# *Multidimensional Synchronous Dataflow*

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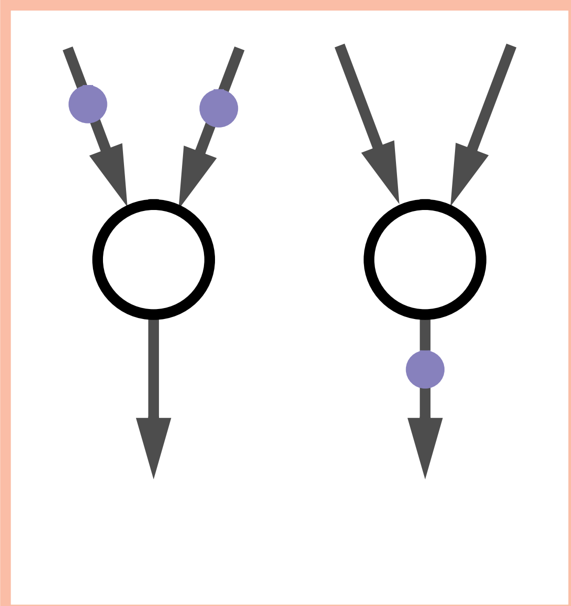
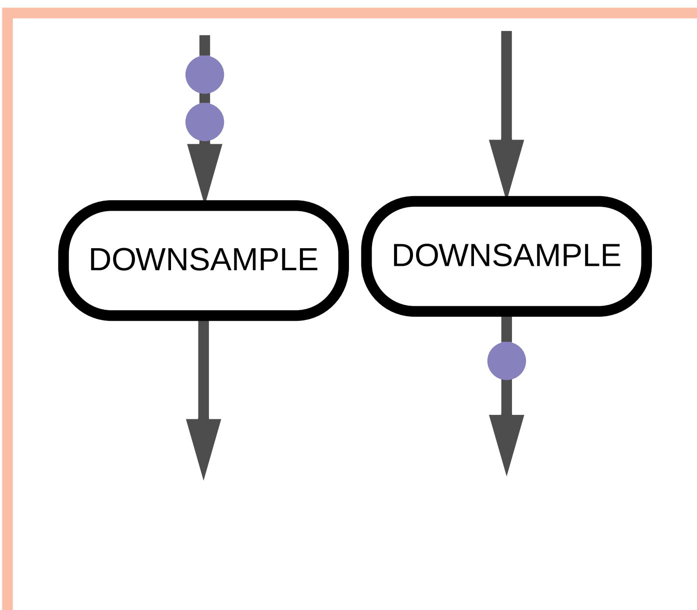
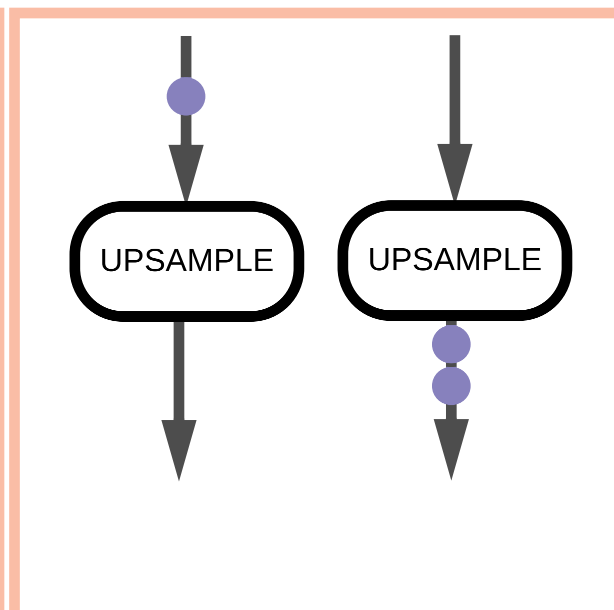
# *Synchronous Dataflow*



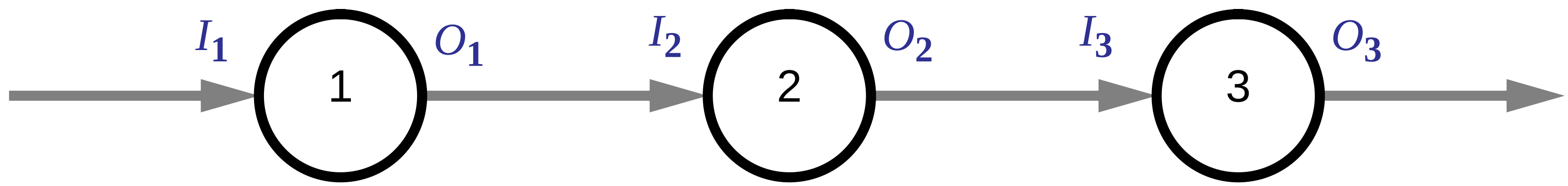
## Properties

- Flow of control is predictable at compile time
- Schedule can be constructed once and repeatedly executed
- Suitable for synchronous multirate signal processing



# *Consistency*




**Balance equations:**

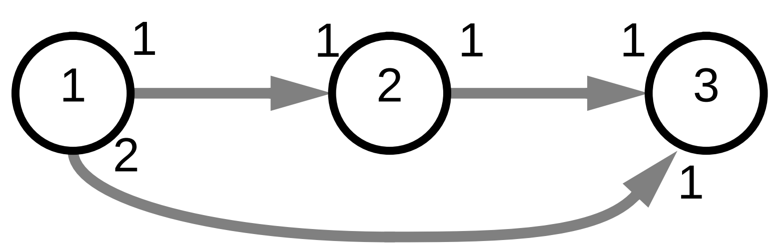
$$r_1 O_1 = r_2 I_2$$

$$r_2 O_2 = r_3 I_3$$

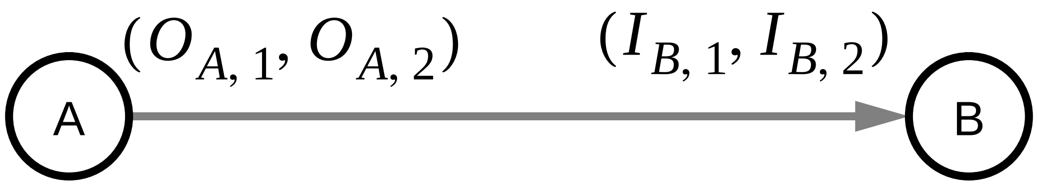
**Solve for the smallest integers $r_i$.**

