

# Real-Time Digital Signal Processing Laboratory

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Lab 0 : Review of the Prerequisites

01/16/2024

T.A. Yongjin Eun, ye2259

# Outline

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## 1. Lab Overview

- Hardware Overview
- Software Overview
- Labs Overview

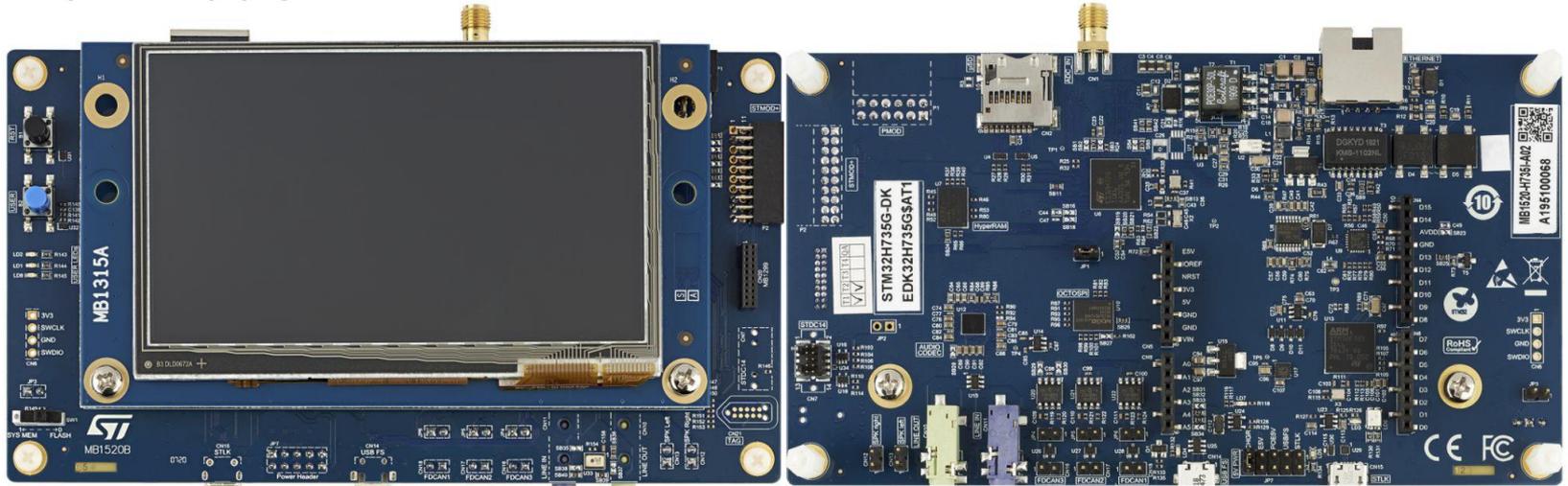
## 2. Programming Review

- **C Programming**
- **Matlab Programming**
- **Floating Point**

## 3. Linear Systems and Signals Review

# Lab Overview - Hardware

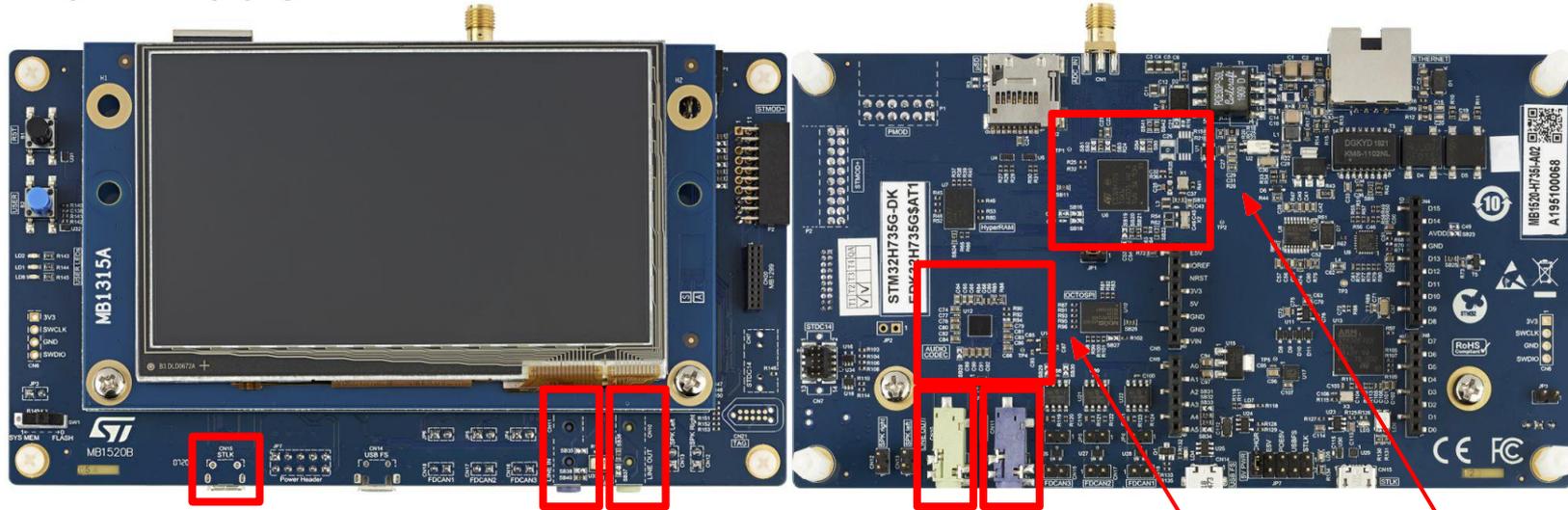
## STM32H735G



- **Arm® Cortex®-M7 core-based**
- **Two 16-bit ADCs and one 12-bit ADC, two 12-bit DACs**
- **Two audio jacks for input/output**
- **4.3-inch RGB TFT-LCD display**

# Lab Overview - Hardware

## STM32H735G



USB port for Programming  
And Debugging

Signal  
Input

Signal  
Output

WM8994  
Codec  
(Cirrus)

STM32H7  
MCU

- For more information, click [HERE](#)

# Lab Overview - Software

## STM32CubeIDE Installation Guide

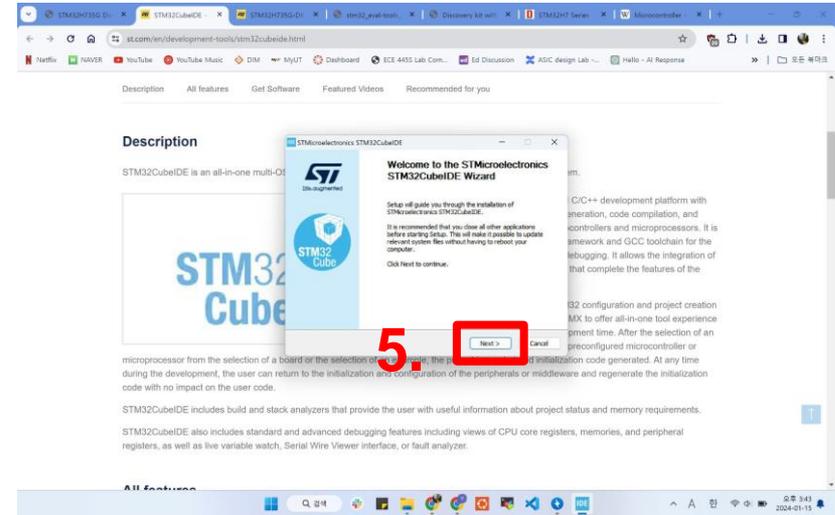
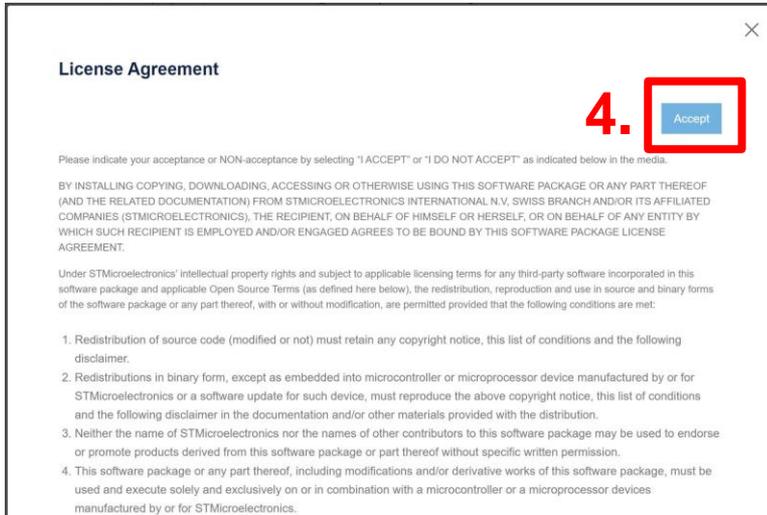
### 2. **Get Software**

