EE445S Real-Time Digital Signal Processing Laboratory

Handout G: Raised Cosine Pulse

Section 7.5, pp. 431–434, Simon Haykin, *Communication Systems*, 4th ed.

We may overcome the practical difficulties encounted with the ideal Nyquist channel by extending the bandwidth from the minimum value $W = R_b/2$ to an adjustable value between W and $2W$. We now specify the frequency function $P(f)$ to satisfy a condition more elaborate than that for the ideal Nyquist channel; specifically, we retain three terms of (7.53) and restrict the frequency band of interest to $[-W, W]$, as shown by

$$P(f) + P(f - 2W) + P(f + 2W) = \frac{1}{W}, -W \leq f \leq W \tag{1}$$

We may devise several band-limited functions to satisfy (1). A particular form of $P(f)$ that embodies many desirable features is provided by a *raised cosine spectrum*. This frequency characteristic consists of a flat portion and a *rolloff* portion that has a sinusoidal form, as follows:

$$P(f) = \begin{cases} \dfrac{1}{2W} & \text{for } 0 \leq |f| < f_1 \\ \dfrac{1}{4W}\left(1 - \sin\dfrac{\pi(|f| - W)}{2W - 2f_1}\right) & \text{for } f_1 \leq |f| < 2W - f_1 \\ 0 & \text{for } |f| \geq 2W - f_1 \end{cases} \tag{2}$$

The frequency parameter f_1 and bandwidth W are related by

$$\alpha = 1 - \frac{f_1}{W} \tag{3}$$

The parameter α is called the *rolloff factor*; it indicates the *excess bandwidth* over the ideal solution, W. Specifically, the transmission bandwidth B_T is defined by $2W - f_1 = W(1+\alpha)$.

The frequency response $P(f)$, normalized by multiplying it by $2W$, is shown plotted in Fig. 1 for three values of α, namely, 0, 0.5, and 1. We see that for $\alpha = 0.5$ or 1, the function $P(f)$ cuts off gradually as compared with the ideal Nyquist channel (i.e., $\alpha = 0$) and is therefore easier to implement in practice. Also the function $P(f)$ exhibits odd symmetry with respect to the Nyquist bandwidth W, making it possible to satisfy the condition of (1). The time response $p(t)$ is the inverse Fourier transform of the function $P(f)$. Hence, using the $P(f)$ defined in (2), we obtain the result (see Problem 7.9)

$$p(t) = \text{sinc}(2Wt)\left(\frac{\cos 2\pi\alpha Wt}{1 - 16\alpha^2W^2t^2}\right) \tag{4}$$

which is shown plotted in Fig. 2 for $\alpha = 0$, 0.5, and 1. The function $p(t)$ consists of the product of two factors: the factor $\text{sinc}(2Wt)$ characterizing the ideal Nyquist channel and a second factor that decreases as $1/|t|^2$ for large $|t|$. The first factor ensures zero crossings of $p(t)$ at the desired sampling instants of time $t = iT$ with i an integer (positive and negative). The second factor reduces the tails of the pulse considerably below that obtained from the ideal Nyquist channel, so that the transmission of binary waves using such pulses is relatively insensitive to sampling time errors. In fact, for $\alpha = 1$, we have the most gradual rolloff in that the amplitudes of the oscillatory tails of $p(t)$ are smallest. Thus, the amount of intersymbol interference resulting from timing error decreases as the rolloff factor α is increased from zero to unity.

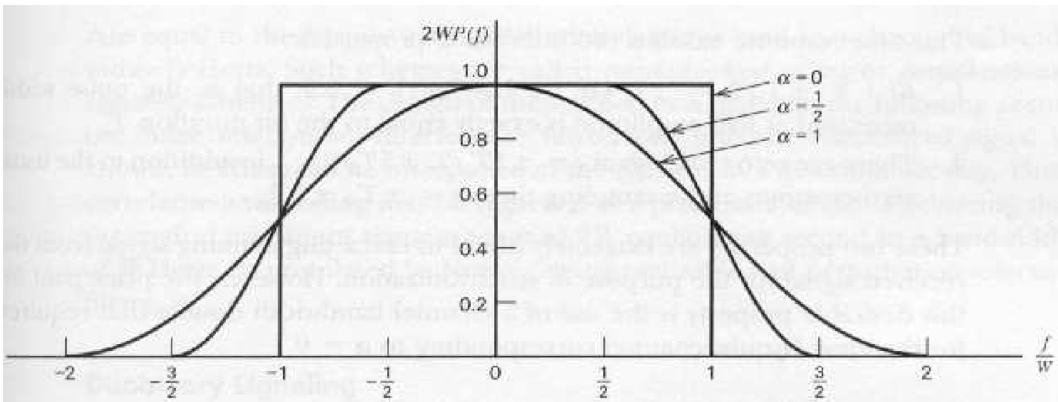

Figure 1: Frequency response for the raised cosine function.