**Then schedule according to data dependencies until repetitions $r_i$ have been met for all actors.**

**The balance equations have no solution if the graph is *inconsistent*. For example:**



# *Multidimensional Dataflow Extension*




**Balance equations:**

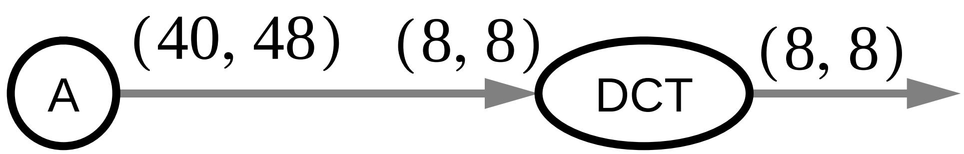
$$r_{A,1}O_{A,1} = r_{B,1}I_{B,1}$$

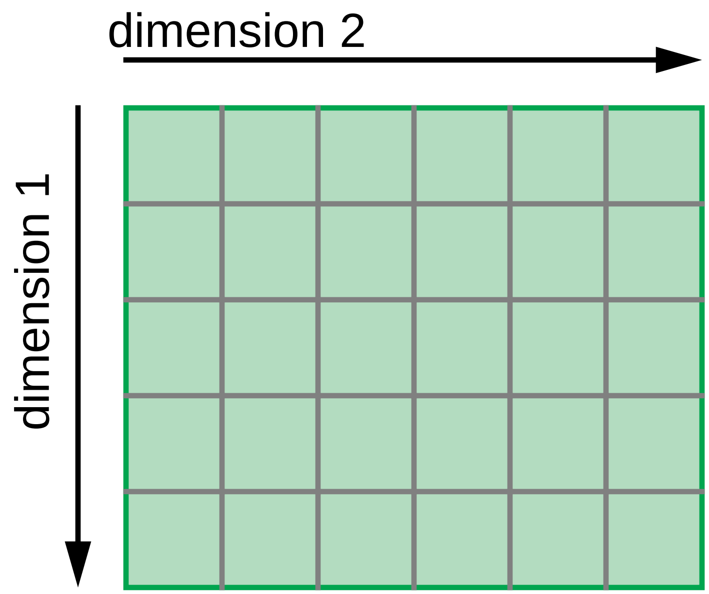
$$r_{A,2}O_{A,2} = r_{B,2}I_{B,2}$$

**Solve for the smallest integers $r_{X,i}$, which then give the number of repetitions of actor $X$ in dimension $i$.**

**Higher dimensionality follows similarly.**

# *Example of Multidimensional Dataflow*



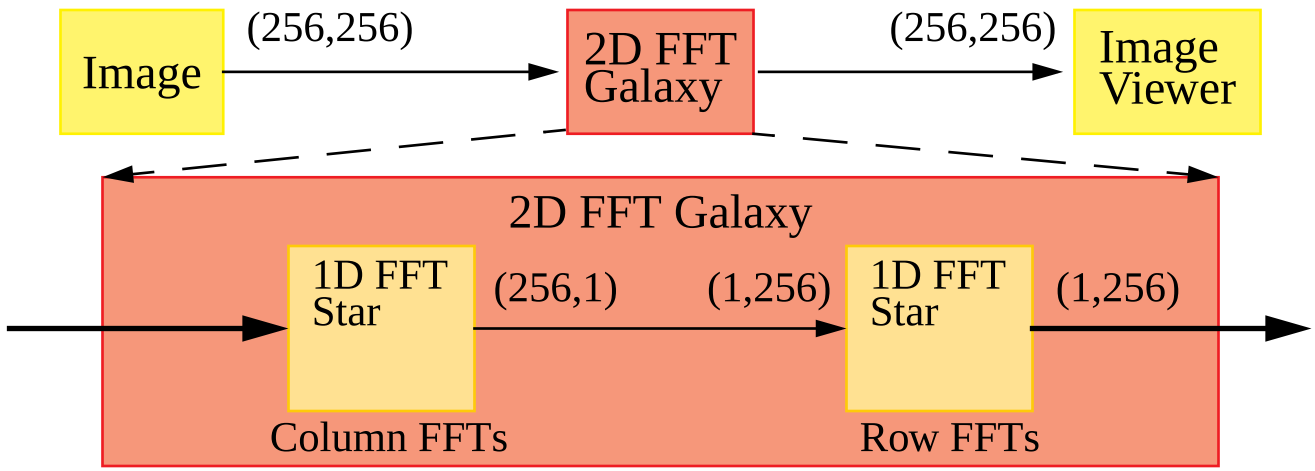

$$r_{A,1} = r_{A,2} = 1$$

$$r_{DCT,1} = 5, \quad r_{DCT,2} = 6$$

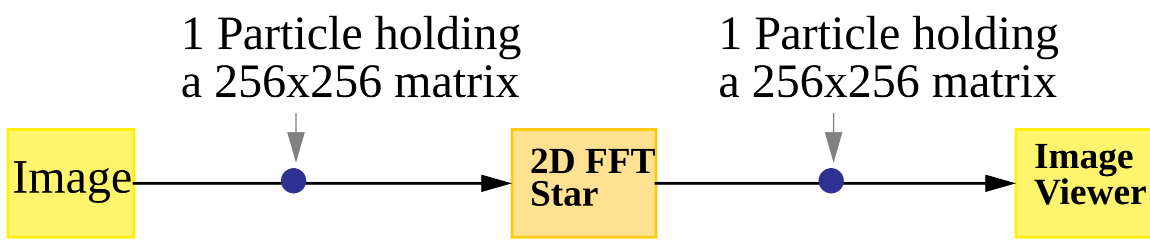
# *Awkwardness of Using SDF for MD Systems*

2D FFT by row-column decomposition

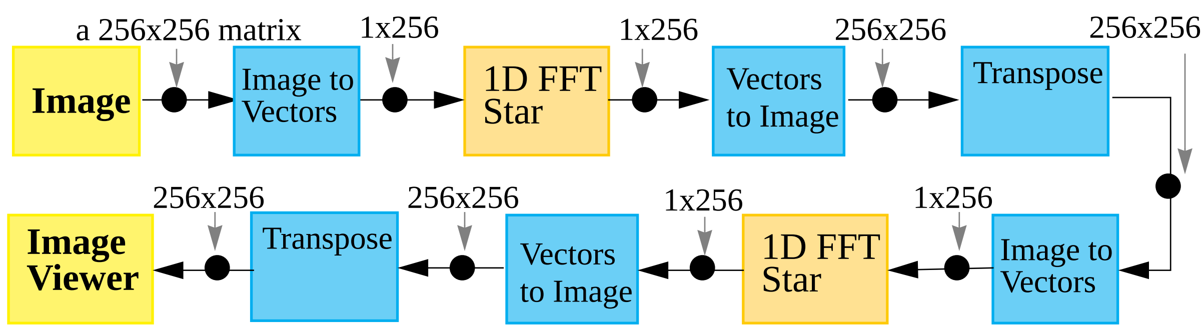
2-D FFT in MDSDF




No data parallelism

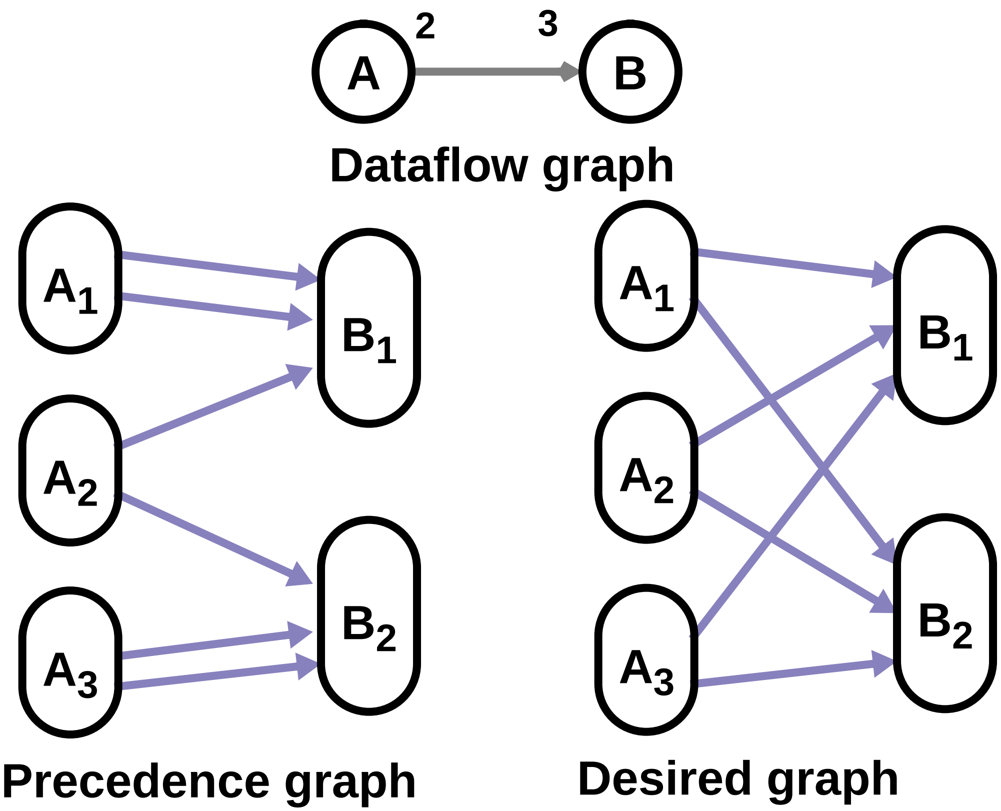



Too many extraneous actors



# *One limitation of 1-D SDF*

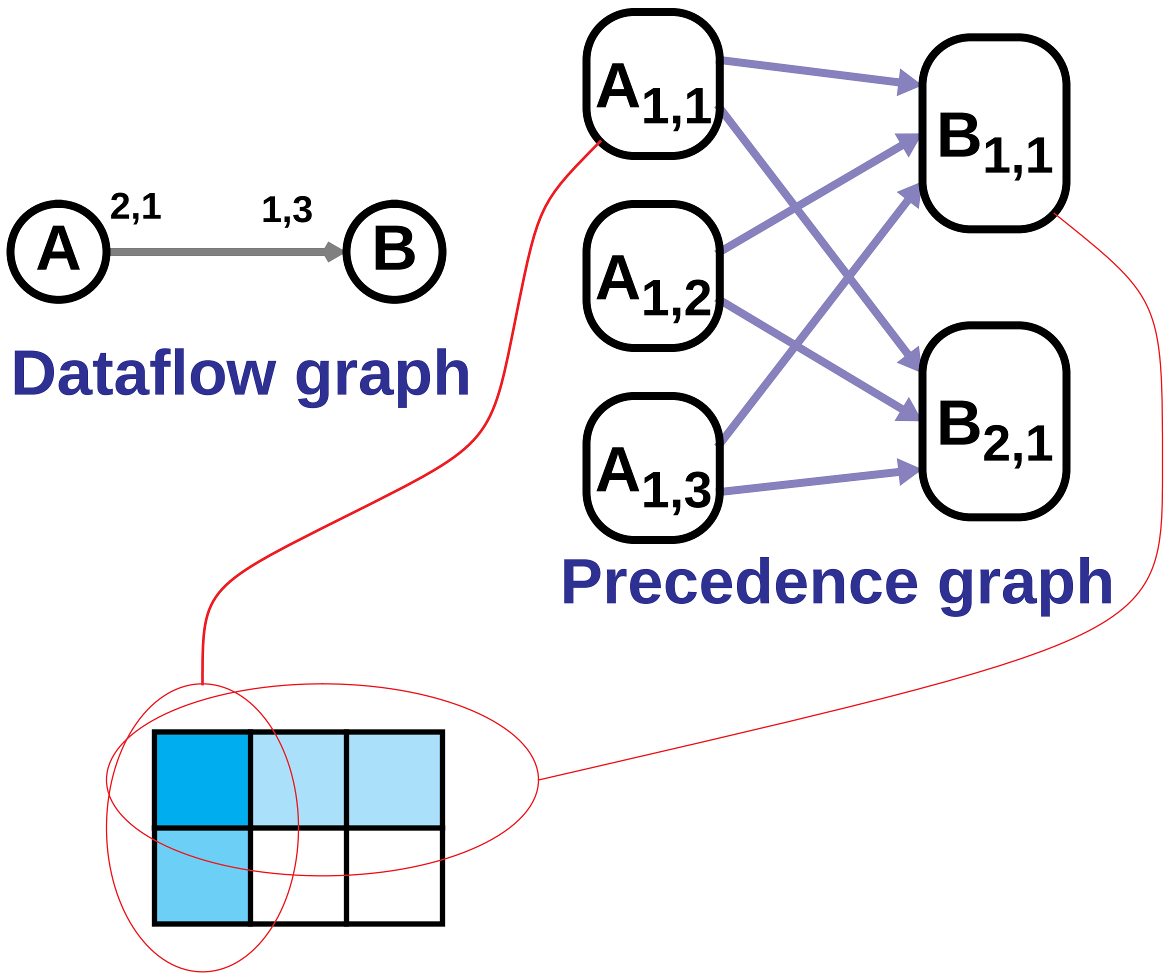
**Suppose we want data exchanged in the following order:**



