DSP 01/18/2023. Wednesday (start 10:30 AM)

I. Announcements

⇒ go to assigned lab sections

⇒ HW 0 due this Friday.

⇒ if you join online lecture, please mute yourself.

II. Takeaway from last lecture on Sinusoidal Generation

$\boxed{\text{Bandwidth}}$ = non-zero extent in positive frequencies.

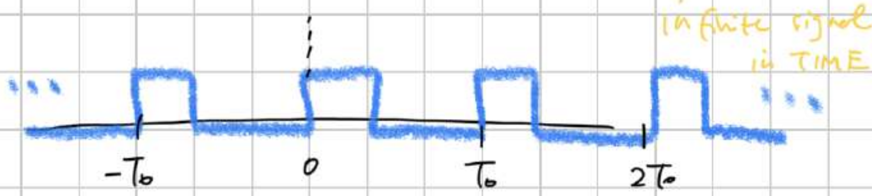
but! what if you have thermal noise? (covers all frequencies)

then how do you define bandwidth?

reminder: finite (signal) in time ⇒ infinite in frequencies

infinite (signal) in time ⇒ finite in frequencies

example: consider square/rect. impulse train...



we have fundamental period T_0

" frequency $f_0 = 1/T_0$

we can use Fourier Series to exactly represent the periodic signal using harmonic frequencies kf_0, $k \in \{\dots, -1, 0, 1, \dots\}$

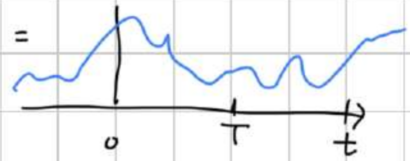
we have finite set of frequencies to represent infinite signal

... back to "how to find $\boxed{\text{Bandwidth}}$"?

we can observe the signal for a <u>finite time</u> and see what it looks like in frequency domain (since finite time $\Rightarrow$ infinite frequency)

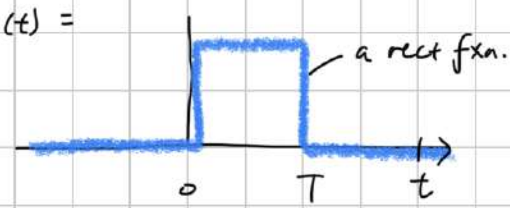
example:

$x(t) =$



signal we want to observe

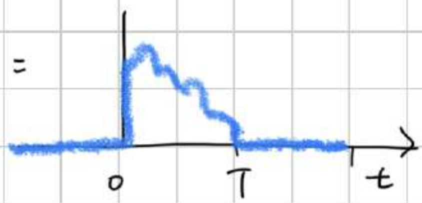
$p(t) =$



finite time to observe

$x_{observed}(t) = x(t) \cdot p(t)$

$=$



then in the freq. domain, $X_{observed}(f) = X(f) * \mathcal{F}\{p(t)\}$

* recall $\mathcal{F}\{\text{rect. function}\}$ = sinc fxn

covers all frequencies.

so we have an issue since we can't use the Bandwidth definition of "non-zero" since noise & finite time signals can ~~(crossed)~~ cover <u>all</u> frequencies

so instead, to determine Bandwidth, we can just set thresholds and estimate Bandwidth

$\Rightarrow$ threshold, then estimate

why is Bandwidth important? $\Rightarrow$ it helps us to set the sampling rate (aka. sampling frequency).

moving on to sinusoidal generation, Amplitude Modulation (with cosine) (slide 1-6)

Modulation: changes parameter of signal.

consider $y_1(t) = x_1(t) \cdot \cos(\omega_c t)$ ⇒ this is sinusoidal amplitude modulation (AM)

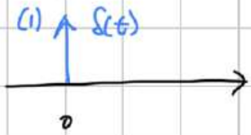
$y_1(t)$ ⇒ we take original signal $x(t)$, then shift its frequency up/down by the sinusoidal frequency, ω_c

⇓ take F.T.

$$Y_1(\omega) = \frac{1}{2\pi} X_1(\omega) * \left(\pi\delta(\omega+\omega_c) + \pi\delta(\omega-\omega_c)\right)$$

⇒ what happens if you convolve by delta? ⇒ Sifting Property.

recall the Dirac Delta ...



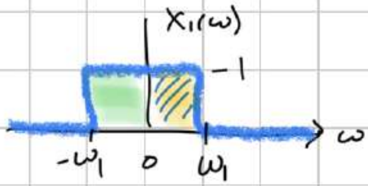
has area $A = \int_{-\infty}^{\infty} \delta(t)\, dt = 1$, amplitude $= \infty$ or undefined

$$Y_1(\omega) = \frac{1}{2\pi} \cdot \left(\pi X_1\underbrace{(\omega+\omega_c) + \pi X_1(\omega-\omega_c)}_{\text{sifting property}}\right)$$

because $\mathcal{F}\{\cos(\omega_c t)\} =$

$$Y_1(\omega) = \frac{1}{2}\left(X_1(\omega+\omega_c) + X_1(\omega-\omega_c)\right)$$

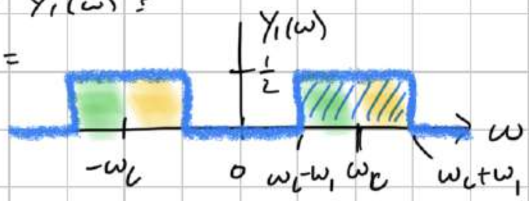
☆ due to sifting property, we shift center frequency of X_1 to ω_c (specified by the sinusoid) for example, let $X_1(\omega) =$



⇒ meaning, Bandwidth of $X_1(\omega) = \omega_1$.