Part Number	General Description	Latest version	Download	All versions
+ STM32CubeIDE-DEB	STM32CubeIDE Debian Linux Installer	1.14.0	<b>3.</b> <a href="#">Get latest</a>	Select version ▾
+ STM32CubeIDE-Lnx	STM32CubeIDE Generic Linux Installer	1.14.0	<a href="#">Get latest</a>	Select version ▾
+ STM32CubeIDE-Mac	STM32CubeIDE macOS Installer	1.14.0	<a href="#">Get latest</a>	Select version ▾
+ STM32CubeIDE-RPM	STM32CubeIDE RPM Linux Installer	1.14.0	<a href="#">Get latest</a>	Select version ▾
+ STM32CubeIDE-Win	STM32CubeIDE Windows Installer	1.14.1	<a href="#">Get latest</a>	Select version ▾

1. To download the program, go to [STM32CubeIDE](#)
2. Scroll down the Website to find 'Get Software'
3. Click and Download the OS Version you are using

# Lab Overview - Software

## STM32CubeIDE Installation Guide

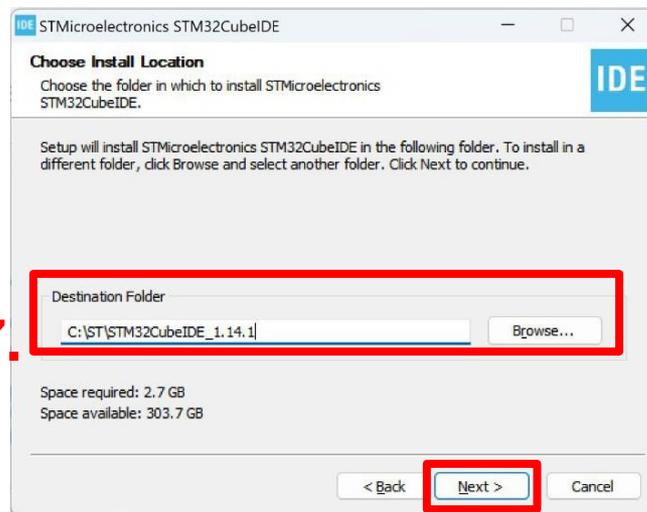


**4. Read and Accept the License Agreement. Accepting it will start the download. You will find a ZIP file downloaded.**

**5. Unzip the ZIP file and Run the .exe file inside. You will see the CubeIDE Wizard**

# Lab Overview - Software

## STM32CubeIDE Installation Guide

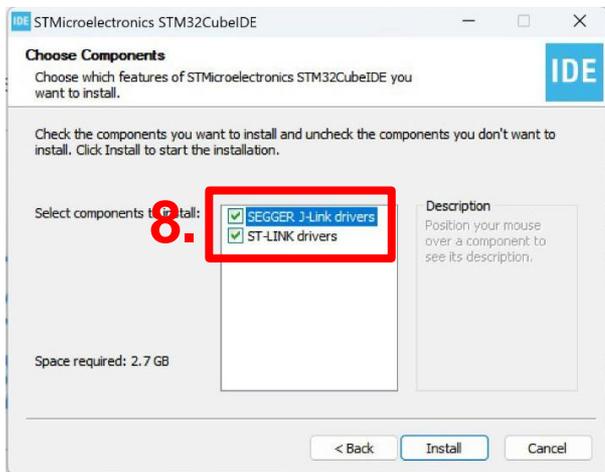


6. Click 'I Agree' to proceed

7. Use the Default Destination Folder or Browse for a specific folder. Click Next

# Lab Overview - Software

## STM32CubeIDE Installation Guide

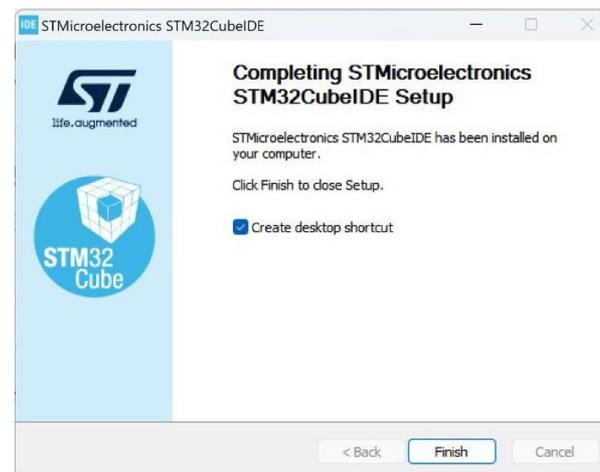
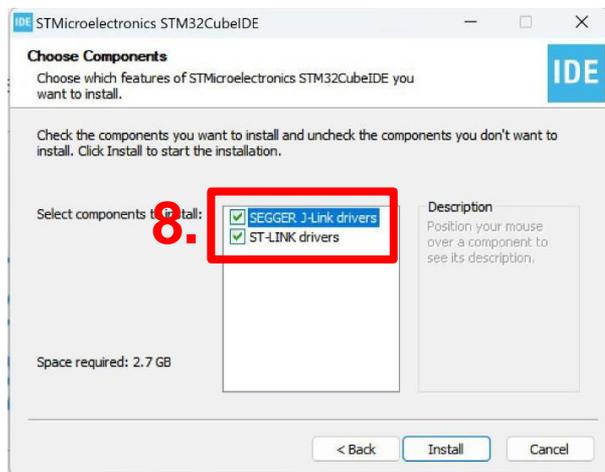


**8. Make sure to check both 'SEGGER J-Link drivers' and 'ST-LINK drivers'. Click Install**

**9. Installation will be completed within few minutes**

# Lab Overview - Software

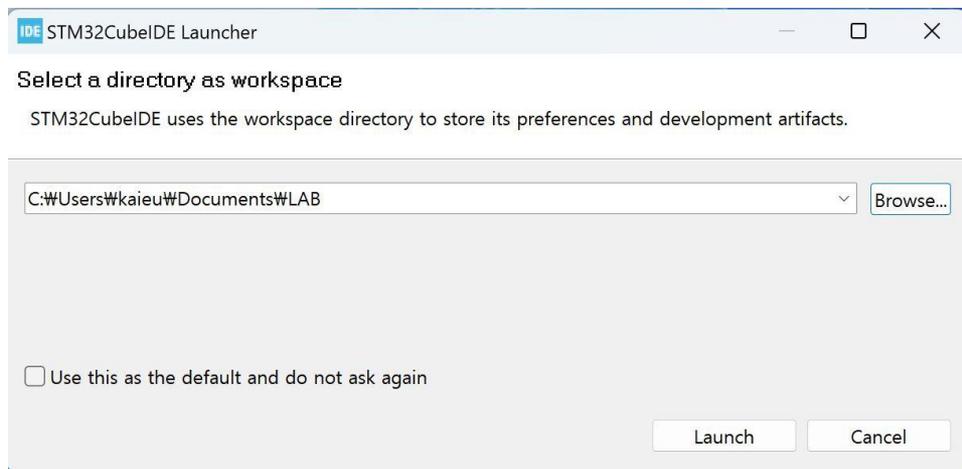
## STM32CubeIDE Installation Guide



8. Make sure to check both 'SEGGER J-Link drivers' and 'ST-LINK drivers'. Click Install
9. Installation will be completed within few minutes