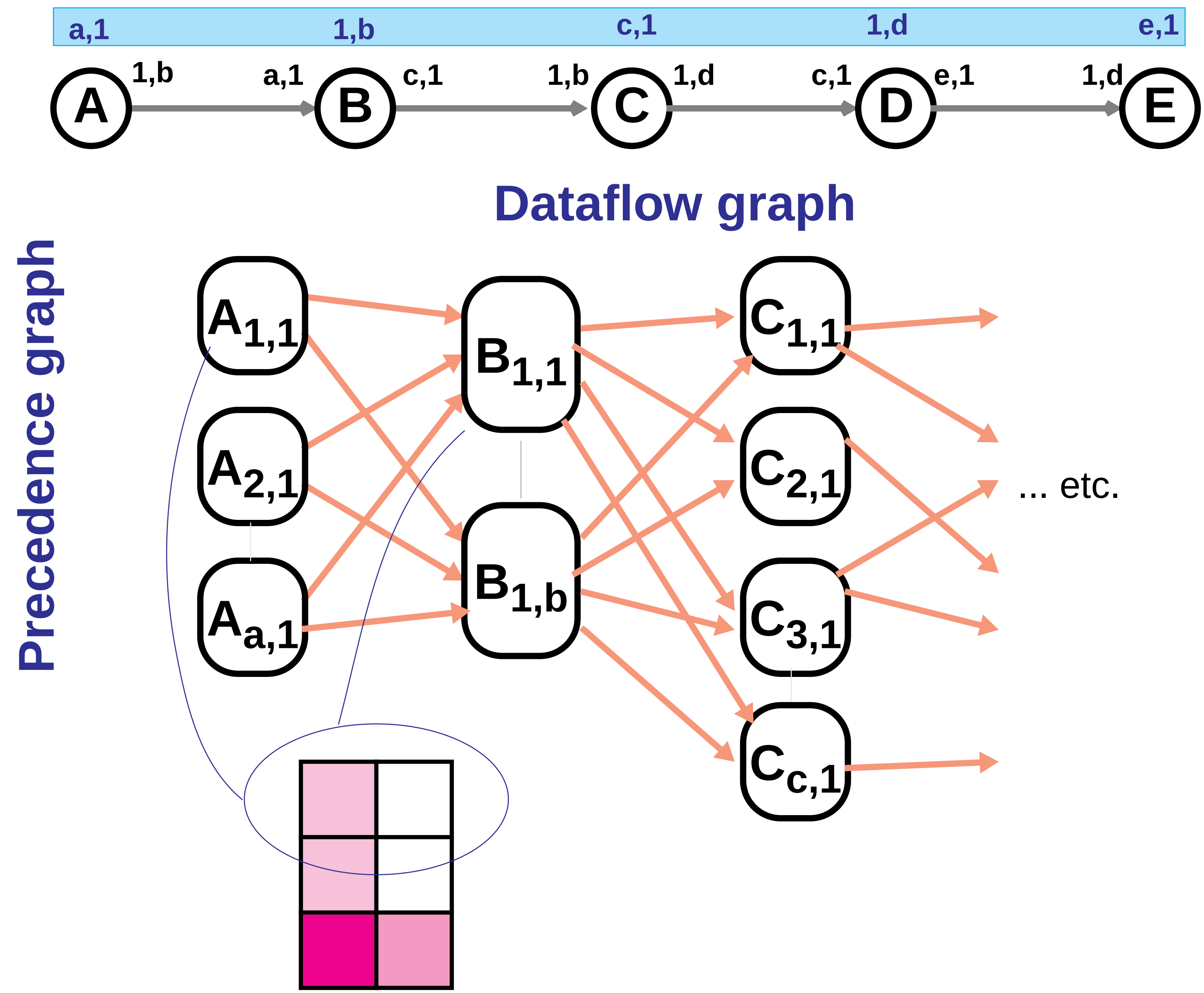

**1-D SDF has no compact, scalable representation of this.**

**Multidimensional dataflow solves this problem.**

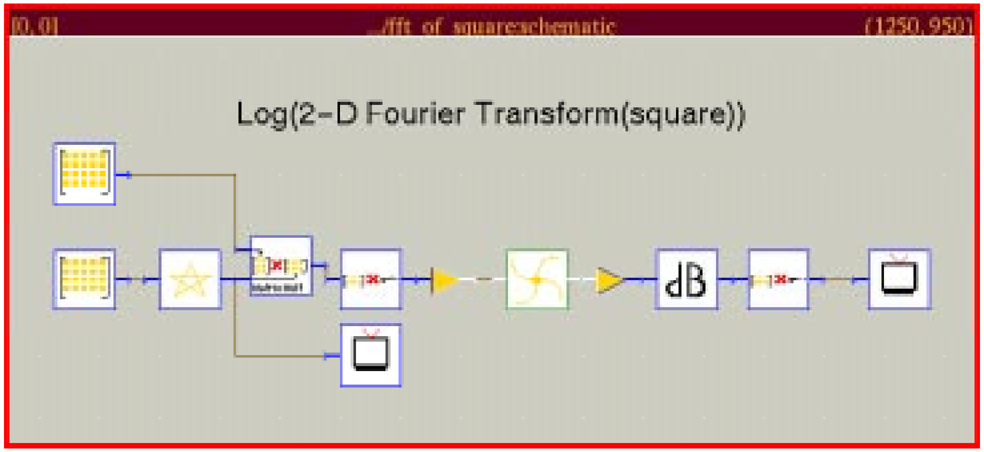
# More Flexible Data Exchange in MDSDF

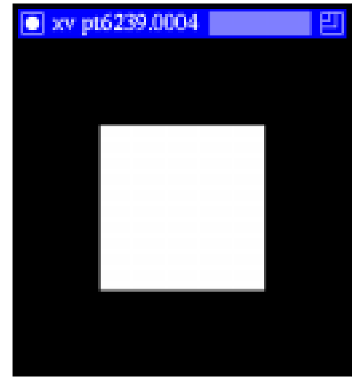


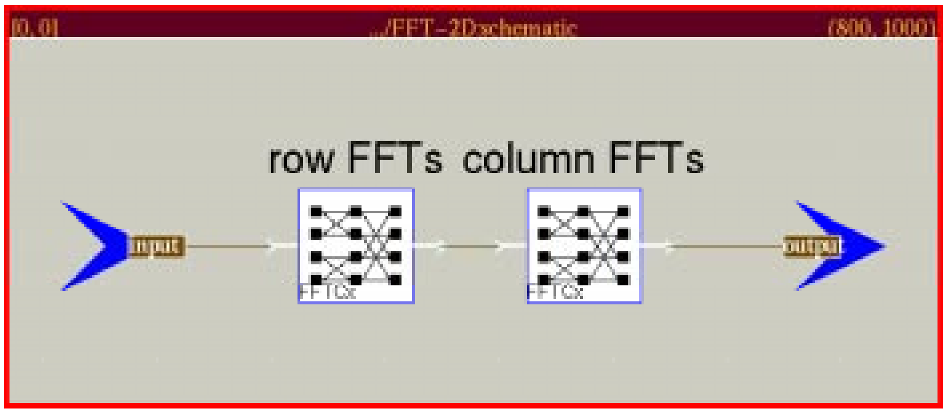
# *Example: Multilayer Perceptron*

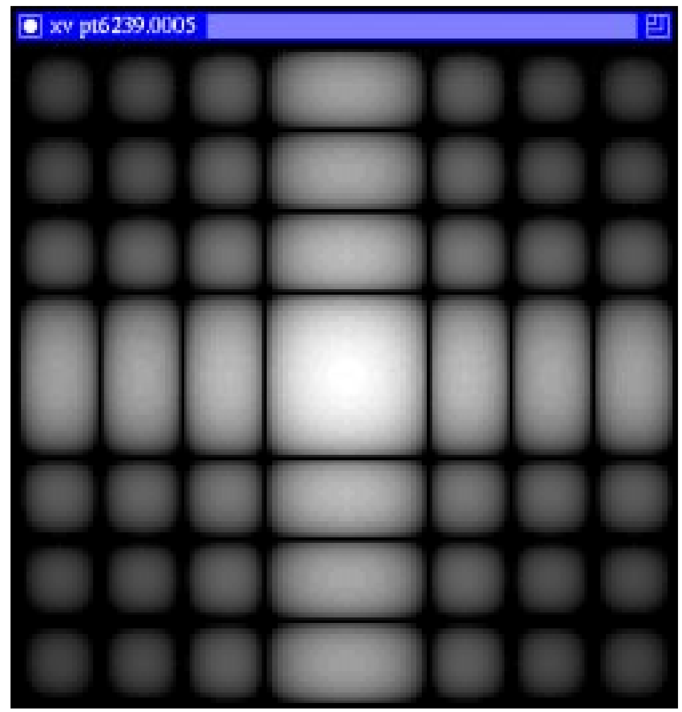


# *MDSDF Example in Ptolemy*



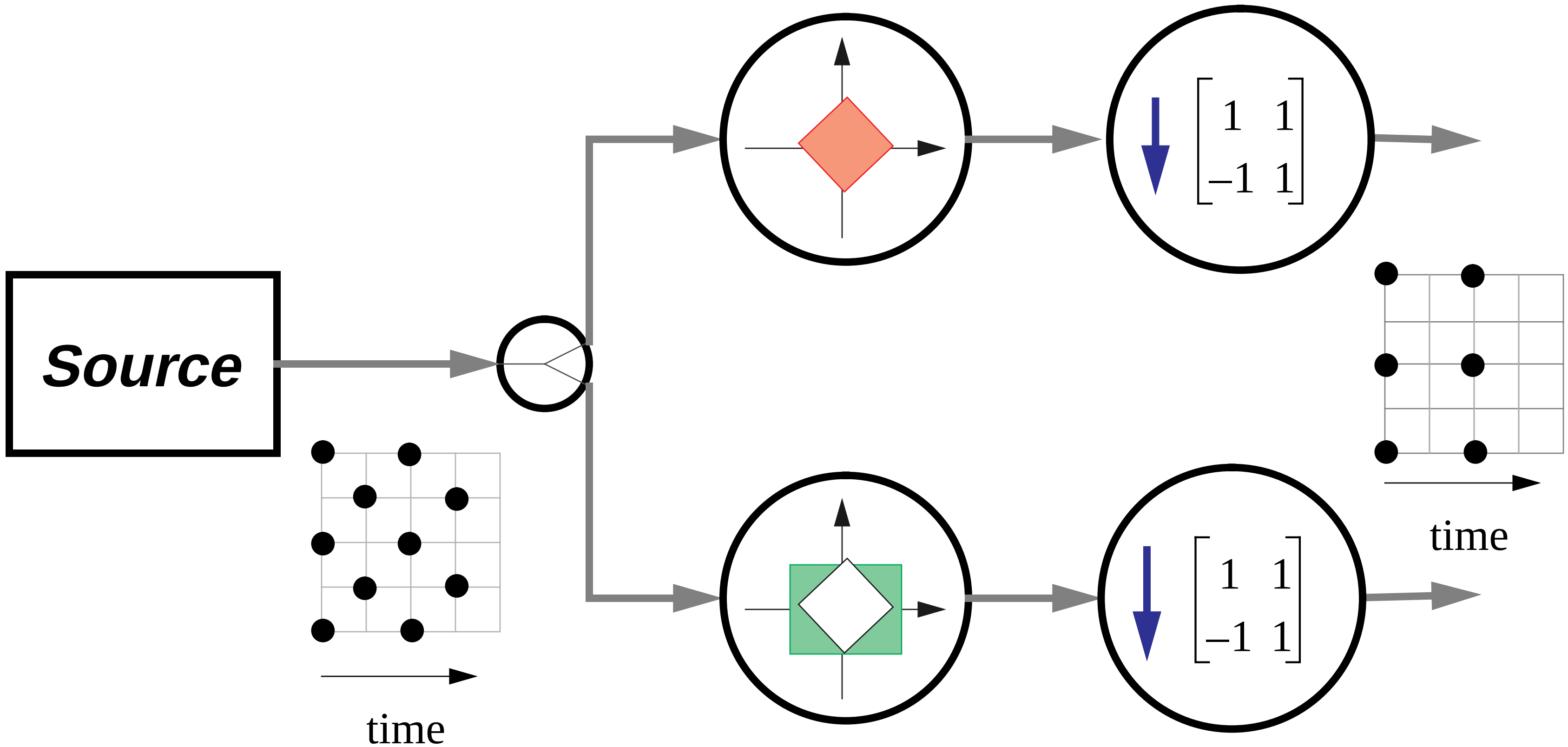




**Generalize streams to multidimensional partial orderings for representing multidimensional operations.**

DOMAIN DEVELOPED BY MICHAEL CHEN

## *Generalization to Arbitrary Lattices*

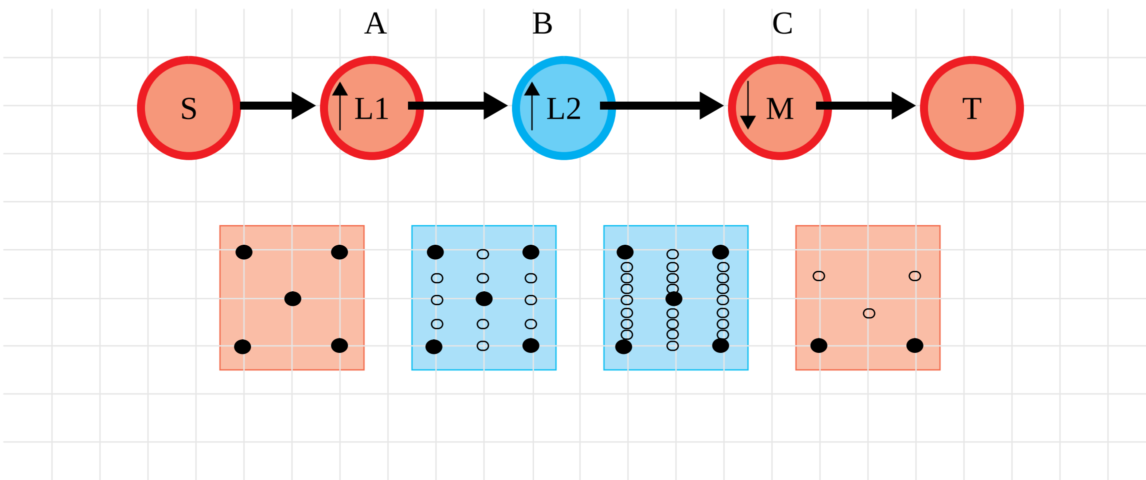
- **MDSDF handles only rectanglularly sampled signals.**
- **GMDSDF handles signals on arbitrary lattices,** ***without sacrificing compile-time schedulability.***



# *Uses of Non-rectangular Systems*

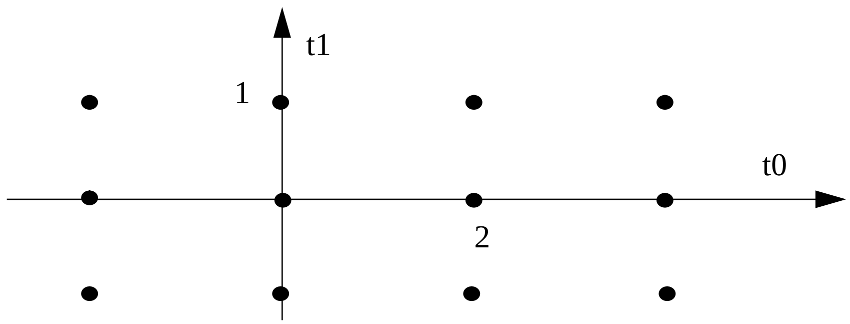
**Non-rectangular systems are used in a variety of contexts:**

- **2:1 interlaced TV (NTSC) [Dub85][ManCorMia93].**
