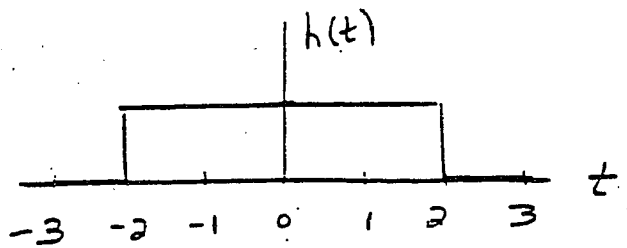
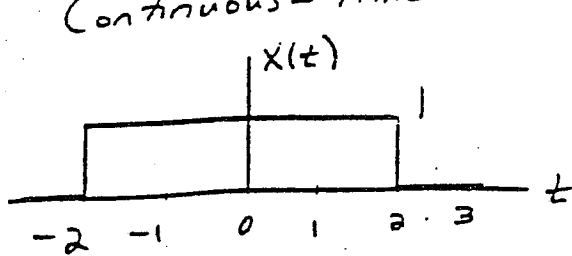
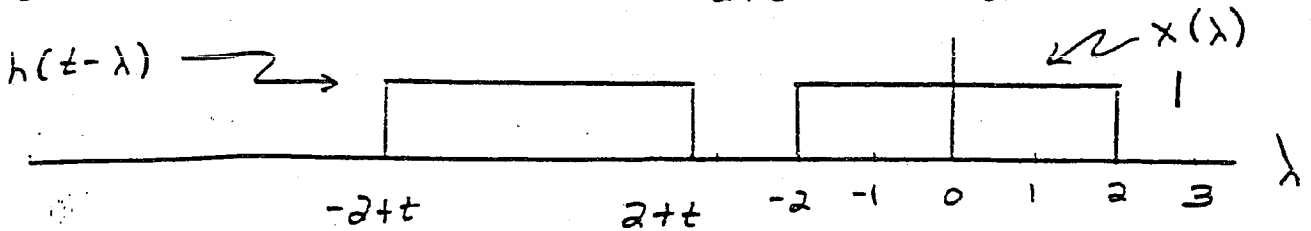
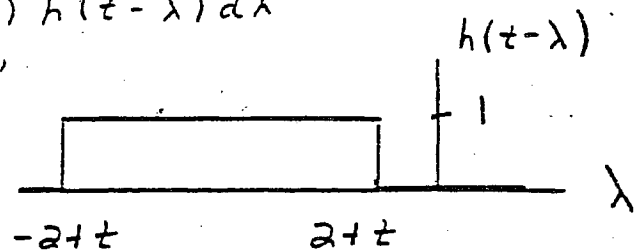
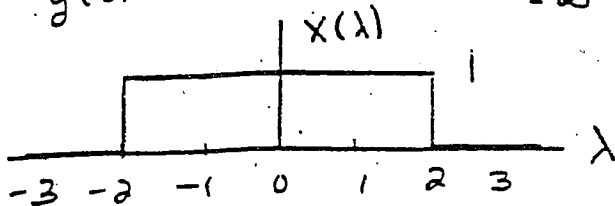


Convolution of Two Rectangular Pulses

1. Continuous-Time Convolution



$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\lambda) h(t-\lambda) d\lambda$$



There is no overlap when $2+t < -2 \Rightarrow t < -4$

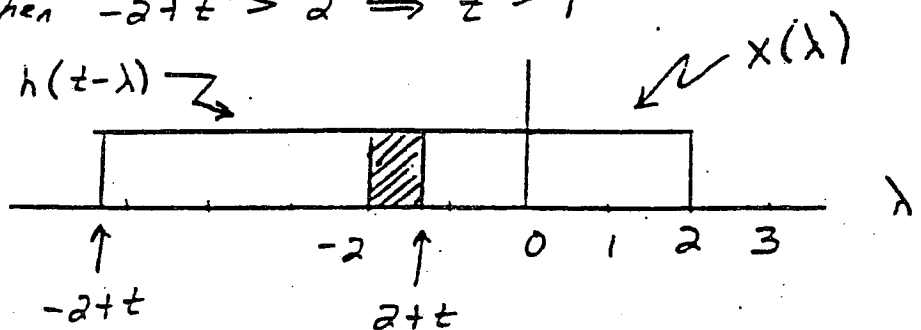
There is no overlap when $-2+t > 2 \Rightarrow t > 4$

For $-4 \leq t \leq 0$,

there is overlap.