# Lab Overview - Software

## STM32CubeIDE Installation Guide

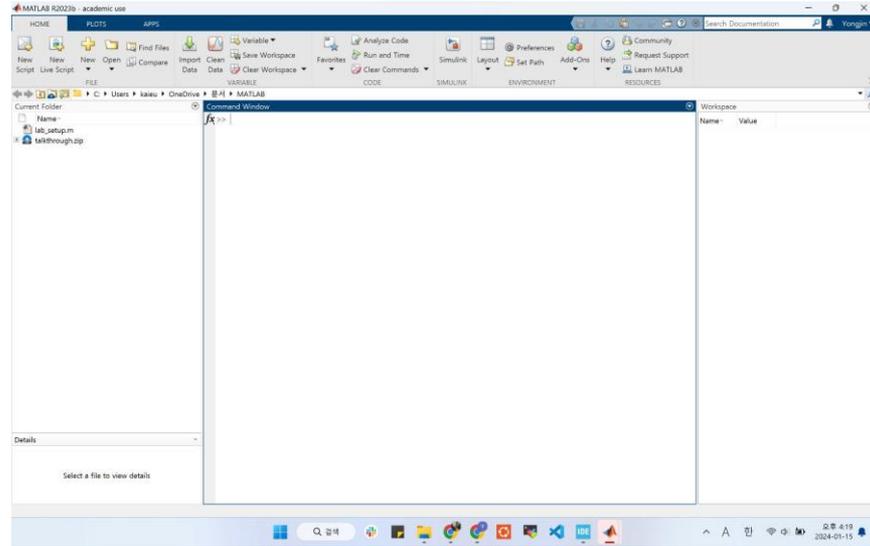


**When opening the STM32CubeIDE for the first time, you will be prompted to choose the workspace directory. Browse and choose the convenient workspace location.**

**Make sure that the directory path only has english! STM32CubeIDE will give errors when there are non-english paths**

# Lab Overview - Software

## MATLAB



You will need Matlab for HWs and Labs.

For the installation of Matlab, refer to the [UT Wiki](#) or the [Course Website](#).

# Lab Overview - Software

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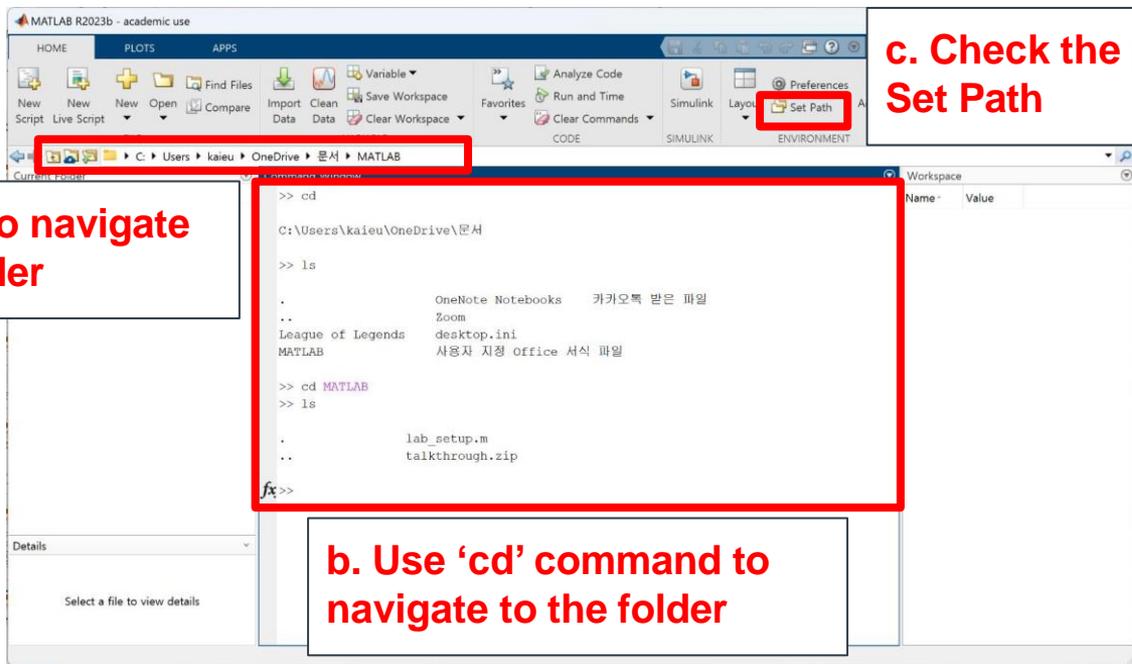
## Using MATLAB to setup Starter Code

For each lab, you would need to generate a new project in CodelIDE with the starter codes. Using Matlab and the instructions in [Here](#), you can simplify the process of making projects.

1. Download [Zip File](#) and [Setup Script](#) to the convenient location. If the script file does not download directly, make a new file named 'lab\_setup.m' and copy/paste the contents to the file.
2. Next, you have to make sure that both files are accessible to Matlab. You can do it by navigating to the folder location using Matlab GUI 'Current Folder' or through commanding 'cd' on the command window. For those who are unfamiliar with 'cd' command, refer to [Linux cd command](#). One easy way is to just move the files to the default Matlab folder. You can check it at the Matlab Toolbar : Home - Environment - Set Path. The first path on the list would be the current directory.

# Lab Overview - Software

## Using MATLAB to setup Starter Code



The screenshot shows the MATLAB R2023b interface. The File menu is open, and the 'Set Path' option is highlighted. The current directory in the Command Window is C:\Users\kaieu\OneDrive\문서\MATLAB. The Command Window shows the following commands and output:

```

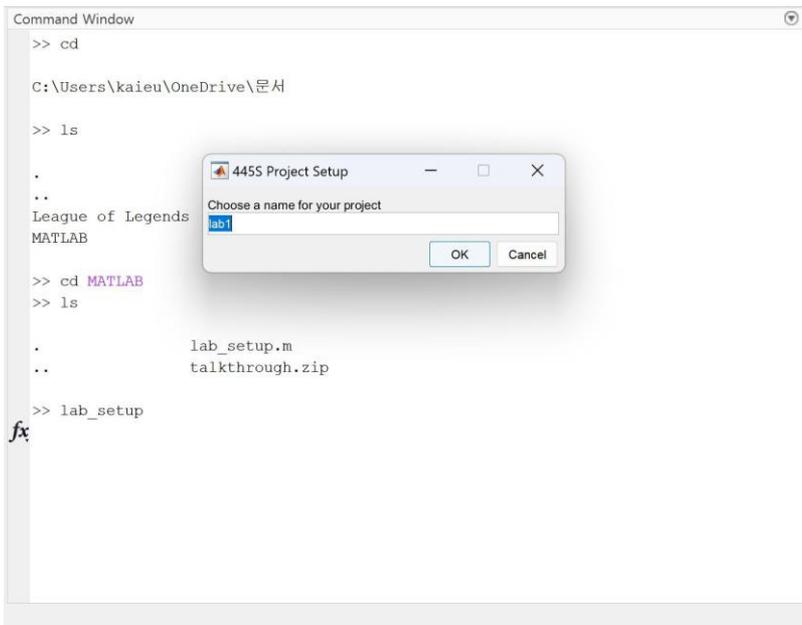
>> cd
C:\Users\kaieu\OneDrive\문서
>> ls
.
..
OneNote Notebooks 카카오톡 받은 파일
Zoom
League of Legends desktop.ini
MATLAB 사용자 지정 Office 서식 파일
>> cd MATLAB
>> ls
.
..
lab_setup.m
talkthrough.zip
fx>>
    
```

Three callout boxes provide instructions:

- a. Use GUI to navigate to the folder**: Points to the file explorer path C:\Users\kaieu\OneDrive\문서\MATLAB.
- b. Use 'cd' command to navigate to the folder**: Points to the Command Window output showing the current directory.
- c. Check the path inside Set Path**: Points to the 'Set Path' option in the File menu.