- **Directional decompositional filterbanks [Bam90].**
- **Digital TV with FCO and quincunx sampling [KovVet93].**
- **Filterbanks for interlaced to progressive conversion [VetKovLeG90].**
- **Array signal processing with hexagonal geometries [DudMer84].**
- **Filter design techniques for non-rectangular lattices [AnsLee91][EvaMcc94].**



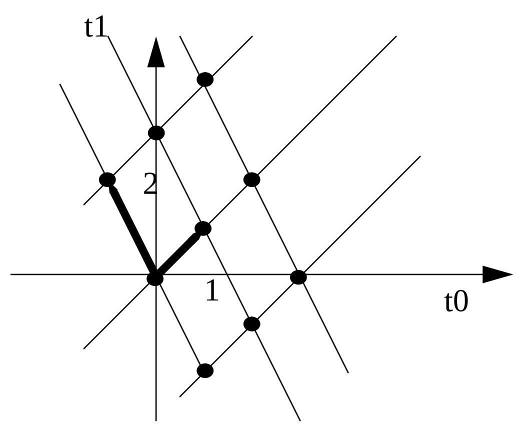
# *Non-rectangular Sampling*

Rectangular sampling




$$V = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

Non-rectangular sampling

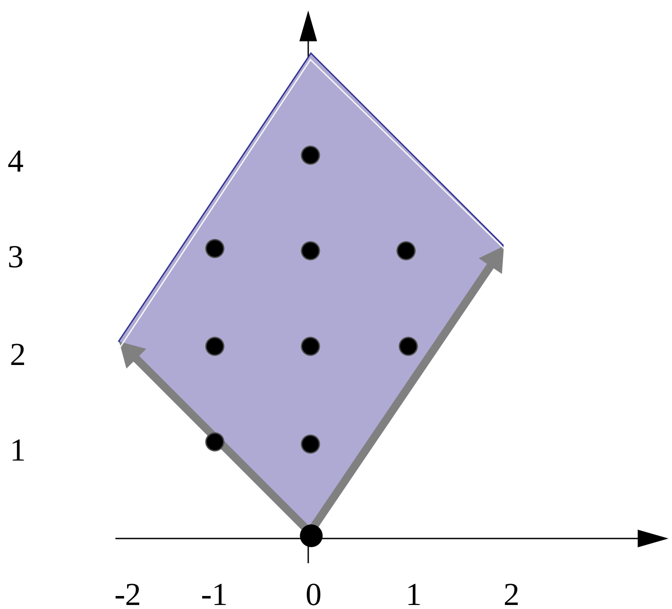



$$V = \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$$

*Definition:* The set of all sample points given by $\hat{t} = V\hat{n},\ \hat{n} \in \aleph$ is called the ***lattice*** generated by $V$. It is denoted $LAT(V)$.

# The Fundamental Parallelpiped

The ***fundamental parallelpiped***, denoted by $FPD(V)$, is the set of points given by $Vx$ where $x = [x_1, x_2]^T$ with $0 \le x_1, x_2 < 1$.




$$L = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix},$$

*Definition:* The set of integer points in $FPD(V)$ is denoted as $N(V)$.

