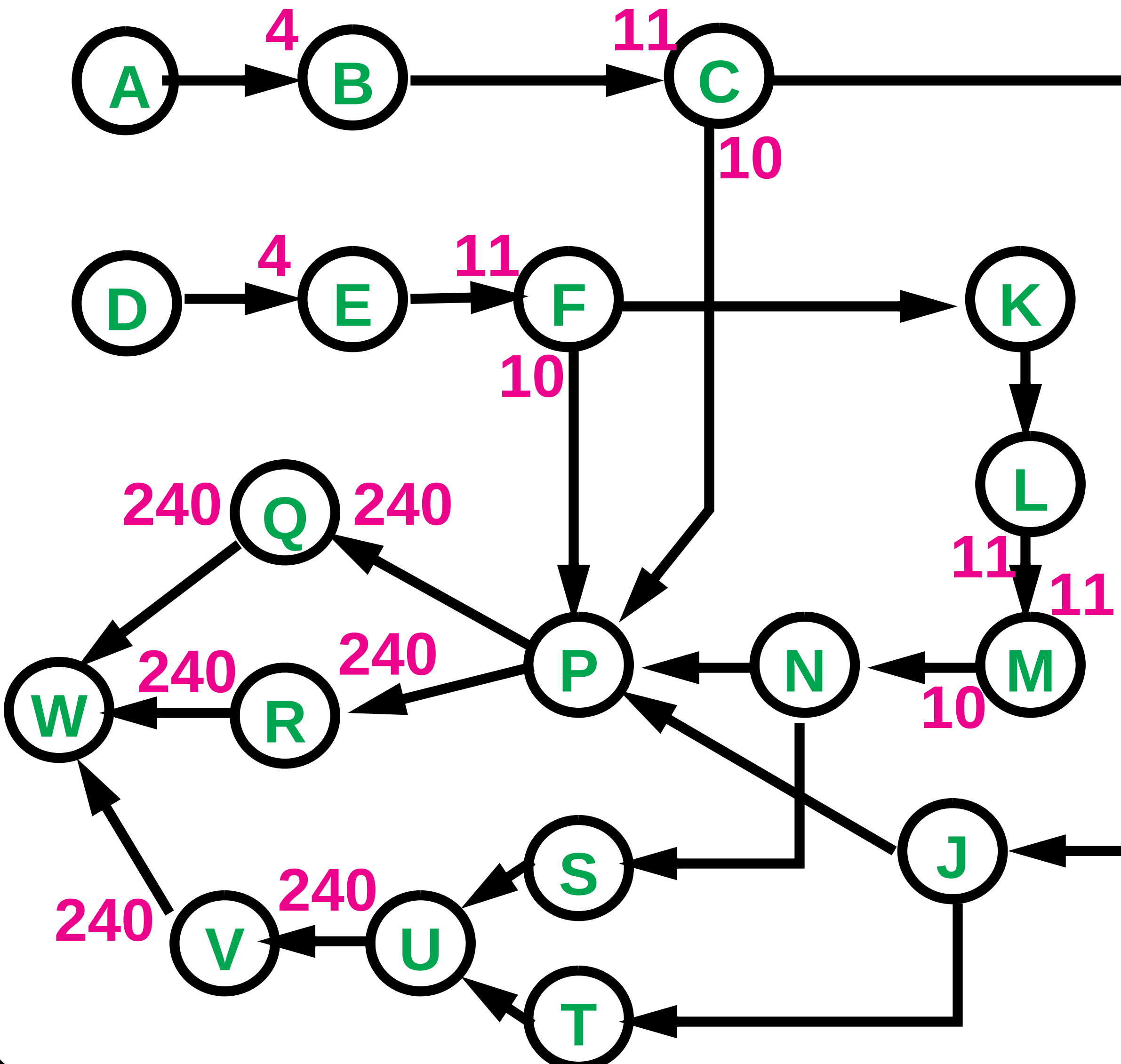
— This represents the least amount of buffering needed on this arc in any single appearance schedule.

Definition: A *BMLB schedule* for an acyclic SDF graph is a single appearance schedule whose buffering cost is equal to the sum of the BMLB costs for each arc.

Theorem: The APGAN algorithm will find a BMLB schedule whenever one exists if $delay(e) < \tau(e)$ for each edge e.

Example: mobile satellite receiver

This example is from [Ritz95]:



BMLB = 1540

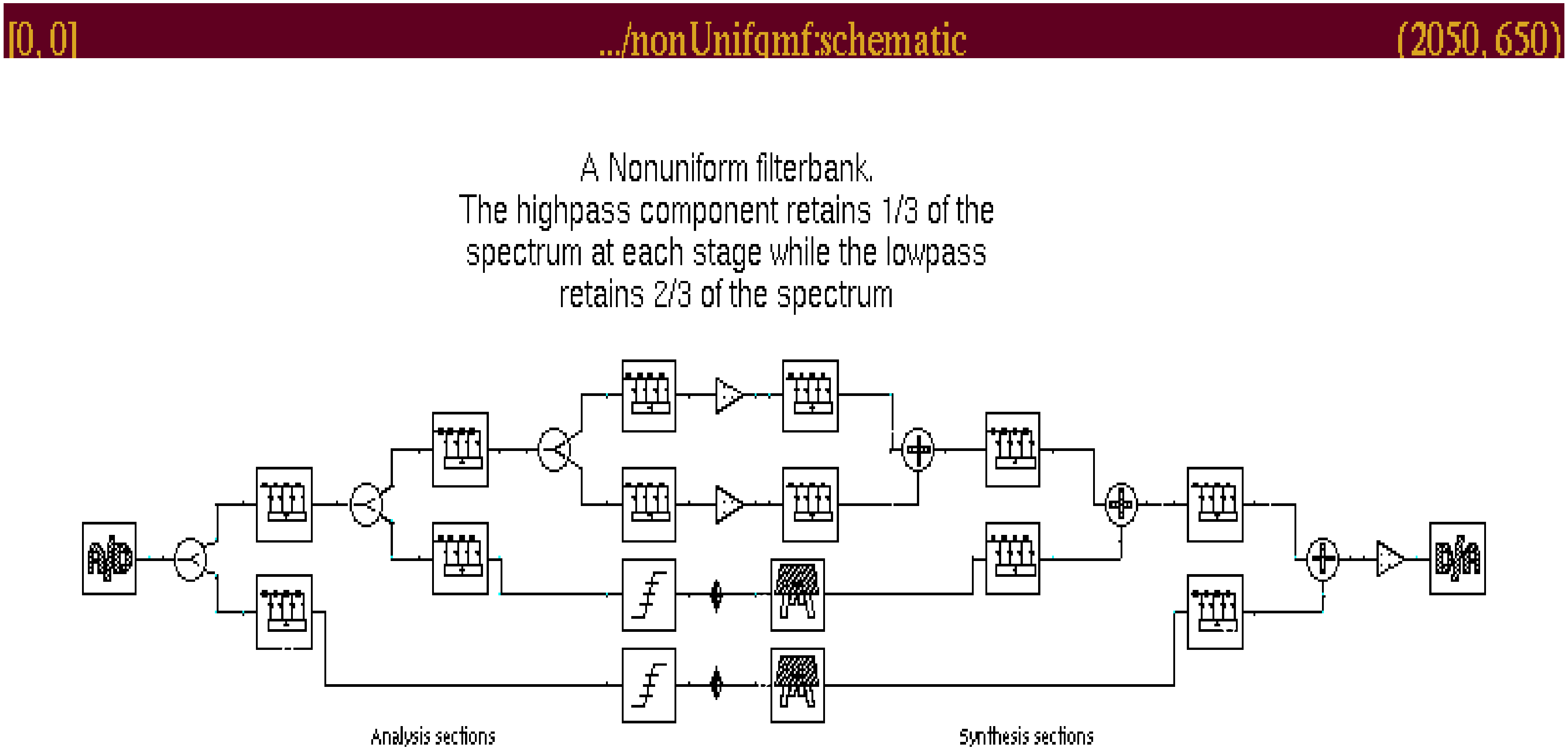
APGAN = 1540

RPMC = 2480

Ritz* = 2040

** Ritz generates a naive single appearance schedule and uses the shared buffer cost.*

Nonuniform filter bank



BMLB = 85

RPMC = 128

APGAN = 137

Performance on practical examples

Performance of the two heuristics on various acyclic graphs.

System	BMUB	BMLB	APGAN	RPMC	Average Random	Graph size(nodes/ arcs)
Fractional decimation	61	47	47	52	52	26/30
Laplacian pyramid	115	95	99	99	102	12/13
Nonuniform filterbank (1/3,2/3 splits, 4 channels)	466	85	137	128	172	27/29
Nonuniform filterbank (1/3,2/3 splits, 6 channels)	4853	224	756	589	1025	43/47
QMF nonuniform-tree filterbank	284	154	160	171	177	42/45
QMF filterbank (one-sided tree)	162	102	108	110	112	20/22
QMF analysis only	248	35	35	35	43	26/25
QMF Tree filterbank (4 channels)	84	46	46	55	53	32/34
QMF Tree filterbank (8 channels)	152	78	78	87	93	44/50
QMF Tree filterbank (16 channels)	400	166	166	200	227	92/106

Performance on random graphs

RPMC < APGAN	63%
APGAN < RPMC	37%
RPMC < min(2 random)	83%
APGAN < min(2 random)	68%
RPMC < min(4 random)	75%
APGAN < min(4 random)	61%
min(RPMC,APGAN) < min(4 random)	87%
RPMC < APGAN by more than 10%	45%
RPMC < APGAN by more than 20%	35%
APGAN < RPMC by more than 10%	23%
APGAN < RPMC by more than 20%	14%

Conclusion

- **Objective: minimizing buffer cost for a minimum code size schedule**
- **Problem is NP-complete, even for acyclic, HSDF graphs.**
- **DPPO: generates an optimum parenthesization for a given lexical ordering. For well-ordered graphs, where there is only one topological sort, DPPO thus generates buffer-optimal single appearance schedules.**
- **Two heuristics are used to generate lexical orderings for arbitrary acyclic SDF graphs:**
  - **RPMC: Does well on some practical examples with irregular topologies and on random graphs**
  - **APGAN: Does well on a lot of practical examples but not as well on random graphs. It is optimal for a class of graphs.**