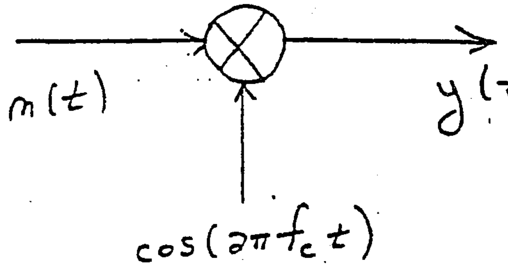
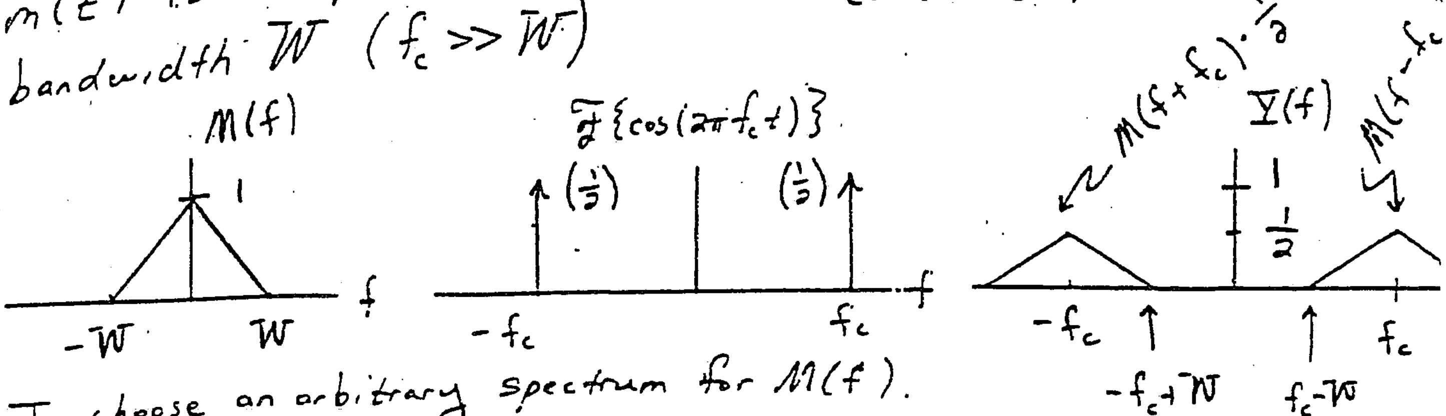
Amplitude Modulation with $\cos(2\pi f_c t)$



$$y(t) = m(t)\cos(2\pi f_c t)$$

$m(t)$ is lowpass with bandwidth W ($f_c >> W$)



I choose an arbitrary spectrum for $M(f)$.

$$Y(f) = M(f) * \mathfrak{F}\{\cos(2\pi f_c t)\}$$

$$Y(f) = M(f) * \tfrac{1}{2}\left(\delta(f+f_c) + \delta(f-f_c)\right)$$

Example uses the normalized Fourier transform

$$Y(f) = \frac{1}{2}\int_{-\infty}^{\infty} \left(\underbrace{\delta(\lambda+f_c)} + \underbrace{\delta(\lambda-f_c)}\right) M(f-\lambda)\, d\lambda$$

Deltas are non-zero @ $\lambda = -f_c$ and $\lambda = f_c$

$$Y(f) = \frac{1}{2}\left(M(f+f_c) + M(f-f_c)\right)$$

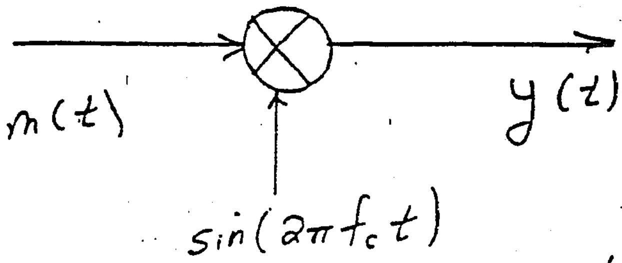
It is easier to work with amplitude modulation to draw pictures in the frequency domain when analyzing the resulting spectrum for amplitude modulation.

Recall that multiplication in time becomes convolution in the frequency domain.

The bandwidth of $Y(f)$ is $2W$

Amplitude Modulation with $\sin(2\pi f_c t)$

$y(t) = m(t)\sin(2\pi f_c t)$



As before, $m(t)$ is lowpass with bandwidth W and $f_c >> W$.

Example uses the normalized Fourier transform

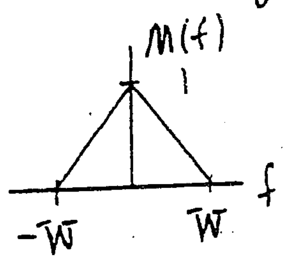
$$Y(f) = M(f) * \mathcal{F}\{\sin(2\pi f_c t)\}$$

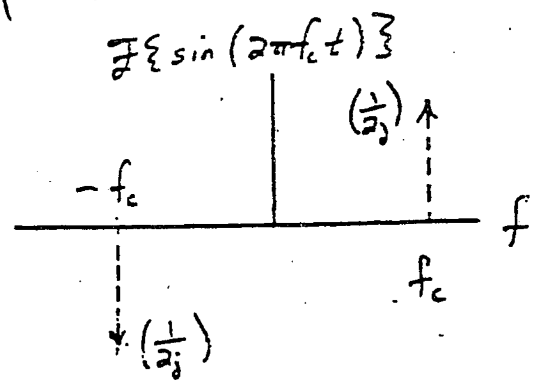
$$Y(f) = M(f) * \frac{1}{2j}\left(-\delta(f+f_c) + \delta(f-f_c)\right)$$

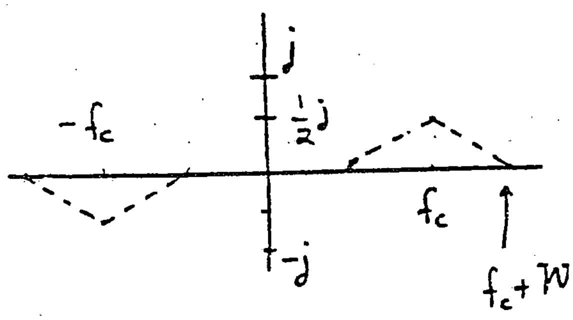
$$Y(f) = \frac{1}{2j}\int_{-\infty}^{\infty}\left(-\delta(\underbrace{\lambda+f_c}) + \delta(\underbrace{\lambda-f_c})\right) M(f-\lambda)\,d\lambda$$

Deltas are non-zero at $\lambda = -f_c$ and $\lambda = f_c$

$$Y(f) = \frac{1}{2j}\left(-M(f+f_c) + M(f-f_c)\right)$$







So, the area under the Dirac delta functionals for $\mathcal{F}\{\sin(2\pi f_c t)\}$ is $-\frac{1}{2j}$, or $+\frac{1}{2j}$, respectively.

So, in the time domain, $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$ are orthogonal. In the frequency domain, $\cos(2\pi f_c t)$ becomes real-valued, and $\sin(2\pi f_c t)$ becomes imaginary.

The bandwidth of $Y(f)$ is $2W$.