# Lab Overview - Software

## Using MATLAB to setup Starter Code

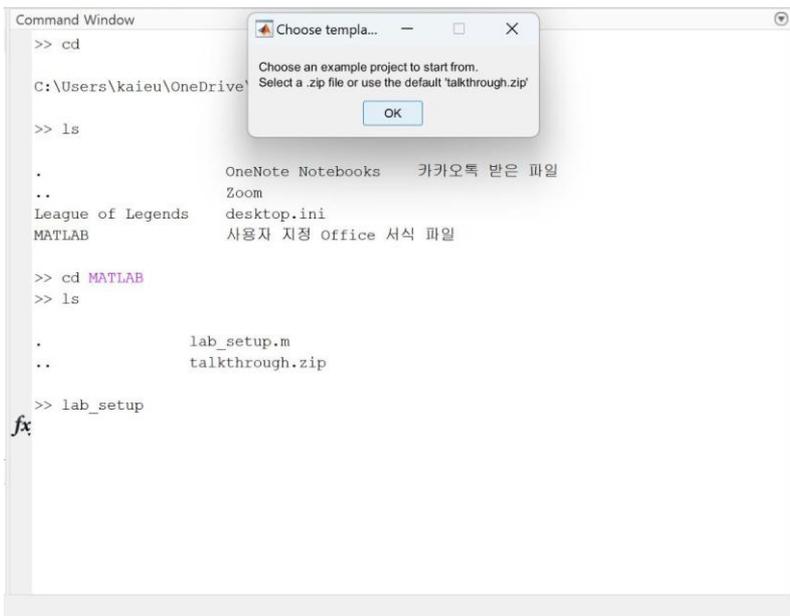


```
Command Window
>> cd
C:\Users\kaieu\OneDrive\문서
>> ls
.
..
League of Legends
MATLAB
>> cd MATLAB
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.
..
lab_setup.m
talkthrough.zip
>> lab_setup
fx
```

3. Run `lab_setup.m` by commanding 'lab\_setup' at the Command Window. A pop-up window will ask you for the name for the project. Choose the name you want.

# Lab Overview - Software

## Using MATLAB to setup Starter Code



```

Command Window
>> cd

C:\Users\kaieu\OneDrive\

>> ls

.           OneNote Notebooks   카카오톡 받은 파일
..          Zoom
League of Legends  desktop.ini
MATLAB        사용자 지정 Office 서식 파일

>> cd MATLAB
>> ls

.           lab_setup.m
..          talkthrough.zip

>> lab_setup
fx
    
```

The screenshot shows a MATLAB Command Window with the following commands and output:

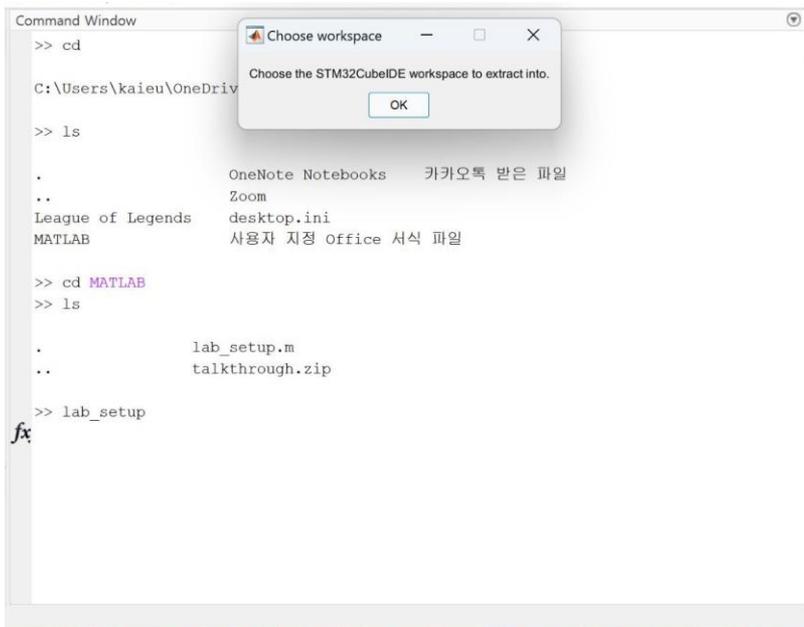
- `>> cd`: Changes the current directory to `C:\Users\kaieu\OneDrive\`.
- `>> ls`: Lists the contents of the current directory, showing folders like `OneNote Notebooks`, `Zoom`, `League of Legends`, and `MATLAB`.
- `>> cd MATLAB`: Changes the current directory to `C:\Users\kaieu\OneDrive\MATLAB`.
- `>> ls`: Lists the contents of the `MATLAB` directory, showing `lab_setup.m` and `talkthrough.zip`.
- `>> lab_setup`: Executes the `lab_setup` script.

A dialog box titled "Choose template..." is overlaid on the Command Window. It contains the text: "Choose an example project to start from. Select a .zip file or use the default 'talkthrough.zip'". There is an "OK" button at the bottom of the dialog box.

4. Next, the window will ask to choose an example .zip file for the project to start from. Choose the 'talkthrough.zip' file you downloaded.