The special case with $\alpha = 1$ (i.e., $f_1 = 0$) is known as the *full-cosine rolloff* characteristic, for which the frequency response of (2) simplifies to

$$P(f) = \begin{cases} \frac{1}{4W}\left(1 + \cos\frac{\pi f}{2W}\right) & \text{for } 0 < |f| < 2W \\ 0 & \text{if } |f| \geq 2W \end{cases}$$

Correspondingly, the time response $p(t)$ simplifies to

$$p(t) = \frac{\text{sinc}(4Wt)}{1 - 16W^2t^2} \tag{5}$$

The time response exhibits two interesting properties:

1. At $t = \pm T_b/2 = \pm 1/4W$, we have $p(t) = 0.5$; that is, the pulse width measured at half amplitude is exactly equal to the bit duration T_b.

2. There are zero crossings at $t = \pm 3T_b/2$, $\pm 5T_b/2$,... in addition to the usual zero crossings at the sampling times $t = \pm T_b, \pm 2T_b, \ldots$

These two properties are extremely useful in extracting a timing signal from the received signal for the purpose of synchronization. However, the price paid for this desirable property is the use of a channel bandwidth double that required for the ideal Nyquist channel corresponding to $\alpha = 0$.

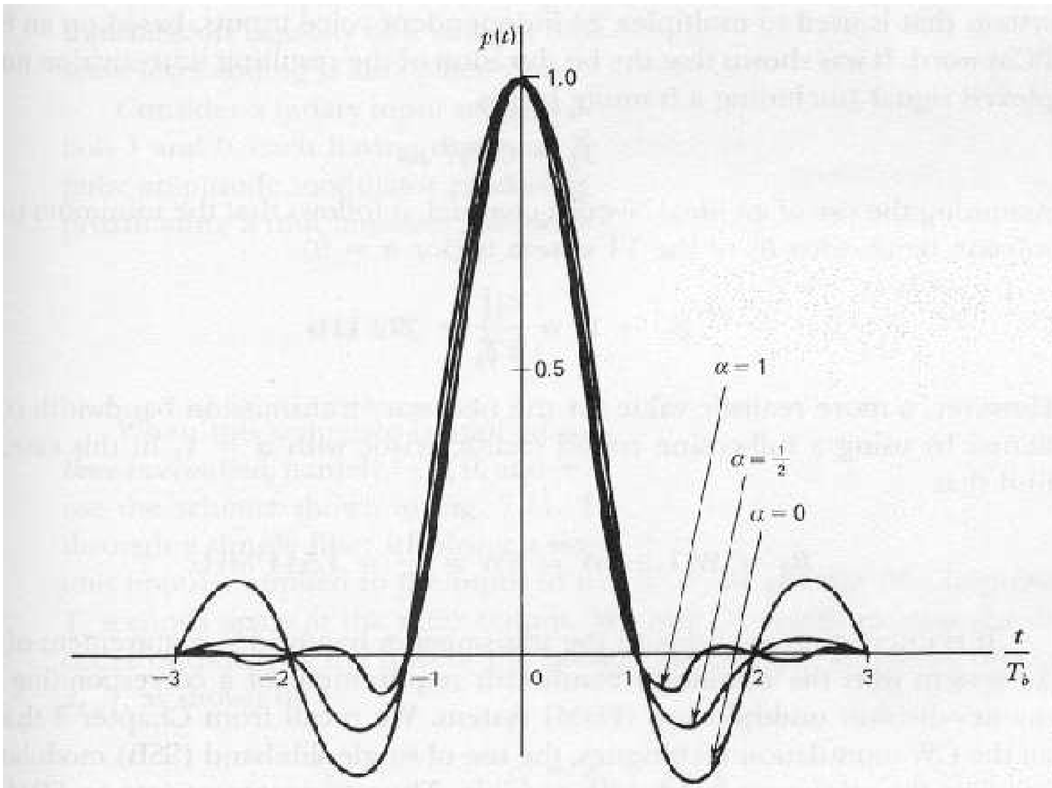

Figure 2: Time response for the raised cosine function.

Placeholder – please ignore.