*Lemma:* $J(V) = |N(V)| = |det(V)|$ for an integer matrix $V$.

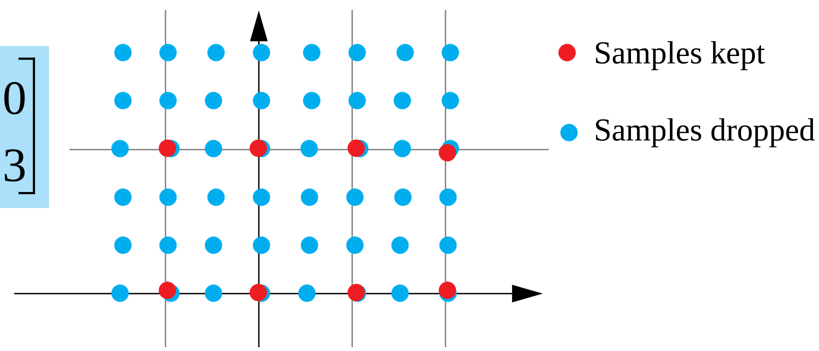
# Multidimensional Decimators

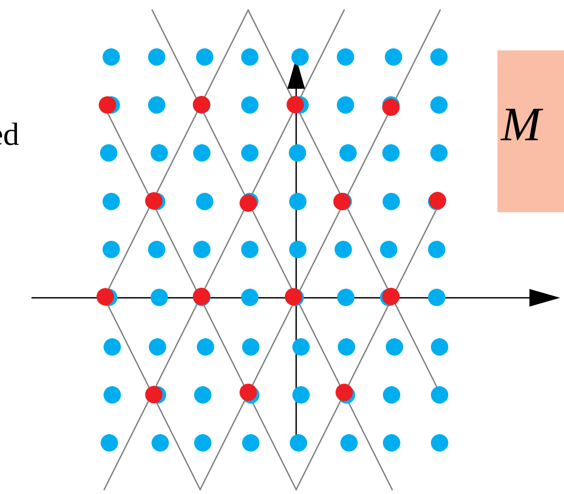
M-D decimation is given by the relationship:

$$y(\hat{n}) = x(\hat{n}), \hat{n} \in LAT(V_I M)$$

where $x$ is defined on the points $V_I k$, $V_I$ being the sampling matrix.

$M = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

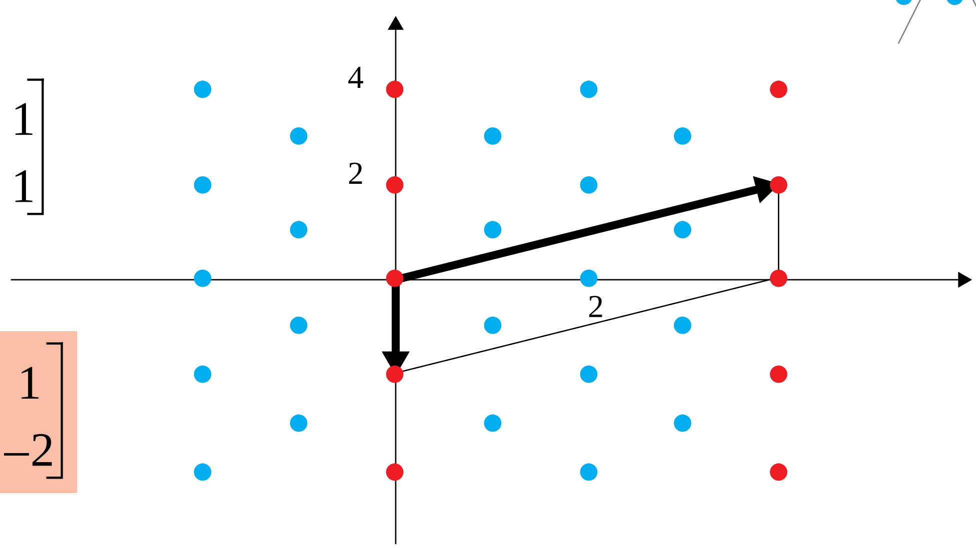


$M = \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}$

$V_I = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$

$M = \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}$




*Decimation ratio*:

$$|det(M)|$$

# Multidimensional Expanders

M-D expander:

$$y(n) = \begin{pmatrix} x(n) & n \in LAT(V_I) \\ 0 & \text{otherwise} \end{pmatrix} \forall n \in LAT(V_I L^{-1})$$

where x is defined at the points $V_I k$, $V_I$ being the sampling matrix.

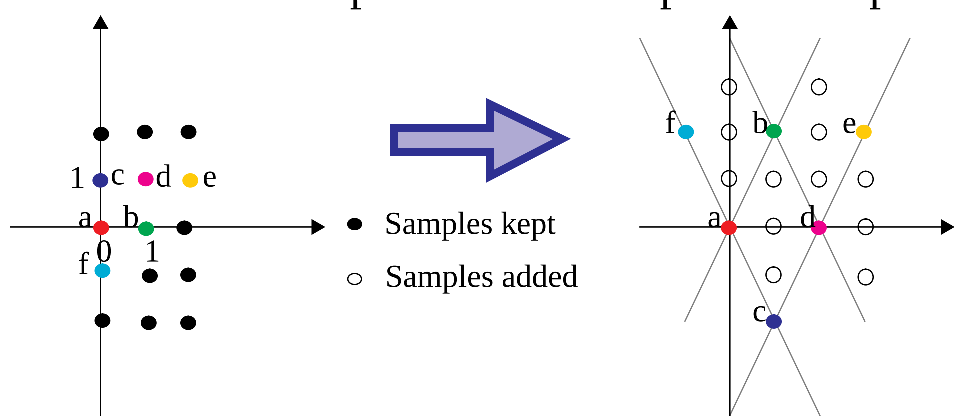
Rectangular expansion Non-rectangular expansion

$$L = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$


$$L = \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}$$

Renumbered samples from the expanders ouput

$$L = \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}$$




*Expansion ratio*:

$$|det(L)|$$

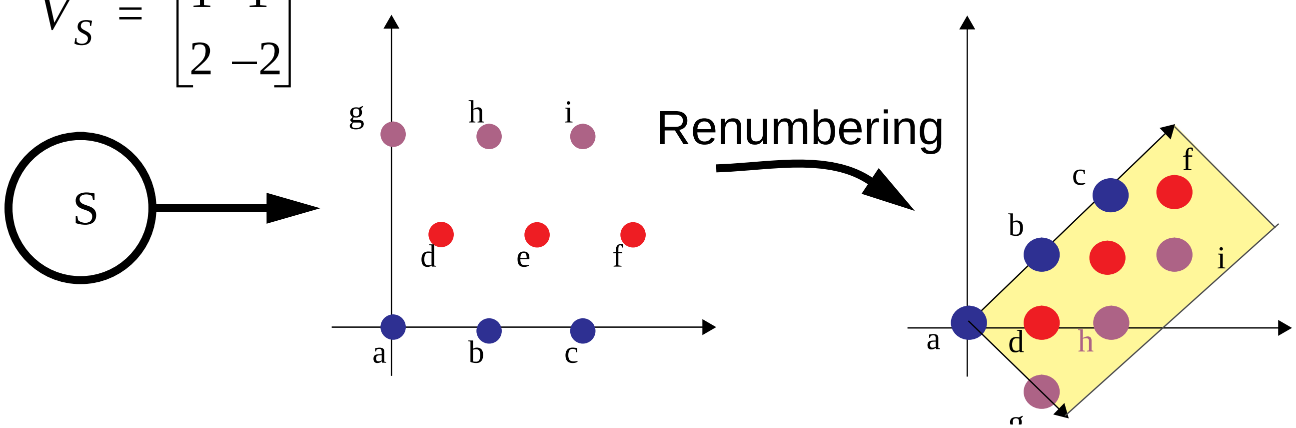
# Genarlized MDSDF (GMDSDF): Sources

*Definition:* The **containability condition:** let $X$ be a set of integer points in $\Re^m$. We say that $X$ satisfies the *containability condition* if there exists an $m \times m$ matrix $W$ such that $N(W) = X$.

