Homework #0 Review of Signals and Systems Material

Assigned Monday, January 13, 2025, and due Friday, January 24, 2025, by 11:59pm *Late homework is subject to a penalty of two points per minute late.*

Through midterm #1, here are key sections from JSK's *Software Receiver Design*, and signals & systems textbooks Oppenheim & Willsky's *Signals & Systems* (2nd ed); McClellan, Schafer & Yoder's *Signal Processing First* and Lathi & Green's *Linear Systems & Signals* (3rd ed). **Bold are topics for this homework**.

Торіс	Lect.	JSK	O&W	SP First	L&G	Handouts
Introduction	0	Ch. 1				
Bandwidth	1	2.2	4.3-4.4	11-4 to 11- 8	6.3-1, 7.2, 7.3, 7.9	Fourier Transforms: <u>Intro</u> , <u>Dictionary</u> and <u>w vs. f</u>
Sinusoidal generation	1	3.2				
Upconversion and downconversion	<u>1</u> & <u>4</u>	2.3-2.6, 3.6; Ch. 5; 6.1-6.4	8.1-8.4	11-8.2	7.3, 7.7	Sinusoidal Ampl. Modulation Example & Summary
Communication sys.	1	Ch. 1-2	Ch. 8		7.7	
Basic CT signals	3	2.10, 4.3	1.3-1.4	2-3, 2-5, 4- 4 & 9-1	1.4	Common Signals in Matlab
CT system properties	<u>3</u>		1.6	5-5	1.7	LTI System Properties
Basic DT signals	<u>3</u>		1.3-1.4	4-2.1 & 5- 3.2	3.3	<u>Common Signals in Matlab;</u> <u>DT Periodicity; Chirp Signals</u>
DT system properties	<u>3</u>		1.6	9-4	3.4-1	Time-Invariance; LTI System Properties
Fundamental Theorem	<u>3</u> & <u>5</u>	3.5	3.2	6-1 & 10-1	2.4-4 &	Fundamental Theorem of
of Linear Systems					3.8-2	Linear Systems
Sampling theorem	<u>4</u>		7.1	4-1 4-2 4-5	8.1	
Sampling and aliasing	<u>4</u>	2.8, 3.4, 6.1	7.3 & 7.4	12-3	8.2	Sampling Unit Step Signal
Bandpass sampling	<u>4</u>					
DT-to-CT conversion	<u>4</u>	2.10 & 6.4	7.2	4-4	8.2	
CT convolution	<u>5</u>	4.4 & 4.5	2.2	9-6 & 9-7	2.4	Convolution Example
DT convolution	<u>5</u>	4.4 & 4.5	2.1	5-3.3 & 5-6	3.8	<u>Convolution Example</u> and <u>Four ways to filter a signal</u>
Z-transforms	<u>5</u> & <u>6</u>	App. A.4 & F	10.1-10.3, 10.5	7-1 to 7-5	5.1-5.2	
Transfer functions	<u>5</u> & <u>6</u>	4.5	10.7 & 10.8	8-3, 8-4 & 8-9	5.3	Designing Averaging Filters; LTI Systems & Freq. Resp.
Relationship between z & Fourier transform	<u>5</u>	App. F.2	10.4	7-6, 8-5, 8-6, 8-10	5.5	
DT FIR filter design and implementation	<u>5</u>	2.12, 3.3, 4.2	10-9	7-7	5.4	Four ways to filter a signal; Designing Averaging Filters
DT FIR filter analysis	<u>5</u>	Ch. 7 & App.G		7-7 to 7-9		Designing Averaging Filters
DT stability of LTI systems	<u>5</u> & <u>6</u>		10.7.2	8-2.4, 8- 4.2, 8-8	3.9 & 3.10	Bounded-Input Bounded- Output Stability
DT IIR filter design by pole-zero placement	<u>6</u>		10.4	8-9 & 8-10	5.6	All-pass Filters
DT classical IIR filter design methods	<u>6</u>				5.10	Elliptic IIR filter design
IIR filter implement.	<u>6</u>		10-9	8-9	5.4	Realizations of IIR Filters

Office hours for the teaching assistants and Prof. Evans; **bold** indicates a 30-minute timeslot. Prof. Evans' weekly coffee hours are on Fridays 12-2pm. The TA office hours start the week of Sept. 2nd.

Time Slot	Monday	Tuesday	Wednesday	Thursday	Friday
10:30am	Evans		Evans		
	(ECJ 1.312)		(ECJ 1.312)		
11:00 am	Evans		Evans		
	(ECJ 1.312)		(ECJ 1.312)		
12:00 pm					Evans coffee
					hours (EER Cafe)
1:00 pm			Evans		Evans coffee
			(EER 6.882 &		hours (EER Cafe)
			Zoom)		
2:00 pm			Evans	Barati	
			(EER 6.882 &	(TBA)	
			Zoom)		
3:00 pm				Barati	
				(TBA)	
3:30 pm				Eun	
				(TBA)	
4:00 pm				Eun	Eun
				(TBA)	(EER 1.810)
4:30 pm			Barati	Eun	Eun
			(EER 1.810)	(TBA)	(EER 1.810)
5:00 pm			Barati		Eun
			(EER 1.810)		(EER 1.810)
5:30 pm			Barati		
			(EER 1.810)		

In your solutions, please put all work for problem 1 together, all work for problem 2 together, etc.