# Lab Overview - Software

## Using MATLAB to setup Starter Code



```
Command Window
>> cd
C:\Users\kaieu\OneDrive
>> ls
.          OneNote Notebooks   카카오톡 받은 파일
..         Zoom
League of Legends  desktop.ini
MATLAB        사용자 지정 Office 서식 파일

>> cd MATLAB
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fx
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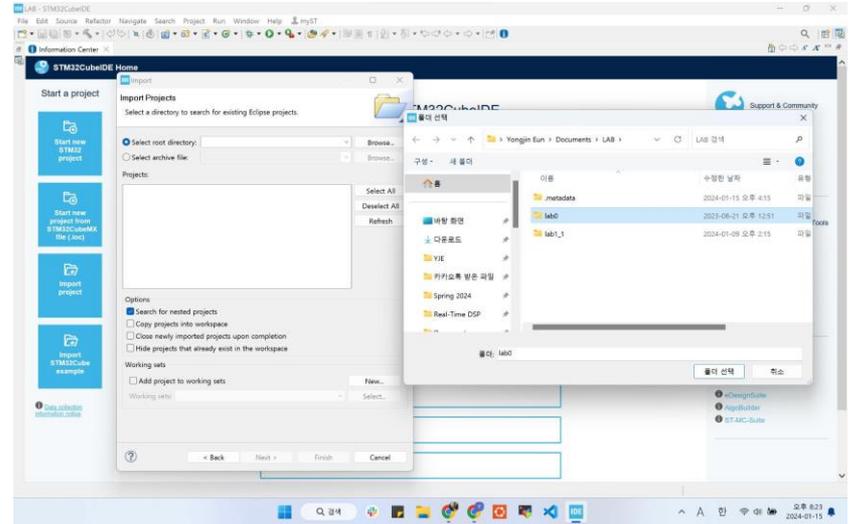
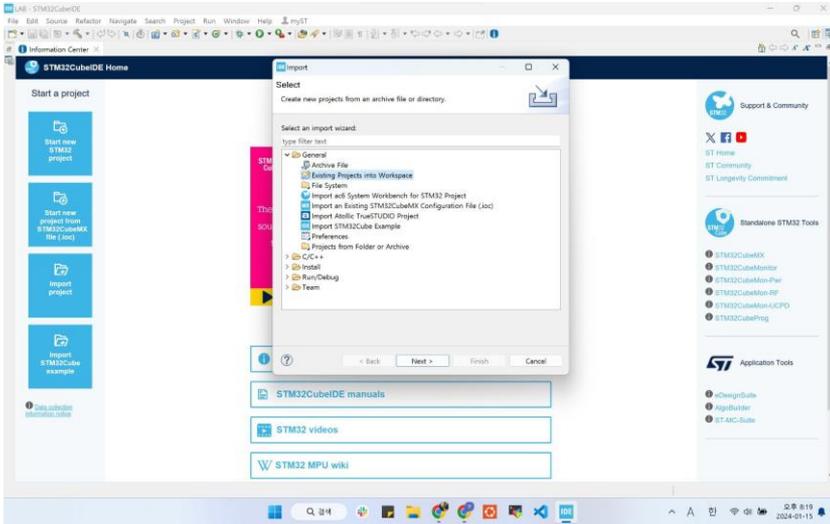
A dialog box titled "Choose workspace" is overlaid on the Command Window. It contains the text "Choose the STM32CubeIDE workspace to extract into." and an "OK" button.

5. Finally choose the STM32CubeIDE workspace to extract into. Choose the workspace you decided to use on the first time opening the CubeIDE.

You will see the lab folder created under the workspace.

# Lab Overview - Software

## Using MATLAB to setup Starter Code



6. Open CubeIDE. Go to File - Import - Existing Projects into Workspace  
Select the root directory as the directory you set in Matlab and click Finish

# Lab Overview - Labs

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- **Lab 0 - Review of Prerequisites**
  - H/W Overview, Programming Reviews
- **Lab 1 - Overview of Hardware and Software Tools**
- **Lab 2 - Generating Cosine and Sine Waves (2 Weeks)**
  - Sinusoidal Generation with Different Methods
- **Lab 3 - Digital Filters (3 Weeks)**
  - FIR / IIR Filter Design

# Lab Overview - Labs

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- **Lab 4 - Pseudo-Random Binary Sequences and Data Scramblers**
  - Pseudonoise sequence, Data scrambler / descrambler
- **Lab 5 - Digital Data Transmission by Baseband Pulse Amplitude Modulation (PAM) (3 Weeks)**
- **Lab 6 - Quadrature Amplitude Modulation (QAM)**
- **Lab 7 - Vocoder and Guitar Effects**
  - Vocoder, Flanger, Distortion

# Programming Review - C

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## Data Types

- C language does not provide exact rules for data types
  - Example: `int`
    - Must be at least 16 bits, but can be larger
    - Must include the range  $[-32,767, +32,767]$ . Often,  $-32,768$  is included as well.
- Toolchain used in lab provides exact specifications for data types
  - `int16_t` : 16-bit signed integer
  - `uint32_t` : 32 bit unsigned integer
  - `float32_t` : 32 bit (single precision) floating point

# Programming Review - C

## Pointers and functions

- Example: measuring clock cycles

```
uint32_t* systick = (uint32_t*) 0xe000e018;
void tic(void)
{
    t = *systick;
}
uint32_t toc(void)
{
    return t - *systick;
}
```



Declaring a pointer 'systick' and initializing as given address, '0xe000e018'



Sets value of 't' as a value pointed by 'systick'



Returns the difference between 't' and the value pointed by 'systick'

# Programming Review - C

## Arrays and For Loops

- Example: Cosine Lookup Table

```
int16_t table[16];  
float32_t amplitude;  
float32_t omega0 = 0.0576;  
  
for (uint32_t n = 0; n < 16; n+=1)  
{  
    amplitude = arm_sin_f32(n * omega0);  
    table[n] = OUTPUT_SCALE_FACTOR * amplitude;  
}
```



Declares an array, 'table', of 16 elements. Each elements are of 'int\_16t' type.



'arm\_sin\_f32' is an ARM math function that computes sine function for float32 type



Scale 'amplitude' with scaling factor and store it the array, 'table'

# Programming Review - C

## Linear Buffers and Circular Buffers

- Example: Linear Buffer Shifting

```

float32_t x[16] = {0};
...
void linear_buffer_shift(float32_t new_element)
{
    for (uint32_t n = 15; n > 0; n--)
    {
        x[n] = x[n-1];
    }
    x[0] = new_element;
}
    
```

$x_{15} = x_{14}$   
 $x_{14} = x_{13}$   
 $\vdots$   
 $x_1 = x_0$   
 $x_0 \leftarrow$



Declares an array, 'x', of 16 elements and initialize all elements to 0



Using for loop, the array is shifted to the right.



First element of 'x[0]' is replaced with a 'new\_element'

# Programming Review - C

## Linear Buffers and Circular Buffers

- Example: Circular Buffer Shifting

```

float32_t x[16] = {0};
int32_t position = 0;
...
void circular_buffer_shift(float32_t new_element)
{
    position = (position-1) % 16
    x[position] = new_element
}
    
```

0 15  
 14  
 13  
 .  
 .  
 .  
 1  
 0



Declares an array, 'x', of 16 elements and initialize all elements to 0



'position' is the index of the buffer for inserting new element



Using modulo operation, update the index



Element of 'x[position]' is replaced with a 'new\_element'

# Programming Review - Matlab

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## Data Types in MATLAB

- By default, every variable in MATLAB is a 2d array of double precision floating point values
- To use another datatype, call the corresponding function (single(), int16(), uint32(), etc)
- View the size and type of variables using the whos() function
- By default, **i** and **j** represent  $\sqrt{-1}$ . Be careful naming variables that overwrite these!
- Data automatically become complex if any multiple of **j** is added, e.g. 1-3j

# Programming Review - Matlab

## Vector and Array Operations

- Matlab Array index starts at '1' unlike C or other languages!

```
A = [1,2,3  
     4,5,6];  
B = [7,8,9  
     3,2,1];  
disp(A);
```

```
1 2 3  
4 5 6
```

```
disp(A(1, 1))
```

```
1
```

```
disp(A(0, 1))
```



Index in position 1 is invalid. Array indices must be positive integers or logical values.

# Programming Review - Matlab

## Vector and Array Operations

- Example: Element-wise product of Two matrices using for loops

```
A = [1,2,3
     4,5,6];
B = [7,8,9
     3,2,1];

for i_row = 1:2
    for i_col = 1:3
        C(i_row,i_col) = A(i_row,i_col) * B(i_row,i_col);
    end
end

disp(C)
```

```
7    16    27
12    10     6
```

# Programming Review - Matlab

## Vector and Array Operations

- Example: Simpler Element-wise product of Two matrices

$$A \ 2 \times 3$$

$$B \ 2 \times 3$$

```
A = [1,2,3
     4,5,6];
B = [7,8,9
     3,2,1];

C_2 = A .* B; % .* returns the element-wise product of two matrices

disp(C_2)
```

```
7 16 27
12 10 6
```

$$\underline{A^2}$$

$$B'$$

$$* \ [M \times N] * [N \times M]$$

```
C_3 = A * B'; % Transpose using ' will allow Matrix Multiplication
disp(C_3)
```