*Definition:* We will assume that any source actor in the system produces data in the following manner. A source $S$ will produce a set of samples $\zeta$ on each firing such that each sample in $\zeta$ will lie on the lattice $LAT(V_S)$. We assume that the renumbered set $\bar{\zeta}$ satisfies the containability condition.

$$V_S = \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}$$

$$Q = \begin{bmatrix} 3 & 1.5 \\ 3 & 1.5 \end{bmatrix}$$



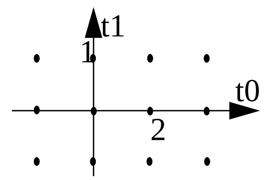

$\zeta$

$$\bar{\zeta} = \{V_S^{-1}x : x \in \zeta\},\ \bar{\zeta} = N(Q)$$

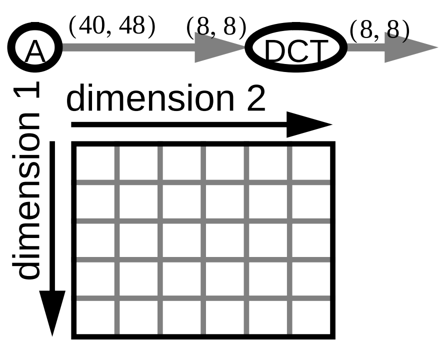
# Concise Problem Statement

## MDSDF

- **Rectangular lattice**




- **Regions of data produced = rectangular arrays**

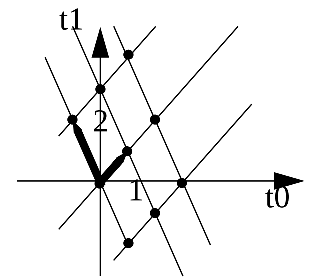



- **Rectangular arrays specified concisely by tuples of produced/ consumed.**

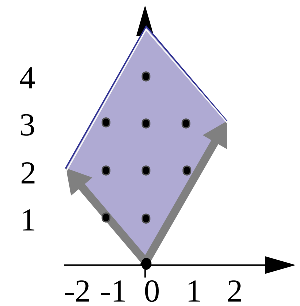
- **Coordinate axes for dataflow along arcs orthogonal to each other (x and y axes).**

## GMDSDF

- **Arbitrary lattice**

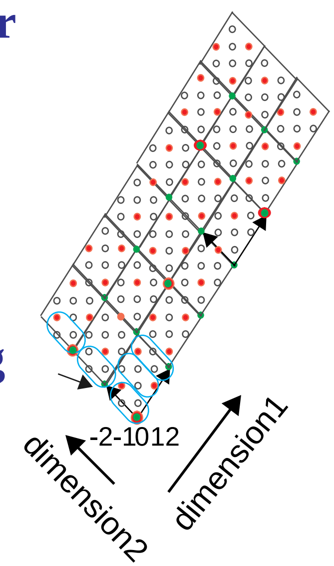



- **Regions of data produced = parallelograms**




- **Parallelograms specified concisely as the set of integer points inside a support matrix.**

- **Coordinate axes for dataflow along arcs not necessarily othogonal.**



# *Support Matrices*

Want to describe regions where the data is contained.

- **In MDSDF, these are ordinary arrays**
- **In the extension, these are *support matrices*.**


**Theorem:**

For the decimator,

$$V_f = V_e M \text{ and } W_f = M^{-1} W_e.$$

For the expander,

$$V_f = V_e L^{-1}, \text{ and } W_f = L W_e.$$

# *Semantics of GMDSDF*

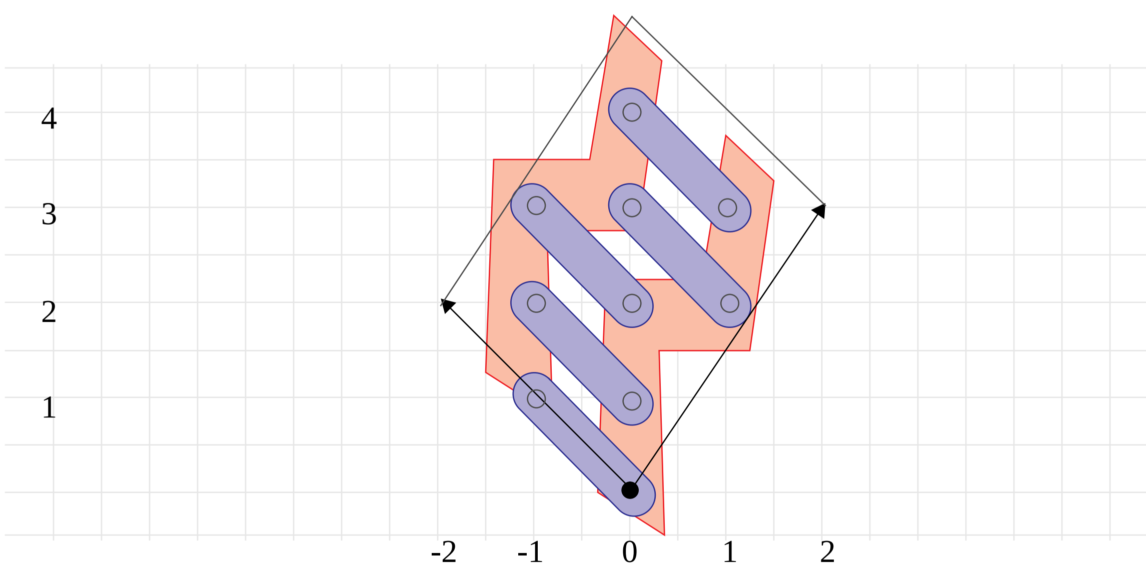

$$V_{SA} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad L = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}, |L| = 5 \times 2 \qquad M = \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}, |M| = 2 \times 2$$

$$W_{SA} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

*A* consumes (1,1) and produces (5,2).

*B* consumes (2,2) and produces (1,1) *on average*.

*T* consumes (1,1)



# GMDSDF — Balance Equations

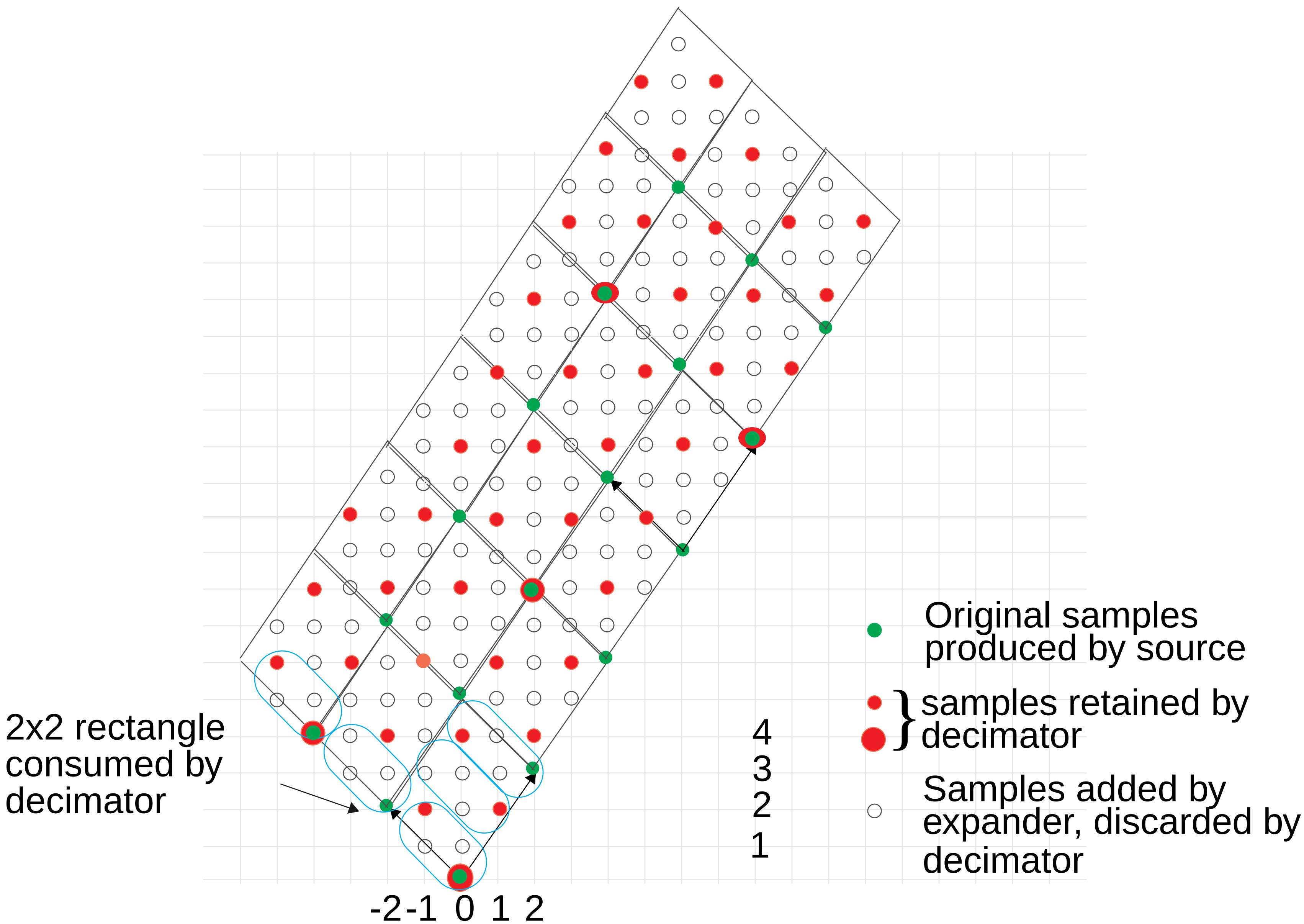
- **We don't know yet exactly how many samples on each firing the decimator will produce.**
- **Idea: *Assume* that it produces (1,1) and solve balance equations:**

$$3r_{S,1} = 1r_{A,1},\ 5r_{A,1} = 2r_{B,1},\ r_{B,1} = r_{T,1}$$
$$3r_{S,2} = 1r_{A,2},\ 2r_{A,2} = 2r_{B,2},\ r_{B,2} = r_{T,2}$$

- **Solution:**

$$r_{S,1} = 2, r_{S,2} = 1$$
$$r_{A,1} = 6, r_{A,2} = 3$$
$$r_{B,1} = 15, r_{B,2} = 3$$
$$r_{T,1} = 15, r_{T,2} = 3$$

# *Dataspace on arc AB*



# *Balance equations cont'd*

**Question**: Have we really "balanced"?

**No**: by counting the number of samples that have been kept in the previous slide.

More systematically:

$$W_{SA} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} r_{S,1} & 0 \\ 0 & r_{S,2} \end{bmatrix} = \begin{bmatrix} 3r_{S,1} & 0 \\ 0 & 3r_{S,2} \end{bmatrix}$$

$$W_{AB} = LW_{SA} = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 3r_{S,1} & 0 \\ 0 & 3r_{S,2} \end{bmatrix} = \begin{bmatrix} 6r_{S,1} & -6r_{S,2} \\ 9r_{S,1} & 6r_{S,2} \end{bmatrix}$$

$$W_{BT} = M^{-1}W_{AB} = \frac{1}{4} \begin{bmatrix} 2 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 6r_{S,1} & -6r_{S,2} \\ 9r_{S,1} & 6r_{S,2} \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 21r_{S,1} & -6r_{S,2} \\ 3r_{S,1} & -18r_{S,2} \end{bmatrix}$$

# Balance equations cont'd

Want to know if

$$|N(W_{BT})| = \frac{|N(W_{AB})|}{|M|}$$

We have

$$|N(W_{AB})| = |det(W_{AB})| = 90r_{S,1}r_{S,2}$$

The right hand side becomes

$$\frac{90r_{S,1}r_{S,2}}{4} = \frac{45r_{S,1}r_{S,2}}{2}$$

Therefore, we need

$$r_{S,1}r_{S,2} = 2k \qquad k = 0, 1, 2, \ldots$$

The balance equations gave us $r_{S,1} = 2, r_{S,2} = 1$.

