EE445S Real-Time Digital Signal Processing Laboratory

Handout I: Analog Sinusoidal Modulation Summary

Many ways exist to modulate a message signal $m(t)$ to produce a modulated (transmitted) signal $x(t)$. For amplitude, frequency, and phase modulation, modulated signals can be expressed in the same form as

$$x(t) = A(t)\cos(2\pi f_c t + \Theta(t))$$

where $A(t)$ is a real-valued amplitude function (a.k.a. the envelope), f_c is the carrier frequency, and $\Theta(t)$ is the real-valued phase function. Using this framework, several common modulation schemes are described below. In the table below, the amplitude modulation methods are double sideband larger carrier (DSB-LC), DSB suppressed carrier (DSB-SC), DSB variable carrier (DSB-VC), and single sideband (SSB). The hybrid amplitude-frequency modulation is quadrature amplitude modulation (QAM). The angle modulation methods are phase and frequency modulation.

Modulation	$A(t)$	$\Theta(t)$	Carrier	Type	Use
DSB-LC	$A_c\,[1 + k_a m(t)]$	Θ_0	Yes	Amplitude	AM radio
DSB-SC	$A_c m(t)$	Θ_0	No	Amplitude	
DSB-VC	$A_c m(t) + \epsilon$	Θ_0	Yes	Amplitude	
SSB	$A_c\sqrt{m^2(t) + [m(t) \star h(t)]^2}$	$\arctan(-\frac{m(t)\star h(t)}{m(t)})$	No	Amplitude †	Marine radios
QAM	$A_c\sqrt{m_1^2(t) + m_2^2(t)}$	$\arctan(-\frac{m_2(t)}{m_1(t)})$	No	Hybrid	Satellite
Phase	A_c	$\Theta_0 + k_p\, m(t)$	No	Angle	Underwater modems
Frequency	A_c	$2\pi k_f \int_0^t m(t)\, dt$	No	Angle	FM radio TV audio

† $h(t)$ is the impulse response of a bandpass filter or phase shifter to effect a cancellation of one pair of redundant sidebands. For ideal filters and phase shifters, the modulation is amplitude modulation because the phase would not carry any information about $m(t)$.

Each analog TV channel is allocated a bandwidth of 6 MHz. The picture intensity and color information are transmitted using vestigal sideband modulation. Vestigal sideband modulation is a variant of amplitude modulation (not shown above) in which the upper sideband is kept and a fraction of the lower sideband is kept, or vice-versa. In an analog TV signal, the audio portion is frequency modulated.

The following quantity is known as the complex envelope

$$\tilde{x}(t) = A(t)\, e^{j\,\Theta(t)} = x_I(t) + j\, x_Q(t)$$

where $x_I(t)$ is called the in-phase component and $x_Q(t)$ is called the quadrature component. Both $x_I(t)$ and $x_Q(t)$ are lowpass signals, and hence, the complex envelope $\tilde{x}(t)$ is a lowpass signal. An alterative representation for the modulated signal $x(t)$ is

$$x(t) = \Re e\{\tilde{x}(t)\, e^{j 2\pi f_c t}\}$$