```
50 10
122 28
```

$$2 \times 3 \Rightarrow 3 \times 2$$

```
C_4 = A * B; % Using * on matrices will perform Matrix Multiplication
```

Error using `*`  
 Incorrect dimensions for matrix multiplication. Check that the number of columns in the first matrix matches the number of rows in the second matrix. To operate on each element of the matrix individually, use `TIMES (.*)` for elementwise multiplication.

### Related documentation

```
% Note that this operation is impossible as the dimensions are incorrect.
% A = 2 x 3 , B = 2 x 3
```

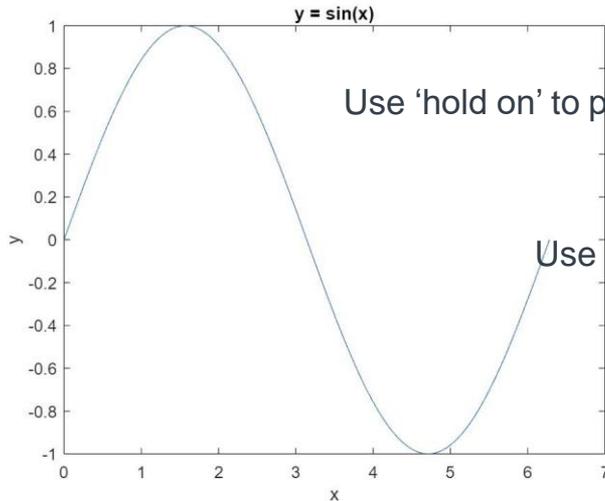
# Programming Review - Matlab

## Creating Plots in Matlab

```
x = linspace(0,2*pi);
y1 = sin(x);
plot(x,y1)
title('y = sin(x)')
xlabel('x')
ylabel('y')
```

→ Use 'plot' to represent continuous signals

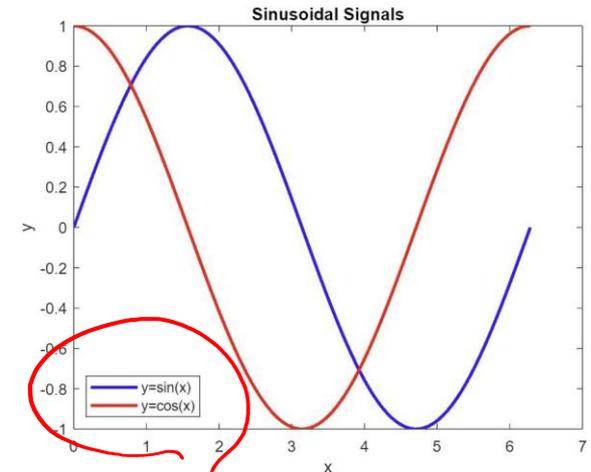
→ Use 'title', 'xlabel', 'ylabel' to label plots and axes



Use 'hold on' to put multiple plots on one figure

Use 'legend' to add legend to plots

```
plot(x,y1,'LineWidth', 2, 'Color','b')
hold on
plot(x,y2, 'LineWidth', 2, 'Color','r')
hold off
legend('y=sin(x)','y=cos(x)', 'Location','southwest')
title('Sinusoidal Signals')
xlabel('x')
ylabel('y')
```



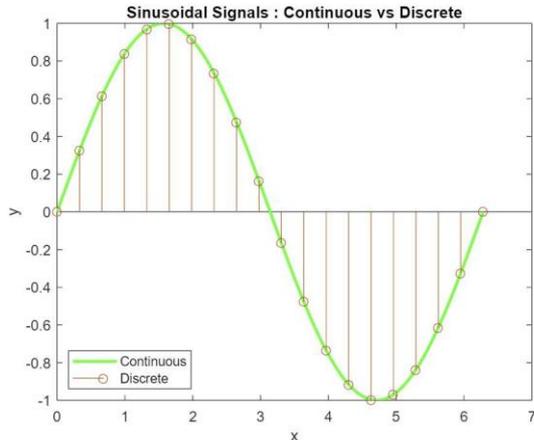
# Programming Review - Matlab

## Creating Plots in Matlab