With these values, we get

$$W_{BT} = \begin{bmatrix} 21/2 & -3/2 \\ 3/2 & -9/2 \end{bmatrix}.$$

This matrix has 47 points inside its FPD (determined by drawing it out).

==> Balance equation solution is not quite right.

# *Augmented Balance Equations*

To get the correct balance, take into account the constraint given by

$$|N(W_{BT})| = \frac{|N(W_{AB})|}{|M|}$$

***Sufficiency***: force $W_{BT}$ to be an integer matrix.

$$\Longrightarrow r_{S,1} = 4k,\ k = 1, 2, \ldots$$

$$\Longrightarrow r_{S,2} = 2k,\ k = 1, 2, \ldots$$

Therefore,

$$r_{S,1} = 4,\ r_{S,2} = 2.$$

- **So decimator produces (1,1) on average but has cyclostatic behavior.**

Production sequence: 2,1,1,2,1,0,1,1,0,1,2,1,1,2,1,...

## Theorem:

**Always possible to solve these *augmented* balance equations.**

# *Effect of Different Factorizations*

Suppose we let $|det(M)| = 1 \times 4$ instead. Balance equations give:

$$r_{S,1} = 1, r_{S,2} = 2$$

$$r_{A,1} = 3, r_{A,2} = 6$$

$$r_{B,1} = 15, r_{B,2} = 3$$

$$r_{T,1} = 15, r_{T,2} = 3$$

Also,

$$W_{BT} = \begin{bmatrix} 21/4 & -3 \\ 3/4 & -9 \end{bmatrix}$$

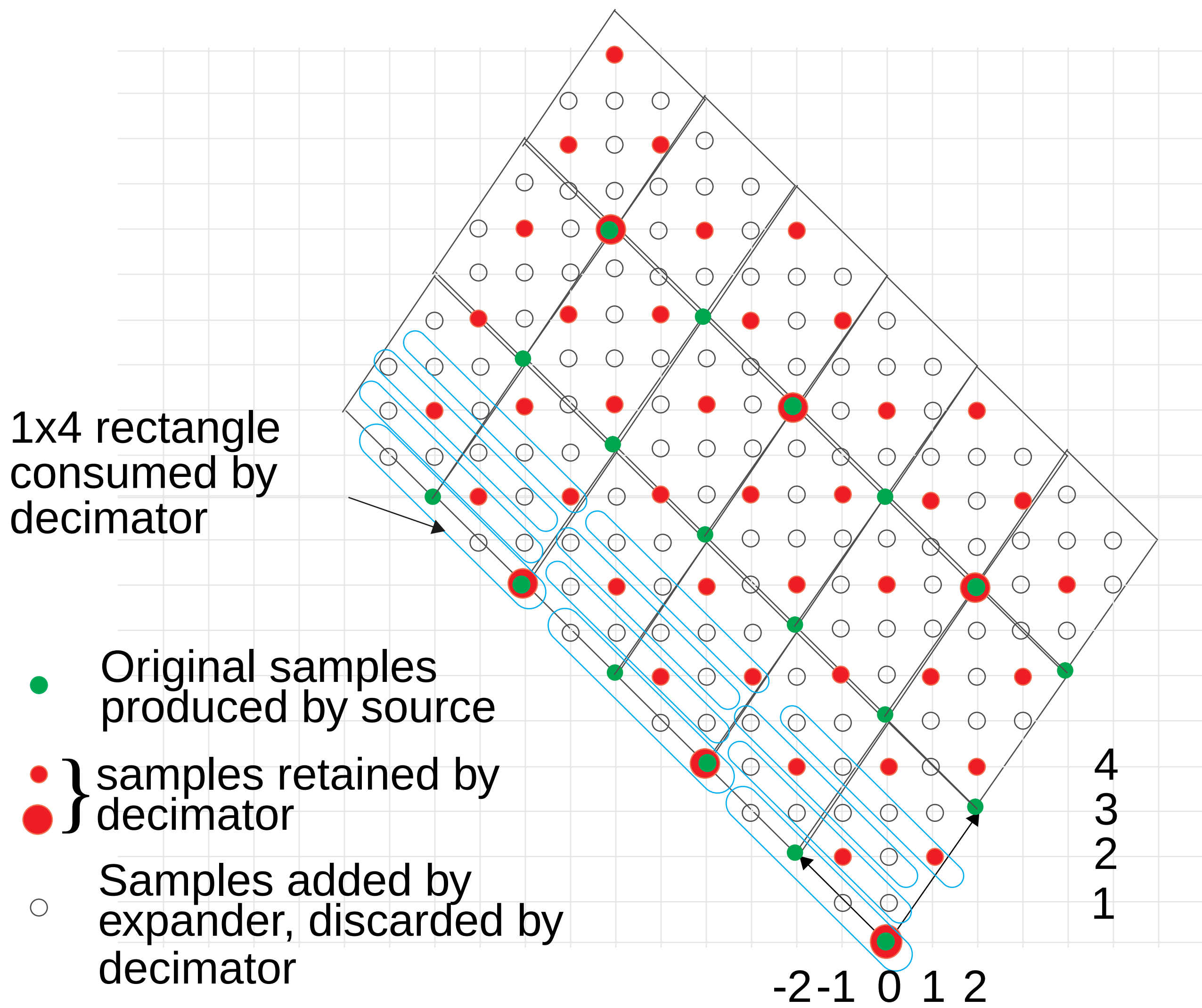
It turns out that

$$|N(W_{BT})| = 45$$

as required.

==> Lower number of overall repetitions with this factoring choice.

# *Dataspace on Arc AB*

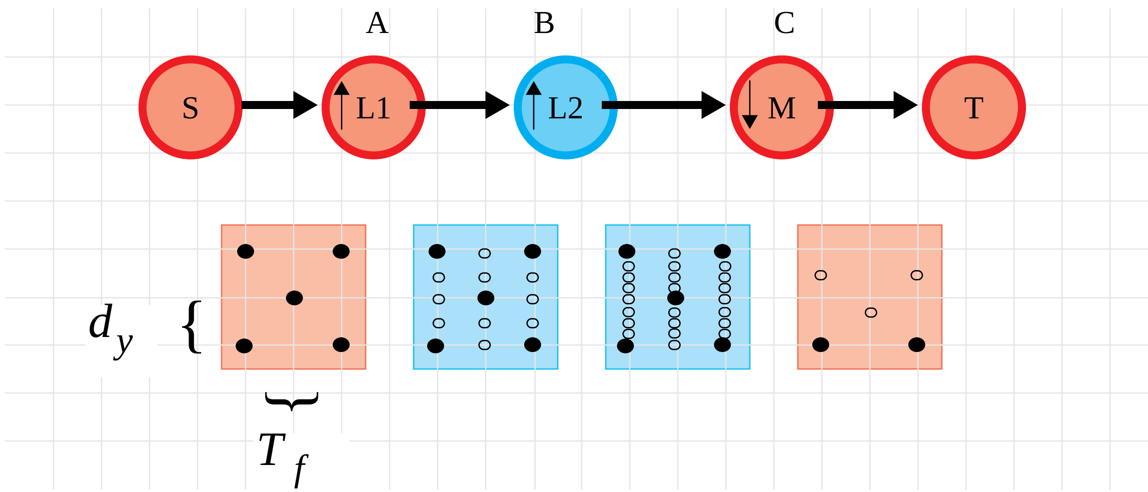


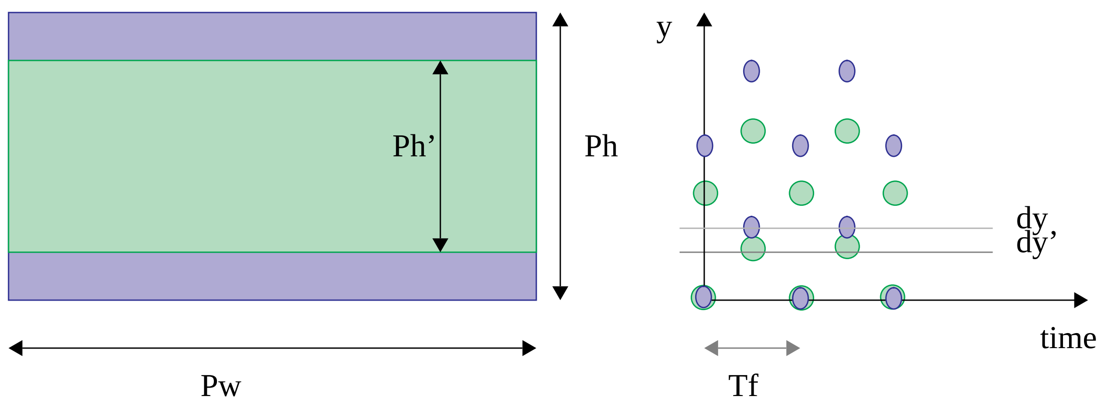
# Summary of Extended Model

- Each arc has associated with it a lattice-generating matrix, and a support matrix.
- The source actor for an arc establishes the ordering of the data on that arc.
- Expander: consumes (1,1) and produces $FPD(L)$, ordered as an $(L_1, L_2)$ rectangle where $L_1 L_2 = |det(L)|$.
- Decimator: consumes an $(M_1, M_2)$ rectangle, where $M_1 M_2 = |det(M)|$ and produces (1,1) on average.
- Write down balance equations.
- Additional equations for support matrices on decimator outputs.
- The above two sets are simultaneously solved to determine the smallest non-zero number of times each node is to be invoked in a periodic schedule.
- Actors are then scheduled as in SDF or MDSDF.

