

Homework #2 Filter Analysis, Simulation, and Design

Assigned Wednesday, Feb. 12, 2025, and due Friday, Feb. 21, 2025, by 11:59pm

Homework submitted after 11:59pm is subject to a penalty of 2 points per minute late.

Through midterm #1, here are key sections from JSK's *Software Receiver Design*, and signals & systems textbooks Oppenheim & Willsky's *Signals & Systems* (2nd ed); McClellan, Schafer & Yoder's *Signal Processing First* and Lathi & Green's *Linear Systems & Signals* (3rd ed). **Bold are topics for this homework.**

Topic	Lect.	JSK	O&W	SP First	L&G	Handouts
Introduction	0	Ch. 1				
Bandwidth	1	2.2	4.3-4.4	11-4 to 11-8	6.3-1, 7.2, 7.3, 7.9	Fourier Transforms: Intro , Dictionary and w vs. f
Sinusoidal generation	1	3.2				
Upconversion and downconversion	1&4	2.3-2.6, 3.6; Ch. 5; 6.1-6.4	8.1-8.4	11-8.2	7.3, 7.7	Sinusoidal Ampl. Modulation Example & Summary
Communication sys.	1	Ch. 1-2	Ch. 8		7.7	
Basic CT signals	3	2.10, 4.3	1.3-1.4	2-3, 2-5, 4-4 & 9-1	1.4	Common Signals in Matlab
CT system properties	3		1.6	5-5	1.7	LTI System Properties
Basic DT signals	3		1.3-1.4	4-2.1 & 5-3.2	3.3	Common Signals in Matlab ; DT Periodicity ; Chirp Signals
DT system properties	3		1.6	9-4	3.4-1	Time-Invariance ; LTI System Properties
Fundamental Theorem of Linear Systems	3&5	3.5	3.2	6-1 & 10-1	2.4-4 & 3.8-2	Fundamental Theorem of Linear Systems
Sampling theorem	4		7.1	4-1 4-2 4-5	8.1	
Sampling and aliasing	4	2.8, 3.4, 6.1	7.3 & 7.4	12-3	8.2	Sampling Unit Step Signal
Bandpass sampling	4					
DT-to-CT conversion	4	2.10 & 6.4	7.2	4-4	8.2	
CT convolution	5	4.4 & 4.5	2.2	9-6 & 9-7	2.4	Convolution Example
DT convolution	5	4.4 & 4.5	2.1	5-3.3 & 5-6	3.8	Convolution Example and Four ways to filter a signal
Z-transforms	5&6	App. A.4 & F	10.1-10.3, 10.5	7-1 to 7-5	5.1-5.2	
Transfer functions	5&6	4.5	10.7 & 10.8	8-3, 8-4 & 8-9	5.3	Designing Averaging Filters ; LTI Systems & Freq. Resp.
Relationship between z & Fourier transform	5	App. F.2	10.4	7-6, 8-5, 8-6, 8-10	5.5	
DT FIR filter design and implementation	5	2.12, 3.3, 4.2	10-9	7-7	5.4	Four ways to filter a signal ; Designing Averaging Filters
DT FIR filter analysis	5	Ch. 7 & App.G		7-7 to 7-9		Designing Averaging Filters
DT stability of LTI systems	5&6		10.7.2	8-2.4, 8-4.2, 8-8	3.9 & 3.10	Bounded-Input Bounded-Output Stability
DT IIR filter design by pole-zero placement	6		10.4	8-9 & 8-10	5.6	All-pass Filters
DT classical IIR filter design methods	6				5.10	Elliptic IIR filter design
IIR filter implement.	6		10-9	8-9	5.4	Realizations of IIR Filters

Please read the [homework hints](#)

Office hours for the teaching assistants and Prof. Evans; **bold** indicates a 30-minute timeslot. Prof. Evans' weekly coffee hours are on Fridays 12-2pm.

Time Slot	Monday	Tuesday	Wednesday	Thursday	Friday
10:30am	Evans (ECJ 1.312)		Evans (ECJ 1.312)		
11:00 am	Evans (ECJ 1.312)		Evans (ECJ 1.312)		
12:00 pm					Evans coffee hours (EER Cafe)
1:00 pm			Evans (EER 6.882 & Zoom)		Evans coffee hours (EER Cafe)
2:00 pm			Evans (EER 6.882 & Zoom)	Barati (TBA)	
3:00 pm				Barati (TBA)	
3:30 pm				Eun (TBA)	
4:00 pm				Eun (TBA)	Eun (EER 1.810)
4:30 pm			Barati (EER 1.810)	Eun (TBA)	Eun (EER 1.810)
5:00 pm			Barati (EER 1.810)		Eun (EER 1.810)
5:30 pm			Barati (EER 1.810)		

**** In inclement weather, including being really hot outside, coffee hours will be in the Innovation Center in EER 2.512, which is 20 feet due East of the EER O's Café outdoor seating.**

In your solutions, please put all work for problem 1 together, all work for problem 2 together, etc.