```
x2 = linspace(0,2*pi,20);  
y3 = sin(x2);  
plot(x, y1, 'LineWidth', 2, 'Color', 'g');  
hold on  
stem(x2, y3)  
hold off  
legend('Continuous','Discrete', 'Location','southwest')  
title('Sinusoidal Signals : Continuous vs Discrete')  
xlabel('x')  
ylabel('y')
```



Use 'stem' to represent discrete signals



Use the incantation `set(0, 'DefaultFigureWindowStyle', 'Docked')` to keep all figures in the same window but make an new tab for each

**If you are unsure of the syntax of functions inside Matlab, search at [Matlab Support](#). They have a very detailed description of functions in their website with examples.**

# Programming Review - Floating Point

## Floating - Point

### Why do we need a floating point?

To represent **real numbers / decimal fractions** with a **wide range of magnitudes**. Broader range with fractional parts.

Floating-point numbers provide **flexibility** and a **wide range of representations**, along with **efficient computational compatibility with hardware**s.

### Two Main parts of Floating-Point Numbers

1. **Significand (Mantissa)** : Digits of Number
2. **Exponent** : Where the decimal point is placed.

3. sign :  $\pm$

# Programming Review - Floating Point

## Floating - Point

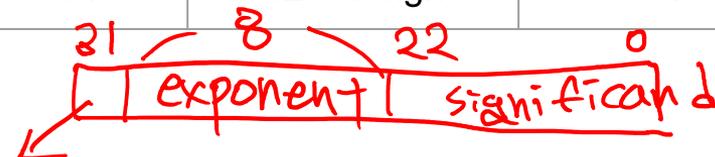
Significand	Exponent	Scientific Notation	Fixed-Point Representation
1.5	4	$1.5 * 10^4$	15000
- 2.001	2	$- 2.001 * 10^2$	-200.1
5	-3	$5 * 10^{-3}$	0.005
6.667	-11	6.667e-11	0.00000000006667

- 1) <https://floating-point-gui.de/formats/fp/>

# Programming Review - Floating Point

## Floating - Point

Format	Total Bits	Significand Bits	Exponent Bits	Smallest Number	Largest Number
Single	32	23 + 1 sign	8	$1.2 * 10^{-38}$	$3.4 * 10^{38}$
Double	64	52 + 1 sign	11	$2.2 * 10^{-308}$	$1.8 * 10^{308}$



$$Value = (-1)^{\text{sign}} \times 2^{(E-127)} \times \left( 1 + \sum_{i=1}^{23} b_{23-i} 2^{-i} \right)$$

- 1) <https://floating-point-gui.de/formats/fp/>

# Programming Review - Floating Point

## Floating - Point

- **Integer and other fixed-point data types** have the **same spacing** between all possible values
  - Useful for representing natural numbers or the value read from ADC
- **Distance between adjacent floating point numbers is variable**
  - Useful for representing filter coefficients
  - Less build up of error for sequences of operations compared to fixed point
- Single precision (near 1.0) is roughly equivalent to 8 decimal points
  - Largest possible value is about  $3.4 \times 10^{38}$
  - Near zero, the smallest distance between values is about  $1.4 \times 10^{-45}$

# Linear Systems and Signals Review

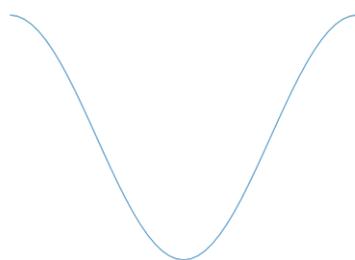
## Sampling and Reconstruction

- **Sampling (continuous-time to discrete-time conversion)**
  - $x[n] = x(nT_s)$
- **Reconstruction (discrete-time to continuous-time conversion)**
  - A continuous-time signal  $x(t)$  with frequencies no higher than  $f_{max}$  can be reconstructed exactly from its samples  $x[n] = x(nT_s)$  if samples are taken at a sampling rate  $f_s > 2f_{max}$

Shannon - Nyquist

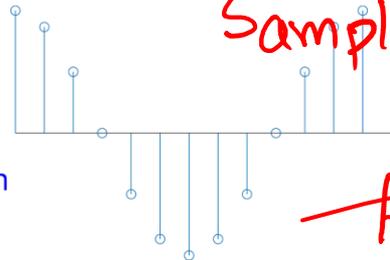
Sampling Theorem.

~~Aliasing~~



sampling

reconstruction



# Linear Systems and Signals Review

## Transforms

Preview of the Different Transforms we will use in this class

- Laplace Transform
- Z-Transform
- Fourier Transform
- Discrete-Time Fourier Transform (DTFT)
- Fourier Series
- Discrete Fourier Transform (DFT)

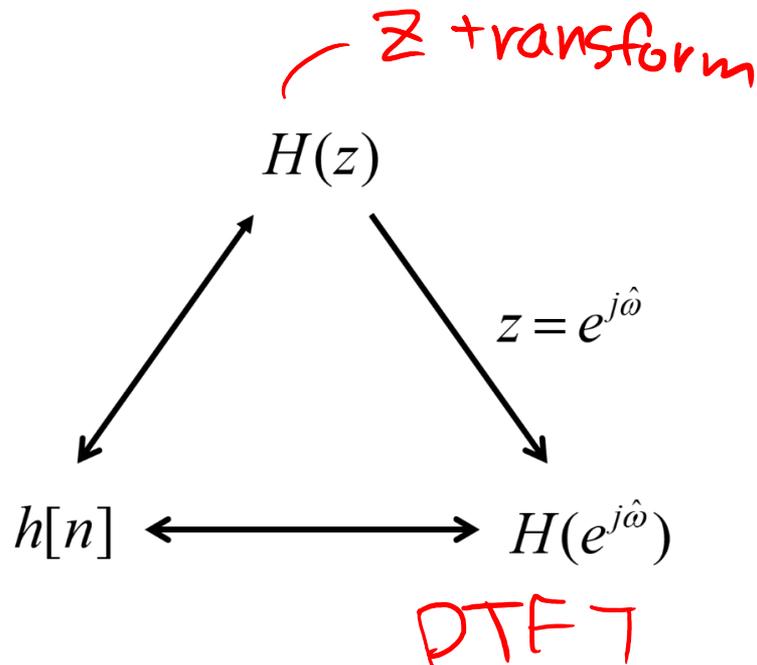
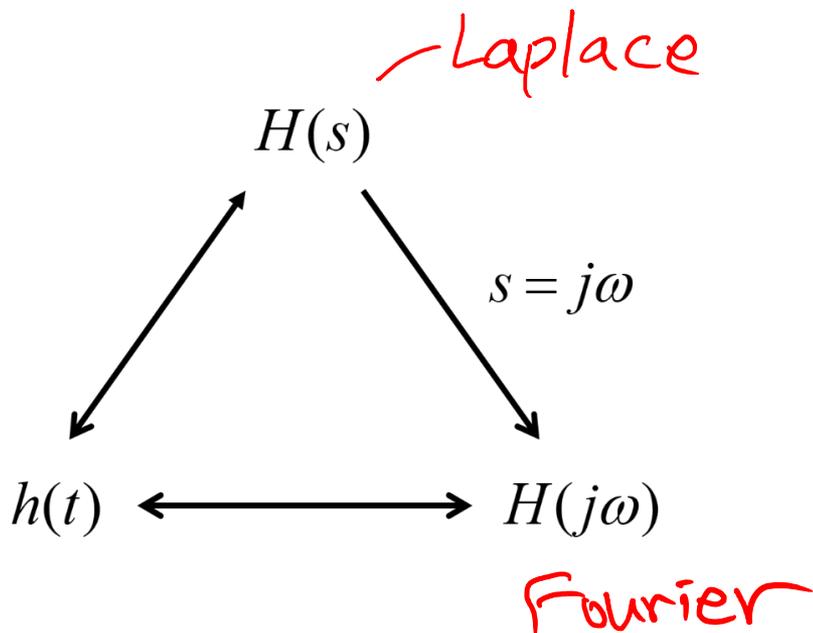
$$\text{rect}(t) \quad \text{sinc}(f)$$

All of these have certain properties in common (with a few caveats)

- Multiplication  $\leftrightarrow$  Convolution
- Stretch  $\leftrightarrow$  Contract

# Linear Systems and Signals Review

## Transforms



# Linear Systems and Signals Review

## Fourier Transform

### Basic Properties of Fourier Transform

#### 1. Linearity

$$a x(t) + b y(t) \iff a X(\omega) + b Y(\omega)$$

#### 2. Time Shifting

$$y(t) = x(t - t_0) \iff Y(\omega) = e^{-j\omega t_0} X(\omega)$$

