## Existence of Single Appearance Schedules

- Every acyclic SDF graph has a single appearance schedule.
- An arbitrary SDF graph has a single appearance schedule iff each strongly connected component has a single appearance schedule.
- A strongly connected SDF graph has a single appearance schedule only if the nodes can be partitioned into $\mathrm{Z}_{1}$ and $\mathrm{Z}_{2}$ such that for any arc $\alpha$ directed from $\mathrm{Z}_{1}$ to $\mathrm{Z}_{2}, \operatorname{delay}(\alpha) \mathbf{q}_{\operatorname{sink}(\alpha)} \times \operatorname{consumed}(\alpha)$.
- "Subindependent Partition"


## Finding a Subindependent Partition

Remove each arc $\alpha$ for which
$\operatorname{delay}(\alpha) \geq \mathbf{q}_{\operatorname{sink}(\alpha)}$


Any "source" strongly connected component is subindependent of the rest of the graph.

$\{\mathrm{C}\}$ is subindependent of $\{\mathrm{A}, \mathrm{B}\}$

## Components of the Scheduling Framework

- Subindependent Partitioning Algorithm.
- Acyclic Scheduling Algorithm.
- Tight Scheduling Algorithm.
- Consolidate each strongly connected component $\left\{\mathrm{C}_{\mathrm{i}}\right\} \rightarrow$ acyclic graph $\mathrm{G}^{\prime}$.
- Apply acyclic scheduling algorithm to $\mathrm{G}^{\prime} \rightarrow$ single appearance schedule of strongly connected components $\mathrm{S}^{\prime}$.
- For each strongly connected component $\mathrm{C}_{\mathrm{i}}$
- Apply the subindependence partitioning algorithm
- If a subindependent partition $\left(\mathrm{P}_{1}, \mathrm{P}_{2}\right)$ is found
—Determine looping factors: $\mathrm{r}_{\mathrm{i}}=\operatorname{gcd}\left(\left\{\mathbf{q}_{\mathrm{N}} \mid \mathrm{N} \in \mathrm{P}_{\mathrm{i}}\right\}\right), \quad$ for $i=1,2$
- Apply schedule-loops() to $P_{1}$ and $P_{2} \rightarrow$ subschedules $S_{1}$ and $S_{2}$
— Replace appearance of $\mathrm{C}_{\mathrm{i}}$ in $\mathrm{S}^{\prime}$ with $\left(\mathrm{r}_{1} \mathrm{~S}_{1}\right)\left(\mathrm{r}_{2} \mathrm{~S}_{2}\right)$
- If no subindependent partition is found
- Apply the tight scheduling algorithm $\rightarrow$ subschedule $\mathrm{S}_{\mathrm{T}}$
- Replace appearance of $\mathrm{C}_{\mathrm{i}}$ in $\mathrm{S}^{\prime}$ with $\mathrm{S}_{\mathrm{T}}$
- We say that a strongly connected SDF graph is tightly interdependent if it does not have a subindependent partition.
- If $Z_{1}$ and $Z_{2}$ are subsets of nodes in an SDF graph such that $Z_{1} \cap Z_{2} \neq \varnothing$ and $\operatorname{subgraph}\left(\mathrm{Z}_{1}\right) \& \operatorname{subgraph}\left(\mathrm{Z}_{2}\right)$ are both tightly interdependent, then $\operatorname{subgraph}\left(\mathrm{Z}_{1} \cup \mathrm{Z}_{2}\right)$ is tightly interdependent.
- Tightly interdependent components.



## Properties of the Scheduling Framework

- Constructs a single appearance schedules whenever one exists.
- Actors outside the tightly interdependent components are scheduled with only one appearance.
- For actors inside tightly interdependent components, the number of appearances is determined entirely by the tight scheduling algorithm.