Please submit any MATLAB code that you have written for the homework solution. In your solutions, please put all work for problem 1 together, then all work for problem 2 together, etc.

As stated on the course descriptor, "Discussion of homework questions is encouraged. Please be sure to submit your own independent homework solution."

2.1. Frequency Responses. 24 points.

For each LTI system in problem 1.1 on homework assignment #1,

- plot the pole-zero diagram for the transfer function. **3 points.**
- is the filter bounded-input bounded-output (BIBO) stable? why or why not? **3 points.**
- give a formula for the frequency response. **9 points.**
- plot the magnitude response. **6 points.**
- if the system is BIBO stable, pick the best one of the following choices to describe the frequency selectivity of the filter: lowpass, highpass, bandpass, or bandstop. **3 points.**

You may use the solution set for problem 1.1 in your solution for this problem.

Please read the [homework hints](#)

2.2. Finite Impulse Response Filter Design for Audio Signals. 30 points.

This problem explores ways to process audio signals.

Please download the audio wave file '[twosignals.wav](#)' from the [homework Web site](#).

This audio file is the sum of two audio signals— a gong sound and a bird chirping. The gong sound and the bird chirping occupy different frequency bands. The gong sound is different from the gong file from the Johnson, Sethares and Klein book.

- (a) Plot the spectrum of the 'twosignals' audio track using `plotspec` and `spectrogram`. Approximately what frequency band does the gong sound occupy? Approximately what frequency band does the bird chirp occupy?
- (b) Design an FIR filter using the Parks-McClellan algorithm (a.k.a. Remez Exchange algorithm and Equiripple design algorithm) to extract the gong signal from the 'twosignals' audio track. Then, apply the filter to the 'twosignals' audio track, play back the filter output to validate that the gong signal has been extracted, and plot the filter output using `plotspec`.
- (c) Design an FIR filter using the Parks-McClellan algorithm (a.k.a. Remez Exchange algorithm and Equiripple design algorithm) to extract the bird chirp from the 'twosignals' audio track. Then, apply the filter to the 'twosignals' audio track, play back the filter output to validate that the bird chirp signal has been extracted, and plot the filter output using `plotspec`.
- (d) Take the extracted gong signal in part (b) and perform downsampling by 2. Downsampling by 2 keeps every other sample and discards the others. Here's Matlab code for downsampling a vector `vec` by 2:

```
vecDownsampledBy2 = vec(1:2:length(vec));
```

- Play the downsampled filtered gong signal at the same playback rate as the filtered gong signal. How does it differ from the gong signal extracted in part (b)?
 - Plot the magnitude spectrum of the downsampled filtered gong signal and compare it against the magnitude spectrum of the gong extracted in part (b).
- (e) Take the extracted gong signal in part (b) and perform upsampling by 2. Upsampling by 2 inserts zero after every sample. Here's Matlab code for upampling row vector `vec` by 2:

```
vec = cumsum( ones(1,10) );  
upsampledLength = 2*length(vec);  
vecUpsampledBy2 = zeros(1,upsampledLength);  
vecUpsampledBy2(1:2:upsampledLength) = vec;
```

- Play the upsampled filtered gong signal at the same playback rate as the filtered gong signal. How does it differ from the gong signal extracted in part (b)?
- Plot the magnitude spectrum of the upsampled filtered gong signal and compare it against the magnitude spectrum of the gong extracted in part (b).

For the sanity of others, you might put in a pair of headphones when working this problem.

The `firpm` command stands for finite impulse response design using the Parks-McClellan algorithm. The Parks-McClellan algorithm was proposed by James McClellan in his 1973 PhD dissertation, which he completed at Rice University under the research supervision of Thomas Parks. James McClellan lived in Austin 1981-1987. From 1987 until his retirement in 2020, James McClellan was a faculty member at Georgia Tech. Thomas Parks retired from Cornell University several years ago.

