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Homework #0: Continuous-time Fourier transforms in ω and f

A question came up last semester about the connection between continuous-time Fourier transforms involving ω in rad/s and *f* in Hz where $\omega = 2\pi f$.

Applications. Many applications use frequencies in Hz instead of rad/s. In audio, notes are assigned frequencies in Hz. We could specify ranges of sub-woofer, woofer and tweeter frequencies in Hz, e.g. 20-200 Hz, 200-2,000 Hz, and 2,000-20,000 Hz, respectively. We talk about Bluetooth and Wi-Fi operating over the 2.4 GHz band, which is 100 MHz wide. (Wi-Fi works over other frequency bands as well.) At other times, we might think about frequency in units of rad/s. This happens at times in circuit design and signal processing theory. Also, rad/s can also be used as an angular frequency.

Visual transform tables. Here is a visual continuous-time Fourier transform table for transforms in both ω and *f* from **Roberts Fundamental Signals & Systems Appendix D** (https://utexas.instructure.com/courses/1312389/files/61372990/download?download_frd=1) (Canvas only). **Fall 2022 link:** https://utexas.instructure.com/files/66782586/download?download_frd=1

Forward transforms. Here is the forward continuous-time Fourier transform in rad/s:

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

Let $\omega = 2\,\pi\,f$:

$$\hat{X}(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$$

Note that both forward continuous-time Fourier transforms look similar.

Inverse transforms. Here is the inverse continuous-time Fourier transform in rad/s:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

Let $\omega = 2\,\pi\,f$. As $\omega \longrightarrow \infty,\ f \longrightarrow \infty$. As $\omega \longrightarrow -\infty,\ f \longrightarrow -\infty$. Also, $d\omega = 2\,\pi\,df$:

$$x(t) = \int_{-\infty}^{\infty} \hat{X}(f) e^{j2\pi f t} df$$

Note that the inverse continuous-time Fourier transform in Hz does not have a scaling factor.

Example #1. Let $X(\omega) = \delta(\omega - \omega_c)$ which is a Dirac delta shifted to the right in the frequency domain in rad/s by ω_c:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \delta(\omega - \omega_c) e^{j\omega t} d\omega = \frac{1}{2\pi} e^{j\omega_c t}$$

Example #2. Let's convert $\delta\left(\omega - \omega_c\right)$ into frequencies in Hz. Let $\omega = 2\,\pi\,f$ and $\omega_c = 2\,\pi\,f_c$,

$$\delta\left(2\pi f - 2\pi f_c\right) = \delta\left(2\pi\left(f - f_c\right)\right) = \frac{1}{2\pi}\delta\left(f - f_c\right)$$

The last step is valid under integration as explained in the **Sampling Unit Step** (http://users.ece.utexas.edu/~bevans/courses/realtime/lectures/01_Sinusoids/SamplingUnitStep.pdf) handout which we'll likely cover in lecture on Wednesday. The expression in Hz is a Dirac delta shifted to the right in the frequency domain in Hz by f_c.

Let's check the inverse transform in Hz of $\frac{1}{2\pi}\delta\left(f - f_c\right)$ to make sure:

$$x(t) = \int_{-\infty}^{\infty} \hat{X}(f) e^{j2\pi f t} df = \int_{-\infty}^{\infty} \frac{1}{2\pi}\delta(f - f_c) e^{j2\pi f t} df = \frac{1}{2\pi} e^{j2\pi f_c t} = \frac{1}{2\pi} e^{j\omega_c t}$$

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