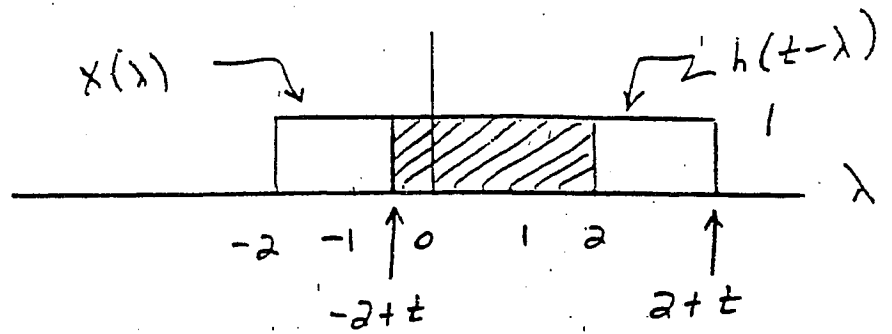
from $\lambda = -2$ to

$$\lambda = 2+t$$



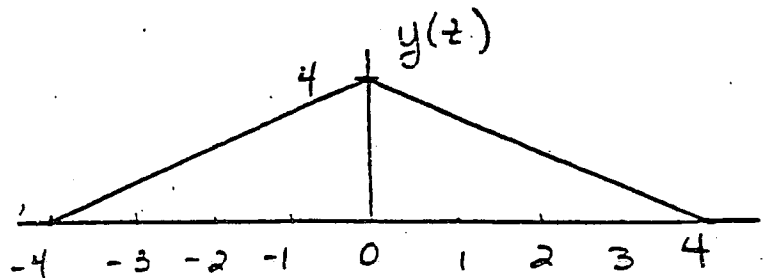
$$y(t) = \int_{-\infty}^{\infty} x(\lambda) h(t-\lambda) d\lambda = \int_{-2}^{2+t} 1 \cdot d\lambda = [\lambda]_{-2}^{2+t} = t+4$$

For $0 < t \leq 4$,
 there is overlap
 from $\lambda = -2 + t$
 to $\lambda = 2$



$$y(t) = \int_{-\infty}^{\infty} x(\lambda) h(t-\lambda) d\lambda = \int_{-2+t}^2 1 \cdot d\lambda = \left[\lambda \right]_{-2+t}^2 = 4 - t$$

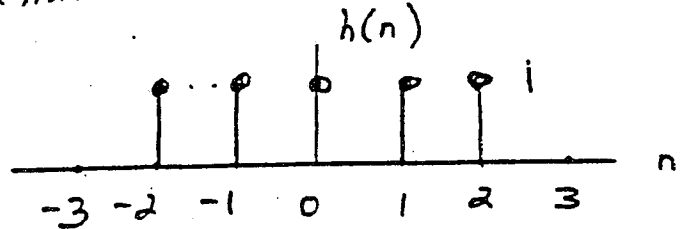
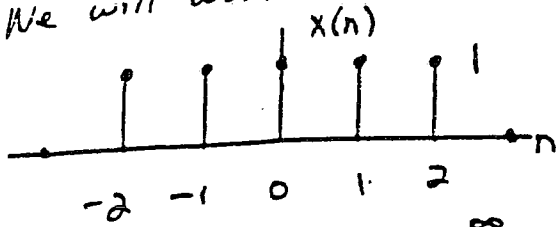
$$y(t) = \begin{cases} 0 & \text{for } t < -4 \\ 4+t & \text{for } -4 \leq t \leq 0 \\ 4-t & \text{for } 0 < t \leq 4 \\ 0 & \text{for } t > 4 \end{cases}$$



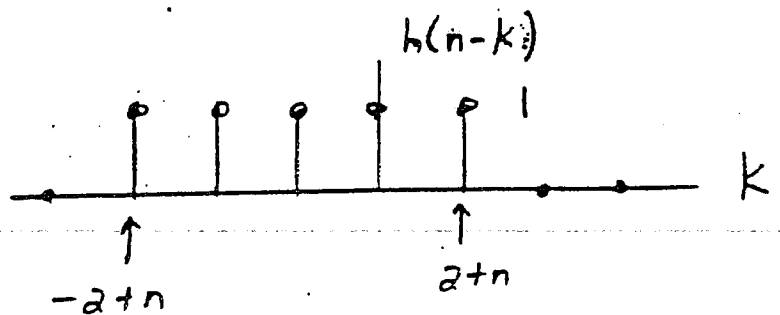
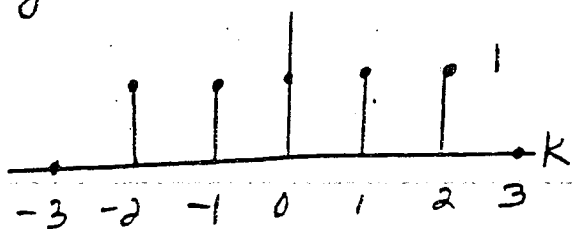
To check a convolution result, check the value of $y(t)$ at the endpoints of each interval - they should agree.