# Aspect Ratio Conversion

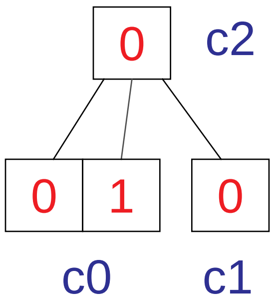
Format conversion of 2:1 interlaced video from 4/3 aspect ratio to 16/9 aspect ratio.



# *Future Work in GMDSDF*

## Concrete Data Structures (Semantics)




Cells

⊢ c0

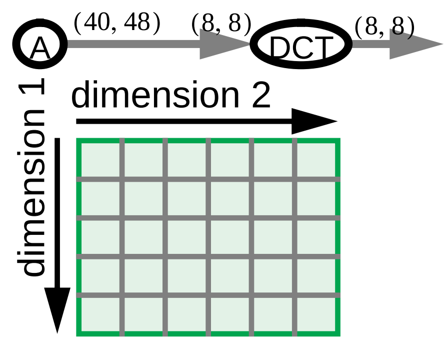
⊢ c1

(c0,0)(c1,0) ⊢ c2

(c0,1) ⊢ c2

Enabling Rules

## GMDSDF (Scheduling)

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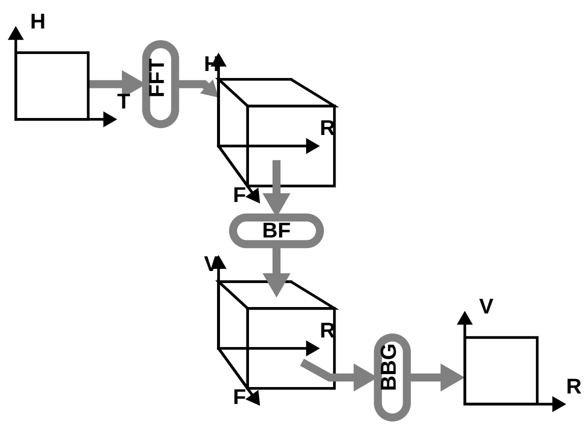
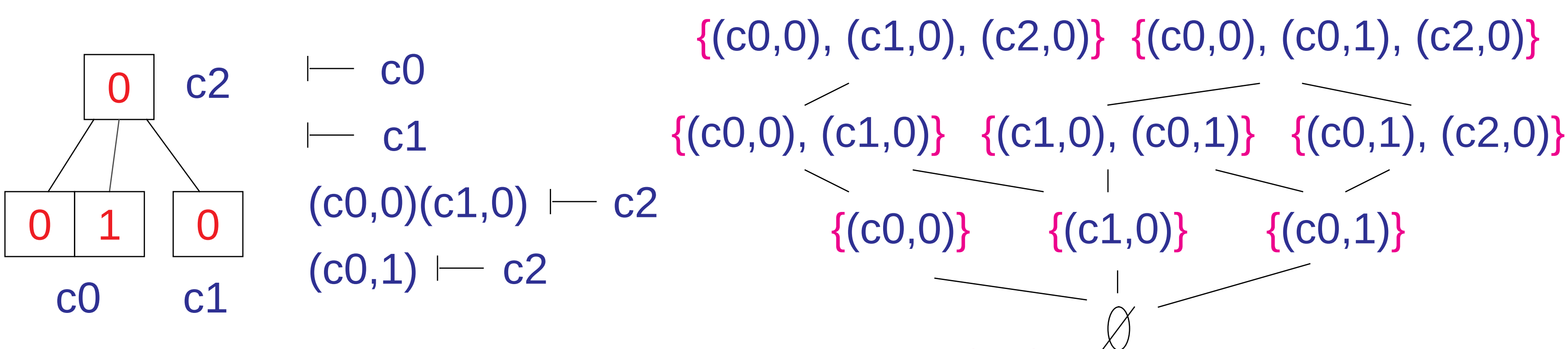
## Array-oriented language (graphical syntax for enabling rules ?)



## *Concrete Data Structures*

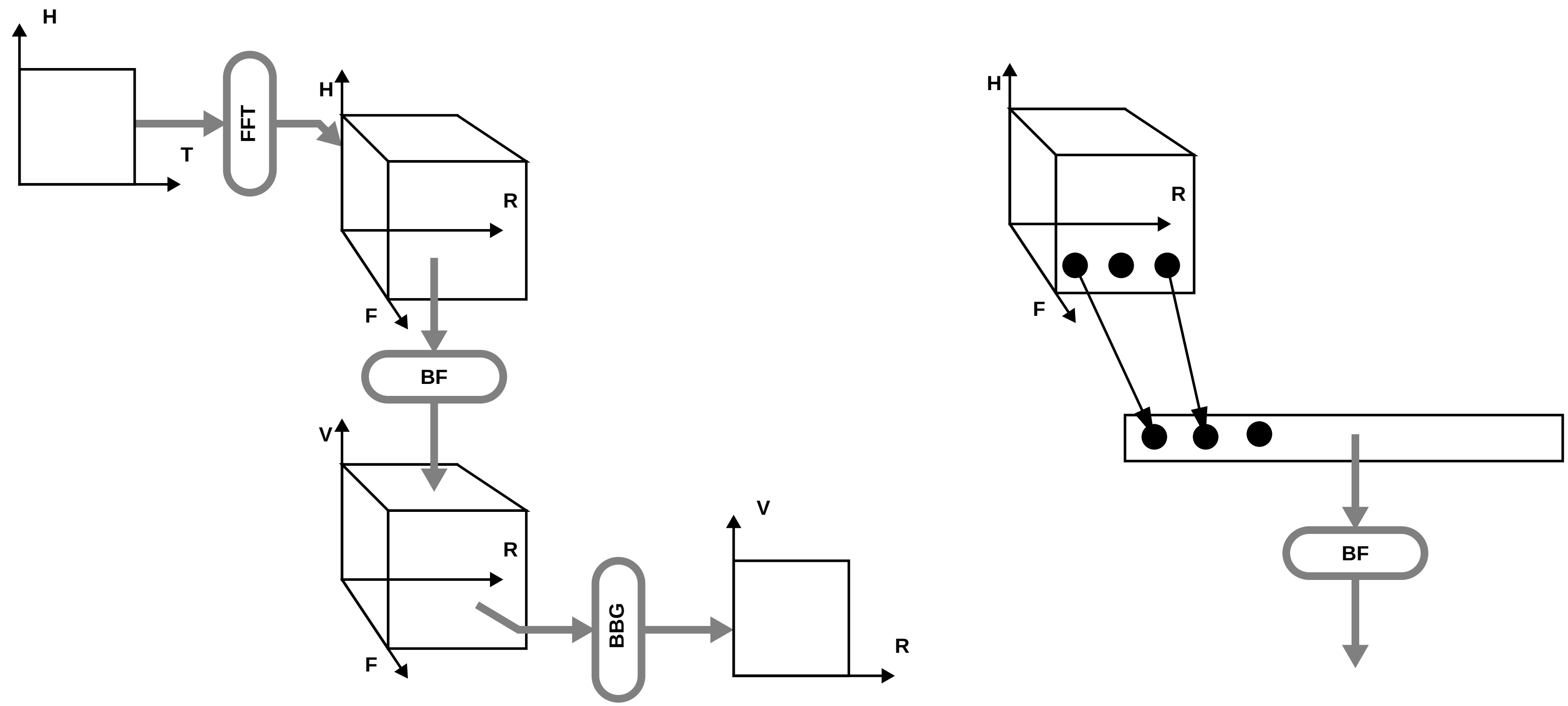
- **"Cells" can have specific "Values"**
- **Enabling relationship says when a cell can be filled.**
- **"Cell" dependency partial order can be arbitrary**
- **Formalizes most forms of "real-world" data structures: lists, trees, arrays etc.**
- **Kahn-Plotkin sequential functions on CDS provide an elegant model of computation with many formal properties, like full abstraction.**
- **CDS approach has been mostly semantic; need to sort out operational issues (like scheduling).**




Cells　　Enabling Rules　　CDS

# *Array-OL*

- **Array-oriented language developed at Thomson**
- **Graphical syntax for specifying "array access patterns"**
  - **In many multidimensional programs, manipulating data aligned in various dimensions is a challenge. For example: Transpose.**
  - **Patterns specified by "fitting" and "paving" relationships.**
- **Combine with MDSDF...**



## *Conclusion*

- MDSDF extension allows modeling of MD DSP systems using rectangular sampling schemes.
- GMDSDF allows modeling of MD DSP systems using arbitrary sampling schemes.
- Both models can be scheduled statically—thus ideally suited for prototyping.
- Integration of AOL concepts, along with CDS generalization might result in a very powerful MoC for multidimensional programming.