Please read the [homework hints](#)

Downsampling is the removal of samples in a regular fashion. The problem uses downsampling by 2. The first sample and every other sample thereafter is kept, and hence, the second sample and every other sample thereafter is removed. Downsampling by would halve the number of samples. Upsampling is the addition of samples in a regular fashion. After every sample in a signal, upsampling by 2 would add a sample of value zero. Upsampling by 2 would double the number of samples. After midterm #1, we'll make heavy use of upsampling in a communications transmitter and downsampling in a communications receiver

2.3. Finite Impulse Response (FIR) Filter Design for Treatment of Tinnitus Loudness. 46 points.

Tinnitus, a.k.a. “ringing of the ears”, is a symptom due to an underlying condition in the auditory system. It could have resulted from injury, infection, or other causes. People with tinnitus hear a tone, clicking, hiss, roaring or buzzing when no external sound is present [1][2]. The tinnitus sound could be at low, medium or high audible frequencies, and may occur in one ear or both ears. The tinnitus sound might be temporary or chronic. Those suffering from chronic tinnitus would hear the same sound in the same frequency range each time. The tinnitus sound has a principal frequency that can be determined through auditory testing. Hearing sound that contains the principal frequency and frequencies close to the principal frequency is particularly painful.

This problem asks you to design a discrete-time filter to alleviate the loudness of tinnitus:

"Maladaptive auditory cortex reorganization may contribute to the generation and maintenance of tinnitus. Because cortical organization can be modified by behavioral training, we attempted to reduce tinnitus loudness by exposing chronic tinnitus patients to self-chosen, enjoyable music, which was modified (“notched”) to contain no energy in the frequency range surrounding the individual tinnitus frequency. After 12 months of regular listening, the target patient group ($n = 8$) showed significantly reduced subjective tinnitus loudness and concomitantly exhibited reduced evoked activity in auditory cortex areas corresponding to the tinnitus frequency compared to patients who had received an analogous placebo notched music treatment ($n = 8$). These findings indicate that tinnitus loudness can be significantly diminished by an enjoyable, low-cost, custom-tailored notched music treatment, potentially via reversing maladaptive auditory cortex reorganization.” [3]

The proposed treatment for tinnitus [3] alters participants' favorite music to remove an octave of frequencies around the tinnitus frequency f_c . An octave means a range of frequencies from f_l to $2 f_l$. Since f_c would be in the middle of the octave, $f_l = (2/3) f_c$. After 12 months of listening to the filtered music, patients reported lessening of tinnitus loudness.

A good rule of thumb in filter design is that the transition region is about 10% of the passband width. In this case, the passband width is $(2/3) f_c$.

Here are the bandstop filter specifications for your design:

- For frequencies 0 Hz to $0.6 f_c$, the passband ripple should be no greater than 1 dB.
- For frequencies $(2/3) f_c$ to $(4/3) f_c$, the stopband attenuation should be at least 80 dB.
- For frequencies above $1.4 f_c$, the passband ripple should be no greater than 1 dB

Please use a tinnitus frequency f_c of 3000 Hz and a sampling rate f_s of 44100 Hz.

- (a) Design FIR filters *with the minimum filter order* to meet the specification by using the Equiripple, Least Squares, and Kaiser Window design methods. FIR equiripple design is also known as Parks-McClellan, Remez Exchange and Chebyshev Design. Please show the specifications that you used for the passband frequencies and ripples, the stopband frequencies and attenuation, and any other

- specifications you provided to the design software (e.g. filter order or weights in the various bands). Please submit a plot of the magnitude and phase response for each filter design. Validate that each filter design meets the filter specifications. *Please see the hints below.* **27 points.**
- (b) Plot the impulse response of the FIR filter designed by the Equiripple (a.k.a. Parks-McClellan, Remez and Chebyshev) design algorithm. What symmetry is in the impulse response? **7 points.**
 - (c) Give the filter lengths required for filters designed for each filter design method. Which method gives the shortest filter length? **6 points.**
 - (d) Analyze the implementation complexity of each FIR filter design:
 - 1) How many multiplication operations are needed? **3 points**
 - 2) How much memory (in words) would it take to store the FIR coefficients and the circular buffer for the current and past inputs? **3 points**

In Matlab, I'd recommend using the filter design and analysis tool, *fdatool*. This tool is particularly useful when exploring different filter structures for implementation. If you type *help fdatool* in Matlab, then you will see how to retrieve the transfer function for the current filter being designed.

References

- [1] R. A. Levine and Y. Oron, "[Tinnitus](#)", *Handbook of Clinical Neurology*, vol. 129, pp. 409–431, 2015.
- [2] "[Tinnitus](#)".
- [3] H. Okamoto, H. Stracke, W. Stoll and C. Pantev, "[Listening to tailor-made notched music reduces tinnitus loudness and tinnitus-related auditory cortex activity](#)", *Proceedings US National Academy of Sciences*, vol. 17, no. 3, pp. 1207-1210, 2010.