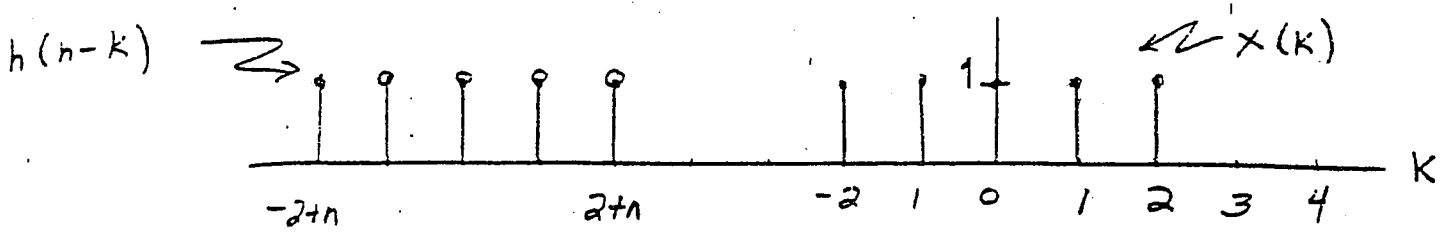
2. Discrete-Time Convolution

We will work a similar convolution in discrete time.



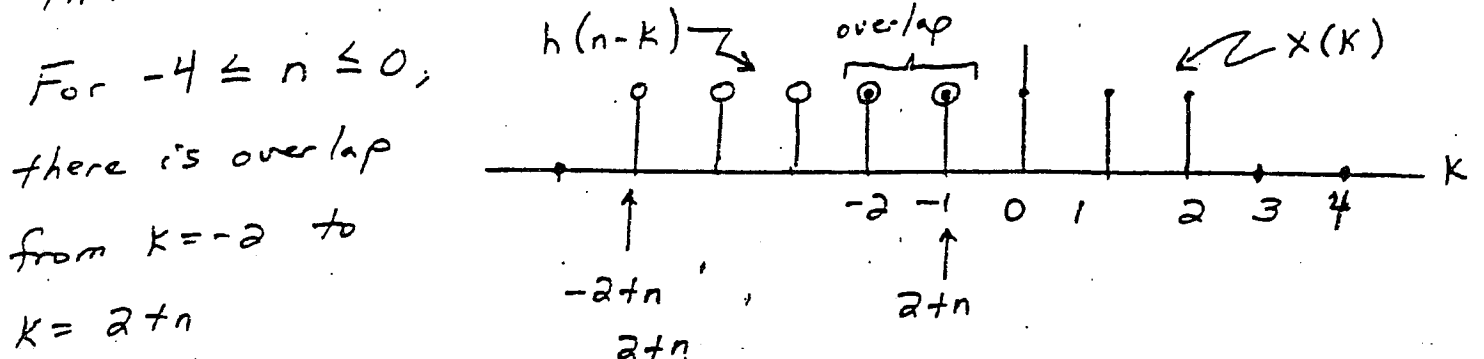
$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k), \text{ so } k \text{ has the same role as } \lambda$$



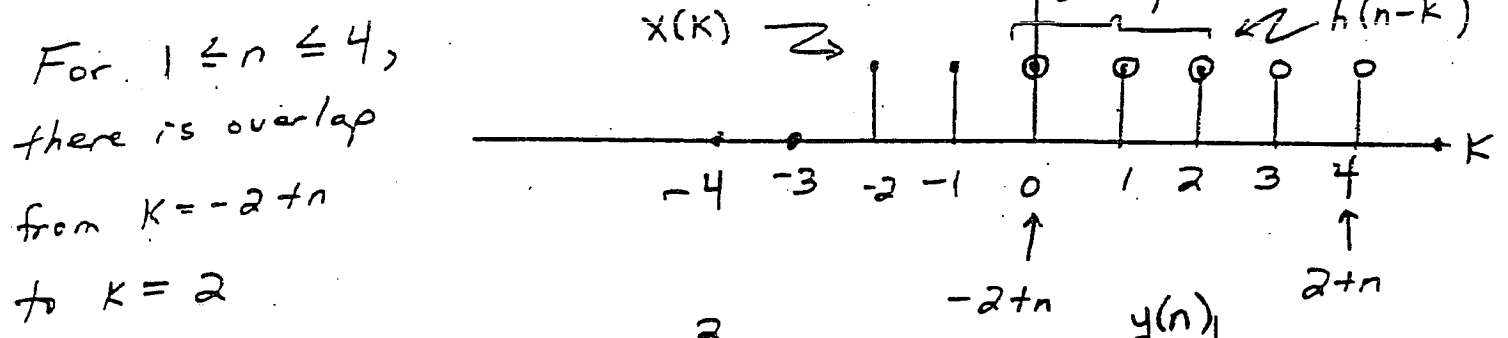


There is no overlap when $2+n < -2 \Rightarrow n < -4$

There is no overlap when $-2+n > 2 \Rightarrow n > 4$



$$y(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k) = \sum_{k=-2}^{2+n} 1 = n+5$$



$$y(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k) = \sum_{k=-2+n}^2 1 = 5-n$$