Please submit any MATLAB code that you have written with the homework solution. In the course reader, <u>Appendix D</u> introduces MATLAB. Here's are slides on <u>Common Signals in Matlab</u>.

As stated on the <u>course descriptor</u>, "Discussion of homework questions is encouraged. Please be sure to submit your own independent homework solution." Cite all your sources, including ChatGPT or other Generative AI tools if used.

Please read <u>homework hints</u>.

1. Continuous-Time Sinusoidal Generation. 27 points.

In practice, we cannot generate a two-sided sinusoid $\sin(2 \pi f_c t)$, nor can we wait until the end of time to observe a one-sided sinusoid $\sin(2 \pi f_c t) u(t)$.

In the lab, we can turn on a signal generator for a short time and observe the output in the time domain on an oscilloscope or in the frequency domain using a spectrum analyzer.

Consider a finite-duration sine that is on from 0 sec to 1 sec given by the equation

 $c(t) = \sin(2 \pi f_c t) \operatorname{rect}(t - \frac{1}{2})$

where f_c is the carrier frequency (in Hz).

- (a) Using MATLAB, plot c(t) for -0.5 < t < 1.5 for $f_c = 10$ Hz. Turn in your code and plot. If you use MATLAB, you may find the rectpuls command useful. 6 points. Give a formula for the Fourier transform of c(t) for a general value of f_c . 6 points.
- (b) Sketch by hand the magnitude of the Fourier transform of c(t) for a general value of f_c . Using MATLAB, plot the magnitude of the Fourier transform of c(t) for $f_c = 10$ Hz. Turn in your code and plot. 9 points.
- (c) Describe the differences between the magnitude of the Fourier transforms of c(t) and a two-sided sine of the same frequency. What is the bandwidth of each signal? 6 points.

2. Downconversion. 19 points.

A signal x(t) is input to a mixer to produce the output y(t) where

$$y(t) = x(t) \cos(\omega_0 t)$$

where $\omega_0 = 2 \pi f_0$ and $f_0 = 5$ kHz. A block diagram of the mixer is shown below on the right. The Fourier transform of x(t) is shown below on the left.



- (a) Using Fourier transform properties, derive an expression for Y(f) in terms of X(f). 6 points.
- (b) Sketch Y(f) vs. f. Label all important points on the horizontal and vertical axes. 6 points.
- (c) What operation would you apply to the signal y(t) in part (b) to obtain a baseband signal? The process of extracting the baseband signal from a bandpass signal is known as *downconversion*. 7 points.

A mixer is a cascade of a sampling circuit operating at sampling rate f_0 and your answer in (c).

3. Sampling in Continuous Time. 24 points.

Sampling the amplitude of an analog, continuous-time signal f(t) every T_s seconds can be modeled in continuous time as

$$y(t) = f(t) p(t)$$

where p(t) is the impulse train defined by

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

 T_s is known as the sampling duration. The Fourier series expansion of the impulse train is

Please read the homework hints

$$p(t) = \frac{1}{T_s} \left(1 + 2\cos(\omega_s t) + 2\cos(2\omega_s t) + \dots \right)$$

where $\omega_s = 2 \pi / T_s$ is the sampling rate in units of radians per second.

- (a) Plot the impulse train $p(t) = \sum_{n=-\infty}^{\infty} \delta(t nT_s)$. 6 points.
- (b) Note that in part (a), p(t) is periodic. What is the period? 6 points.
- (c) Using the Fourier series representation of p(t) given above, please give a formula for $P(\omega)$, which is the Fourier transform of p(t). Express your answer for $P(\omega)$ as an impulse train in the Fourier domain. 6 points.
- (d) What is the spacing of adjacent impulses in the impulse train in $P(\omega)$ with respect to frequency ω in rad/s? 6 points.

4. Discrete-Time Sinusoidal Generation. 30 points.

Consider a causal discrete-time linear time-invariant system with input x[n] and output y[n] being governed by the following difference equation:

$$y[n] = (2 \cos \omega_0) y[n-1] - y[n-2] + x[n] - (\cos \omega_0) x[n-1]$$

The impulse response of the above system is a *causal sinusoid* with discrete-time frequency ω_0 in units of rad/sample. Normally, ω_0 would be in the interval [- π , π). In lab #2, you'll implement the difference equation in C on a programmable digital signal processor for real-time sinusoidal generation.

- (a) Draw the block diagram for this system using add (or summation), multiplication (or gain), and delay blocks. Please label delay blocks with the text z^{-M} to denote a delay of M samples. Use arrowheads to indicate direction of the flow of signals. *6 points*.
- (b) Please state all initial conditions. Please give values for the initial conditions to satisfy the stated system properties. *6 points*.
- (c) Find transfer function equation in the *z*-domain, including the region of convergence. 6 points.
- (d) Compute the inverse z-transform of the transfer function in part (c) to find the impulse response of the system. *6 points*.
- (e) Using MATLAB, plot the impulse response obtained in part (d) for ω_0 equal to 0, π , and a value in the interval (0, π) of your choosing. Turn in your code and plots. *6 points*.