What about $Y_1(\omega)$?

$Y_1(\omega) =$



⇒ so Bandwidth of $Y_1(\omega) = 2\omega_1$ (Bandwidth is 2x!!) increase

⇒ Bandwidth (BW) = expensive resource.

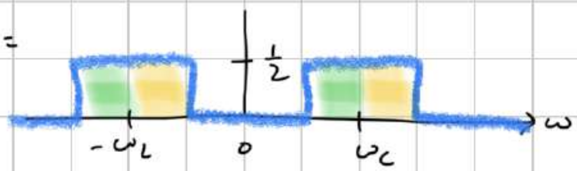
using 2x BW is expensive !! (it is also a waste, because we send the same info twice. could just send it 1x)

application of AM? you can Transmit (Tx) information more effectively. instead of transmit a low frequency, then modulate to higher frequency.

... now consider <u>Amplitude Demodulation</u> by cosine

⇒ goal: get back $x_1(t)$ from $y_1(t) = x_1(t)\cos(\omega_c t)$.

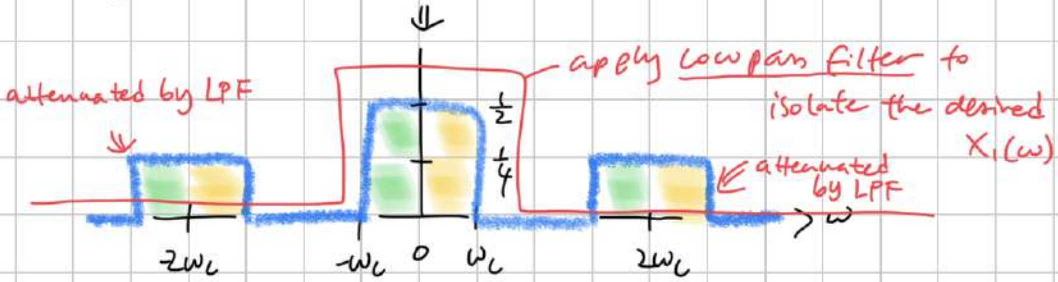
$Y_1(\omega) =$



let's just multiply by same cosine to demodulate!

$$\mathcal{F}\{\, y_1(t) \cdot \cos(\omega_c t) \,\} = \frac{1}{2} Y_1(\omega + \omega_c) + \frac{1}{2} Y_1(\omega - \omega_c)$$

$$\Downarrow$$



(reference lecture slides for more accurate representation)

slide 1-7

Amplitude Modulation by Sine

$$y_2(t) = x_2(t)\sin(\omega_c t)$$

$\Downarrow$ F.T.

$$Y_2(\omega) = \frac{j}{2} \cdot X_2(\omega + \omega_c) - \frac{j}{2} X_2(\omega - \omega_c)$$

Similar to using cosine!! except it's all in imaginary domain!

(reference slides 1-8)

just like w/ AM by cosine, the Bandwidth doubles ($\frac{1}{2}$ is wasted!)

☆ modulate by cosine ⇒ real-valued signal $Y_1(\omega)$
modulate by sine ⇒ imaginary-valued signal $Y_2(\omega)$
modulate by cosine/sine ⇒ orthogonal!!

useful for QAM.

to get back $X_2(t)$ from $y_2(t)$, = demodulate by sine, (slide 1-9)
we do same thing as in demodulate by cosine!

$$\mathcal{F}\{y_2(t) \cdot \sin(\omega_c t)\} = \frac{j}{2} Y_2(\omega + \omega_c) - \frac{j}{2} Y_2(\omega - \omega_c)$$

then apply Low Pass Filter (LPF) to isolate the original $X_2(\omega)$

... demonstrations at 11:28 AM. (slide 1-10)
(recommended to lower headphone volume)

audio demonstrations of modulating to higher frequencies.
also have additional visual demonstrations of modulating.

and also demonstration of demodulation.

Q: any constraints on carrier frequency (the frequency of the sinusoid)

A: yes, want carrier frequency ω_c > bandwidth of original signal (i.e. ω_1 or ω_2)

A: also, want to ensure your f_c or ω_c satisfies Nyquist Theorem for the sampling frequency f_s.

5:41 PM Wed Jan 18

All on My iPad

How to Use Bandwidth Efficiently? (slide 1-15)

recall modulation by sine, cosine ⇒ orthogonal !!

so we can modulate 1 signal by cosine
another signal by sine ⇒ using same carrier frequency
f_c or ω_c.

⇒ essentially, you send 2x information of same ~~...~~ carrier frequency.
(primary benefit)

⇒ this is called QAM (quadrature amplitude modulation).
discussed later in the course.