#### 3. Frequency Shifting

$$e^{j\omega_0 t} x(t) \iff X(\omega - \omega_0)$$

#### 4. Time Scaling

$$x(at) \iff \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$$

$$1 \xrightarrow{f} \delta(f)$$

$$2. \cos at \Rightarrow \frac{1}{2} \delta\left(f - \frac{a}{2\pi}\right) + \frac{1}{2} \delta\left(f + \frac{a}{2\pi}\right)$$

$$3. \sin at \Rightarrow \frac{1}{2j} \delta\left(f - \frac{a}{2\pi}\right) - \frac{1}{2j} \delta\left(f + \frac{a}{2\pi}\right)$$

$$4. \text{rect}(t) \xrightarrow{f} \text{sinc}(f)$$

$$\delta(f - f_0) * X(f) = X(f - f_0)$$

# Linear Systems and Signals Review

## Fourier Transform

Example 1. What is a Fourier Transform of

$$x(t) = (1 + \cos(2\pi t)) \text{rect}(t)$$

$$\mathcal{F}(x(t)) = \mathcal{F}(1 + \cos 2\pi t) * \mathcal{F}(\text{rect}(t))$$

$$\left\{ \mathcal{F}(1) + \mathcal{F}(\cos 2\pi t) \right\} * \text{sinc}(f)$$

$$\left\{ \delta(f) + \frac{1}{2}\delta(f-1) + \frac{1}{2}\delta(f+1) \right\} * \text{sinc}(f)$$

$$= \text{sinc}(f) + \frac{1}{2} \text{sinc}(f-1) + \frac{1}{2} \text{sinc}(f+1)$$

# Linear Systems and Signals Review

## Fourier Transform

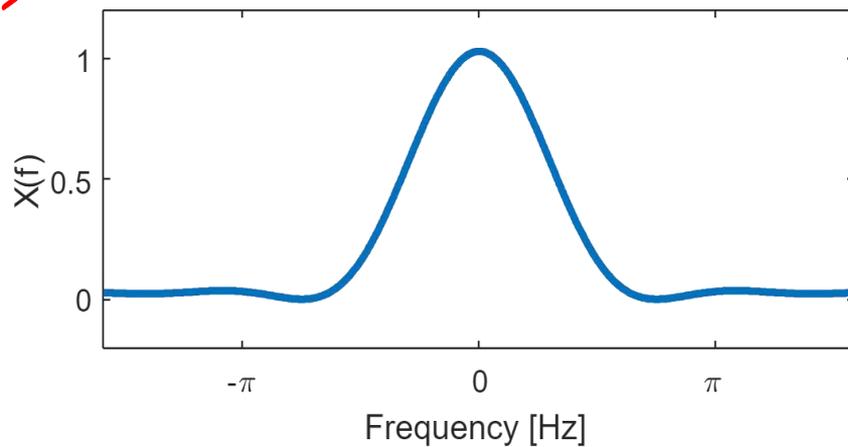
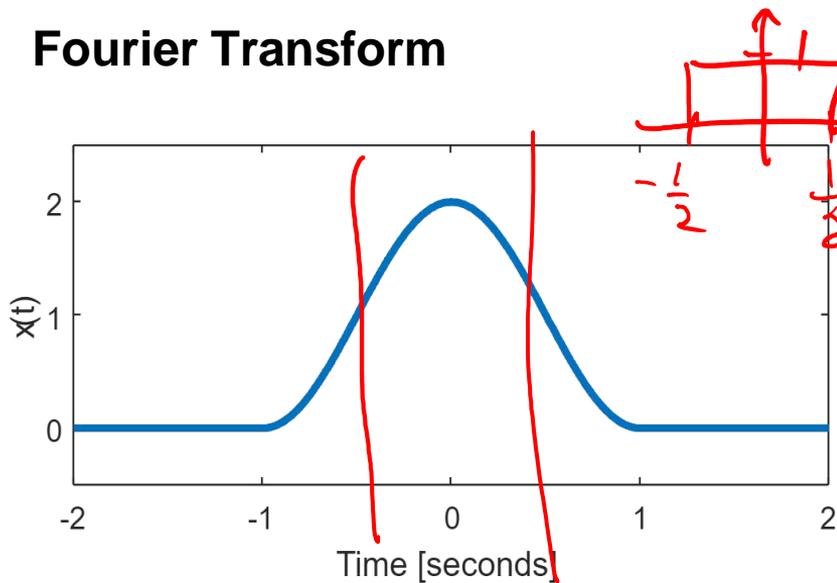
Example 1. What is a Fourier Transform of

$$x(t) = (1 + \cos(2\pi t)) (\text{rect}(t))$$

$$\begin{aligned}\mathcal{F}(x(t)) &= \mathcal{F}((1 + \cos(2\pi t)) (\text{rect}(t))) \\ &= \mathcal{F}(1 + \cos(2\pi t)) * \mathcal{F}(\text{rect}(t)) \\ &= \{\mathcal{F}(1) + \mathcal{F}(\cos(2\pi t))\} * \mathcal{F}(\text{rect}(t)) \\ &= \left\{ \delta(f) + \frac{1}{2} \delta(f - 1) + \frac{1}{2} \delta(f + 1) \right\} * \text{sinc}(f) \\ &= \frac{1}{2} \text{sinc}(f - 1) + \text{sinc}(f) + \frac{1}{2} \text{sinc}(f + 1)\end{aligned}$$

# Linear Systems and Signals Review

## Fourier Transform



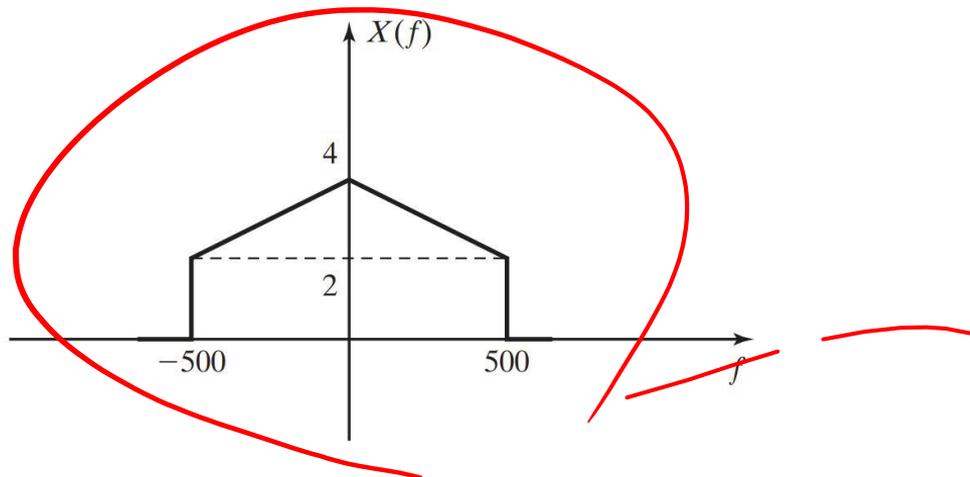
$$x(t) = (1 + \cos(2\pi t)) \text{rect}(t)$$

$$\mathcal{F}(x(t)) = \frac{1}{2} \text{sinc}(f - 1) + \text{sinc}(f) + \frac{1}{2} \text{sinc}(f + 1)$$

# Linear Systems and Signals Review

## Fourier Transform

**Example 2.** The Fourier transform of a signal is as the following figure. Determine and sketch the Fourier transform of the signal  $x_1(t) = -x(t) + x(t) \cos(3000\pi t) + 2x(t) \cos^2(5000\pi t)$



# Linear Systems and Signals Review

## Fourier Transform

Example 2.

$$x_1(t) = -x(t) + x(t) \cos(3000\pi t) + 2x(t) \cos^2(5000\pi t)$$

$$\mathcal{F}(x_1(t)) = \underbrace{-\mathcal{F}(x(t))} + \underbrace{\mathcal{F}(x(t) \cdot \cos(3000\pi t))} + \underbrace{2 \cdot \mathcal{F}(x(t) \cdot \cos^2(5000\pi t))}$$

$$= -X(f) + \underbrace{\mathcal{F}(X(f)) * \mathcal{F}(\cos 3000\pi t)}$$



$$+ 2 \mathcal{F}(X(f)) * \mathcal{F}(\cos^2 5000\pi t)$$

$$\underbrace{\frac{X(f-1500)}{2} + \frac{X(f+1500)}{2}}$$

$$\cos^2 a\pi t = \frac{1 + \cos 2a\pi t}{2}$$

# Linear Systems and Signals Review

## Fourier Transform

**Example 2.**  $x_1(t) = -x(t) + x(t) \cos(3000\pi t) + 2x(t) \cos^2(5000\pi t)$

$$\begin{aligned} x_1(t) &= -x(t) + \cos(3000\pi t)x(t) + 2\cos^2(5000\pi t)x(t) \\ &= -x(t) + \cos(3000\pi t)x(t) + (1 + \cos(10000\pi t))x(t) \end{aligned}$$

$$\begin{aligned} \mathcal{F}(x_1(t)) &= \mathcal{F}(-x(t)) + \mathcal{F}(\cos(3000\pi t)x(t)) + \mathcal{F}((1 + \cos(10000\pi t))x(t)) \\ &= -X(f) + \mathcal{F}(\cos(3000\pi t)) * X(f) + \mathcal{F}(1 + \cos(10000\pi t)) * X(f) \\ &= -X(f) + \left\{ \frac{1}{2}\delta(f - 1500) + \frac{1}{2}\delta(f + 1500) \right\} * X(f) + \mathcal{F}(1 + \cos(10000\pi t)) * X(f) \\ &= -X(f) + \frac{1}{2}X(f - 1500) + \frac{1}{2}X(f + 1500) + \left\{ \delta(f) + \frac{1}{2}\delta(f - 5000) + \frac{1}{2}\delta(f + 5000) \right\} * X(f) \\ &= -\cancel{X(f)} + \frac{1}{2}X(f - 1500) + \frac{1}{2}X(f + 1500) + \cancel{X(f)} + \frac{1}{2}X(f - 5000) + \frac{1}{2}X(f + 5000) \\ &= \frac{1}{2}X(f - 1500) + \frac{1}{2}X(f + 1500) + \frac{1}{2}X(f - 5000) + \frac{1}{2}X(f + 5000) \end{aligned}$$

# Linear Systems and Signals Review

## Fourier Transform

Example 2.

